12.333 Problem Set 3: Vorticity and Rossby waves

(Due: April 14)

1. Consider free barotropic Rossby waves on a “beta-plane” centered on latitude 45°N, with a uniform mean flow $U = 20\text{ms}^{-1}$. Calculate the phase speeds of Rossby waves with zonal wavenumbers corresponding to waves 1, 2, 3, 4, 5 around the world at this latitude. [In each case, assume the latitudinal component of wavenumber to be $l = 5000\text{km}$].

2. Consider steady inviscid flow of an incompressible fluid of uniform density, on a rotating plane (of uniform Coriolis parameter $f > 0$), over a circular mountain of height $d(x, y)$, with shape given by

$$d(x, y) = d_0 \exp \left( -\frac{1}{2} \frac{r^2}{R^2} \right)$$

where $r = \sqrt{x^2 + y^2}$. (We have chosen to the mountain peak to be at the origin in $(x, y)$ space.) The top surface of the fluid is constrained by a rigid horizontal lid such that the depth, far from the mountain, has the uniform value $D > d_0$. Well upstream of the mountain, the flow is purely in the $x-$direction, and uniform (i.e., $(u, v) = (U, 0)$ far upstream, where $U$ is constant), as shown in the figure.

Recall that, for barotropic flow in a fluid of varying depth $H$, $d\Pi/dt = 0$ where $\Pi = \zeta_a/H$ is the potential vorticity and $\zeta_a$ is the absolute vorticity. Assuming the flow goes over the mountain:

(a) Determine the distribution of relative vorticity;

(b) Calculate the circulation around a circular contour of radius $r$ centered on the origin (i.e., the center of the mountain) [note that the perturbation flow (that induced by the mountain) is azimuthally symmetric around the mountain]. Sketch the total flow (including the background $U$).

3. Consider stationary waves forced by a mountain on a midlatitude $\beta$-plane centered on $45^\circ$N, in the presence of a uniform westerly wind of $20\text{ms}^{-1}$. Calculate (i) the
magnitude of the total wavenumber of the wavetrain produced by the mountain, and (ii) the maximum value of eastward group velocity (and the zonal and meridional components of wavenumber corresponding to that maximum).