12.333 Problem Set 4: Synoptic systems, deformation and fronts

(3 questions, 2 pages.) Due Tuesday April 28.

1. A series of baroclinic eddies has, at the Earth’s surface, a pressure perturbation

\[ p' = P_0 \sin \left[ k \left( x - ct \right) \right] \sin ly \]

where \( P_0 = 20\text{hPa}, \) and \( k = 2\pi/L \) and \( l = \pi/L, \) where \( L = 1000\text{km}. \) Determine the maximum northward geostrophic flow at 45° latitude. (Density of air at STP = 1.293kg m⁻³.)

2. Consider a baroclinic wave on an \( f \)-plane (i.e., \( f \) is constant); its geopotential height fluctuation on pressure surfaces is

\[ z'(x, p, t) = Z_0(t) \cos \left( x - \Lambda p - ct \right), \]

where \( \Lambda \) is constant. Thus, at a given time, \( z' \) is constant along sloping surfaces (of constant phase) on which \( x - \Lambda p \) is constant. Using the geostrophic relationship to get \( v' \) from \( z' \), and the hydrostatic relationship to get \( T' \) from \( z' \), show that

\[ v'T' = \frac{g^2 \rho}{f R} \Lambda \left( \frac{\partial z'}{\partial x} \right)^2. \]

[Hint: use the fact that for any function \( F(x - Ap - Bt), \) where \( A \) and \( B \) are constant, \( \partial F/\partial t = -A \partial F/\partial x. \) Hence, for a growing disturbance, deduce that the surfaces of constant phase must slope westward with height—in both the northern and southern hemispheres—if the basic state temperature decreases poleward.

3. Consider the steady two-dimensional, nondivergent, irrotational, deformation flow with streamfunction \( \psi = -Kxy. \)
Consider a rectangle, of dimensions $\delta x \times \delta y$, centered on the origin. The walls of this rectangle are simply advected by the flow. Show that the rectangle remains rectangular, but stretches exponentially in the $x$-direction and collapses exponentially in the $y$-direction, as $\exp (Kt)$ and $\exp (-Kt)$, respectively, and thereby preserves its area.