Plan view of horizontal velocity and height perturbations associated with an equatorial Rossby-gravity wave. (Adapted from Matsuno, 1966.)

Plan view of horizontal velocity and height perturbations associated with an equatorial Kelvin wave. (Adapted from Matsuno, 1966.)

<table>
<thead>
<tr>
<th>$n$</th>
<th>$h_n$ (km)</th>
<th>$c_n$ (m/s)</th>
<th>$L_{E,n}$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8.471</td>
<td>288.3</td>
<td>3620</td>
</tr>
<tr>
<td>1</td>
<td>0.316</td>
<td>55.7</td>
<td>1590</td>
</tr>
<tr>
<td>2</td>
<td>0.085</td>
<td>28.8</td>
<td>1140</td>
</tr>
</tbody>
</table>
Fig. 4.25. Time-height sections for the equatorial lower stratosphere, showing evidence of Kelvin-wave activity. (a) Zonal wind and (b) temperature at Canton Island (3°S). Note the westerly phase of the QBO encroaching from upper levels in (a): see Chapter 8. [From Gia
Fig. 4.2. Contours of $[u]$ and $[\theta]$ for (a) DJF; and (b) JJA, based on six ye:
ECMWF data. Contour interval for $[u]$ as in Fig. 4.1. Contour interval for $\theta$ i

Fig. 4.1. The zonal mean wind $[u]$ and vectors of the meridional wind for (a)
December–January–February (DJF); (b) June–July–August (JJA).
Fig. 6.1.3. Circulation over the global tropics in January. Resultant wind streamlines and isotachs in knots. (a) 200 mb, and (b) gradient wind level. Sources: Atkinson and Sadler (1970), Atkinson (1971), Sadler (1975b).
Fig. 6.1: Circulation over the global tropics in July. Resultant wind streamlines and isotachs in knots. (a) 200 mb, and (b) gradient wind level. Sources: Atkinson and Sadler (1970), Atkinson (1971), Sadler (1975b).
Figure 1.6. The annual mean rainfall of Taylor (1973) and the first harmonic of the annual cycle in rainfall. The contour interval for the annual mean is 2 m. The amplitudes of the first harmonic have been divided by the annual mean rainfall. The normalized amplitudes are denoted by the lengths of the arrows according to the scale in the figure. A plus sign is plotted at stations where the amplitudes are less than one-tenth of the annual mean. The phase is indicated by the direction of the arrow. An arrow that points downwards means maximum rainfall on 1 January; a horizontal arrow pointing left means maximum rainfall on 1 April. [From Horel (1982).]
Walker circulation
Walker circulation
Fig. 11.13  Plan view of horizontal velocity and height perturbations associated with an equatorial Rossby-gravity wave. (Adapted from Matsuno, 1966.)

Fig. 11.15  Plan view of horizontal velocity and height perturbations associated with an equatorial Kelvin wave. (Adapted from Matsuno, 1966.)

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UNFORCED

\[
\frac{\partial u'}{\partial t} - \beta y v' = -g \frac{\partial h'}{\partial x} \\
\frac{\partial v'}{\partial t} + \beta y u' = -g \frac{\partial h'}{\partial y} \\
\frac{\partial h'}{\partial t} + D \left( \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = 0
\]

FORCED

\[
\frac{\partial u'}{\partial t} - \beta y v' = -g \frac{\partial h'}{\partial x} \\
\frac{\partial v'}{\partial t} + \beta y u' = -g \frac{\partial h'}{\partial y} \\
\frac{\partial h'}{\partial t} + D \left( \frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} \right) = -DQ
\]

FORCED DIMENSIONLESS

\[
\frac{\partial u}{\partial t} - \frac{1}{2} y v = -\frac{\partial h}{\partial x} \\
\frac{\partial v}{\partial t} + \frac{1}{2} y u = -\frac{\partial h}{\partial y} \\
\frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q
\]
FORCED DIMENSIONLESS

\[ \frac{\partial u}{\partial t} - \frac{1}{2}yv = -\frac{\partial h}{\partial x} \]
\[ \frac{\partial v}{\partial t} + \frac{1}{2}yu = -\frac{\partial h}{\partial y} \]
\[ \frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q \]

Add damping (friction+thermal)

\[ \frac{\partial u}{\partial t} - \frac{1}{2}yv = -\frac{\partial h}{\partial x} - \epsilon u \]
\[ \frac{\partial v}{\partial t} + \frac{1}{2}yu = -\frac{\partial h}{\partial y} - \epsilon v \]
\[ \frac{\partial h}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q - \epsilon h \]

Steady, damped problem

\[ \epsilon u - \frac{1}{2}yv = -\frac{\partial h}{\partial x} \]
\[ \epsilon v + \frac{1}{2}yu = -\frac{\partial h}{\partial y} \]
\[ \epsilon h + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q \]

Long wave approx \((L_x \ll L)\)

\[ \epsilon u - \frac{1}{2}yv = -\frac{\partial h}{\partial x} \]
\[ \frac{1}{2}yu = -\frac{\partial h}{\partial y} \]
\[ \epsilon h + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -Q \]
\[
\begin{align*}
\varepsilon u - \frac{1}{2} y v v &= -\frac{\partial h}{\partial x} \\
\frac{1}{2} y u &= -\frac{\partial h}{\partial y} \\
\varepsilon h + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= -Q
\end{align*}
\]

\[
q = h + u \\
r = h - u
\]

\[
\begin{align*}
\varepsilon q + \frac{\partial q}{\partial x} + \frac{\partial v}{\partial y} - \frac{1}{2} y v v &= -Q \\
\varepsilon r - \frac{\partial r}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{2} y v v &= -Q \\
\frac{\partial q}{\partial y} + \frac{1}{2} y q + \frac{\partial r}{\partial y} - \frac{1}{2} y r &= 0
\end{align*}
\]

\[
q(x,y) = \sum_{n=0}^{\infty} q_n(x) D_n(y)
\]

\[
D_n(y) = H e_n(y) \exp\left(-\frac{1}{4} y^2\right)
\]

\[
= 2^{-n/2} H_n(\xi) \exp\left(-\frac{1}{2} \xi^2\right) \quad (\xi = \sqrt{2} y)
\]

\[
\begin{pmatrix}
D_0(y) \\
D_1(y) \\
D_2(y)
\end{pmatrix} = \begin{pmatrix}
1 \\
y \\
y^2 - 1
\end{pmatrix} \exp\left(-\frac{1}{4} y^2\right)
\]

\[
\int_{-\infty}^{\infty} D_n(y) D_m(y) \, dy = n! \sqrt{2\pi} \, \delta_{nm}
\]

\[
\begin{align*}
\frac{dD_n}{dy} + \frac{1}{2} y D_n &= n D_{n-1} \\
\frac{dD_n}{dy} - \frac{1}{2} y D_n &= -D_{n+1}
\end{align*}
\]

\[
D_{n+1} - y D_n + n D_{n-1} = 0
\]
Symmetric forcing

\[ Q(x,y) = F(x) \exp \left(-\frac{1}{4}y^2 \right) \]

\[ F(x) = \begin{cases} 
\cos \frac{\pi x}{2L} , & |x| < L \\
0 , & |x| > L 
\end{cases} \]

\[ q(x,y) = \sum_{n=0}^{\infty} q_n(x) D_n(y) \]

\[ \begin{align*}
\varepsilon q_n + \frac{dq_n}{dx} + \frac{dv}{dy} - \frac{1}{2} yv &= -Q_n , \quad n \geq 1 \\
\varepsilon q_0 + \frac{dq_0}{dx} &= -Q_0 \\
\varepsilon r_n - \frac{dr_n}{dx} + (n+1)v_{n+1} &= -Q_n \\
(n+1)q_{n+1} - r_{n-1} &= 0 , \quad n \geq 1 \\
q_1 &= 0 \\
Q_0 &= F(x) \\
Q_n &= 0 , \quad n > 0
\end{align*} \]
\[
\begin{align*}
\{ & \varepsilon q_n + \frac{dq_n}{dx} - v_{n-1} = -Q_n, \quad n \geq 1 \\
& \varepsilon q_0 + \frac{dq_0}{dx} = -Q_0 \\
& \varepsilon r_n - \frac{dr_n}{dx} + (n+1)v_{n+1} = -Q_n \\
& (n+1)q_{n+1} - r_{n-1} = 0, \quad n \geq 1 \\
& q_1 = 0 
\end{align*}
\]

n=0 solution: \[
\varepsilon q_0 + \frac{dq_0}{dx} = -Q_0
\]

\[
q_0(x) = \begin{cases} 
0 , & x < -L \\
-(\varepsilon^2 + k^2)^{-1}[\varepsilon \cos kx + k(\sin kx + e^{-\varepsilon(x+L)})] , & -L < x < L \\
-k(\varepsilon^2 + k^2)^{-1}(1 + e^{-2\varepsilon L})e^{\varepsilon(L-x)} , & x > L 
\end{cases}
\]

No corresponding \(v,r\):

\[
u = 0 \quad \text{and} \quad \nu = 0
\]

Kelvin wave component:

\[
e^{-ex/c_g} \quad [c_g = +1]
\]

\[
\begin{align*}
u(x, y) &= \frac{1}{2} q(x, y) = \frac{1}{2} q_0(x) \exp\left(-\frac{1}{4}y^2\right) \\
u(x) &= 0
\end{align*}
\]
\[
\begin{align*}
\varepsilon q_n + \frac{dq_n}{dx} - v_{n-1} &= -Q_n, \quad n \geq 1 \\
\varepsilon q_0 + \frac{dq_0}{dx} &= -Q_0 \\
\varepsilon r_n - \frac{dr_n}{dx} + (n + 1)v_{n+1} &= -Q_n \\
(n + 1)q_{n+1} - r_{n-1} &= 0, \quad n \geq 1 \\
q_1 &= 0
\end{align*}
\]

\[\text{n=1 Rossby wave}\]

\[
\begin{align*}
(q_2, v_1, r_0) \\
\varepsilon q_2 + \frac{dq_2}{dx} - v_1 &= 0 \\
\varepsilon r_0 - \frac{dr_0}{dx} + v_1 &= -F(x) \\
2q_2 &= r_0 \\
\rightarrow \frac{dq_2}{dx} - 3\varepsilon q_2 &= F(x)
\end{align*}
\]

\[
q_2 = \begin{cases} 
-(9\varepsilon^2 + k^2)^{-1} k (1 + e^{-6\varepsilon L}) e^{3\varepsilon(x+L)} , & x < -L \\
-(9\varepsilon^2 + k^2)^{-1} [3\varepsilon \cos kx - k (\sin kx - e^{3\varepsilon(x-L)})] , & -L < x < L \\
0 & , \quad x > L
\end{cases}
\]

\[
v_1 = \varepsilon q_2 + \frac{dq_2}{dx}
\]

\[
r_0 = 2q_2
\]
Kelvin:

\[ u = h = \frac{1}{2} q_0(x) D_0(y) \]
\[ v = 0 \]

Rossby:

\[ h = \frac{1}{2} (q_2(x) D_2(y) + r_0(x) D_0(y)) \]
\[ u = \frac{1}{2} (q_2(x) D_2(y) - r_0(x) D_0(y)) \]
\[ v = v_1(x) D_1(y) \]
Kelvin:

\[ u = h = \frac{1}{2} q_0(x) D_0(y) \]
\[ v = 0 \]

Rossby:

\[ h = \frac{1}{2} (q_2(x) D_2(y) + r_0(x) D_0(y)) \]
\[ u = \frac{1}{2} (q_2(x) D_2(y) - r_0(x) D_0(y)) \]
\[ v = v_1(x) D_1(y) \]
Figure 1. Solution for heating symmetric about the equator in the region $|x| < 2$ for decay factor $z = 0.1$.

(a) Contours of vertical velocity $w$ (solid contours are 0, 0.1, 0.6, broken contour is -0.1) superimposed on the velocity field for the lower layer. The field is dominated by the upward motion in the heating region where it has approximately the same shape as the heating function. Elsewhere there is subsidence with the same pattern as the pressure field.

(b) Contours of perturbation pressure $p$ (contour interval 0.3) which is everywhere negative. There is a trough at the equator in the easterly régime to the east of the forcing region. On the other hand, the pressure in the westerlies to the west of the forcing region, though depressed, is high relative to its value off the equator. Two cyclones are found on the north-west and south-west flanks of the forcing region.

(c) The meridionally integrated flow showing (i) stream function contours, and (ii) perturbation pressure. Note the rising motion in the heating region (where there is a trough) and subsidence elsewhere. The circulation in the right-hand (Walker) cell is five times that in each of the Hadley cells shown in (c).
Figure 2. Solution for heating antisymmetric about the equator in the region $|x| < 2$ for decay factor $\delta = 0.1$.
(a) Contours of vertical velocity $w$ (contour interval 0.3) superimposed on the velocity field for the lower layer. The field is dominated by the motion in the heating region where it is approximately the same shape as the heating function (positive in northern hemisphere). Outside the forcing region, the pattern is the same as for the pressure field with subsidence in the northern hemisphere and weak upward motion in the southern hemisphere. There is no motion for $x > 2$.
(b) Contours of perturbation pressure $p$ (contour interval 0.3) which is positive in the north (where there is an anticyclone) and negative south of the equator (where there is a cyclone). The flow has the expected sense of rotation around the pressure centres and flows down the pressure gradient where it crosses the equator. All fields are zero for $x > 2$. 
Figure 3. Solution obtained by adding the solutions shown in the two previous figures, corresponding to heating which is confined to the region $|x| < 2$ and is mainly concentrated to the north of the equator.
(a) Contours of vertical velocity $w$ (contour interval 0.3) showing the dominance of the heating north of the equator. The flow to the east of the forcing region is the same as in Fig. 1, this being provided entirely by the symmetric part of the heating. West of the forcing, the westerly inflow is concentrated between the equator and $y = 2$. An easterly flow is found south of the equator.
(b) Contours of perturbation pressure $p$ (contour interval 0.3). The pattern is dominated by a low on the western flank of the heating region and by the equatorial trough. A high is found in the southern hemisphere.