Absolute Barotropic Instability and Monsoon Depressions

R. S. LINDZEN AND B. FARRELL

Center for Earth and Planetary Physics, Harvard University, Cambridge, MA 02138

A. J. ROSENTHAL

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139

(Manuscript received 5 October 1982, in final form 24 January 1983)

ABSTRACT

We have approached the barotropic instability of mean zonal flows over the Bay of Bengal for the months of June, July and August from the perspective of pulse asymptotics rather than most rapidly growing normal modes. The results are in good agreement with observations of monsoon depressions.

1. Introduction

The origin of monsoon depressions over the Bay of Bengal has been discussed in numerous papers (Shukla, 1977, 1978; Krishnamurti *et al.*, 1977; Sikka *et al.*, 1977; Goswami *et al.*, 1981; Pearce, 1981). These depressions develop almost entirely in July and August. Some suggestion exists that these disturbances originate in Southeast Asia and propagate westward (Saha *et al.*, 1981). We shall remark on this later, but such identifications are uncertain. What is clear is that regardless of the origin of the perturbation, strong development takes place over the Bay of Bengal. Three obvious candidates for this development are barotropic instability, baroclinic instability, and wave CISK. The last two can be plausibly dismissed. Stevens and Lindzen (1978) have demonstrated that cumulus momentum exchange effectively eliminates vertical wave propagation and wave CISK—at least in the presence of substantial precipitation—except for short period disturbances such as squalls. The situation with respect to baroclinic instability is as follows: Goswami *et al.* (1981) have found that for winds typical of the Bay of Bengal region, the inclusion of baroclinicity barely modifies barotropic results. In addition, Lindzen *et al.* (1980) have shown that baroclinic instability consists in the interaction of vertically propagating Rossby waves with the mean flow. However, as already mentioned, Stevens and Lindzen (1978) have shown that such propagation is eliminated by cumulus momentum exchange in the presence of cumulus precipitation.

This leaves us with only barotropic instability as a mechanism for the development of the wave disturbances associated with monsoon depressions. Barotropic instability which involves horizontal propagation is not so strongly affected by vertical exchange processes. However, the normal mode instabilities calculated for this mechanism also have difficulties. Calculations by Krishnamurti (1972), Shukla (1977) and others show that the most rapidly growing instabilities are associated with 200 mb winds. These instabilities do not penetrate the lower troposphere sufficiently to organize convection; they travel to the west far more rapidly than do the observed depressions, their scales are much too large and their growth rates show no peak in July. Many of these difficulties were noted by Pearce (1981). These discrepancies led Shukla (1977) to suggest that barotropic instability, alone, was unable to explain the depressions. The purpose of the present paper is to show that barotropic instability is able to explain the wave aspects of monsoon depressions, but that normal mode analysis is inadequate.

It is obvious that we never have initial perturbations in the form of normal modes. A more plausible initial perturbation would consist of a regionally confined pulse—presumably in the Bay of Bengal. We shall discuss the initiating perturbation later in this paper. The usual assumption is that such a pulse can be synthesized with an infinite number of normal modes and that normal mode instabilities, therefore, remain relevant. In an inviscid, axially symmetric flow the most unstable normal mode may in fact emerge clearly as it travels around the earth. However, this process would take a long time during which the assumptions of linear stability theory would break down. Moreover, the fluid is not inviscid and the basic state is not axially symmetric. More realistically a normal mode may be perceived in a WKB sense downstream of the original perturbation. However, if it is traveling sufficiently fast it will not be seen at any point as a steadily growing perturbation because it will have left that point before the growth is perceived. Clearly, in the neighborhood of the original perturbation, the disturbance field will be dominated not, in general by the most rapidly growing normal mode, but by the mode whose group velocity is sufficiently small so that the rate at which it grows is faster than the rate at which it is "moving away." Such modes do not always exist.

The above features emerge automatically from a study which considers the evolution of the initial pulse as an initial value problem. Such an approach might use Fourier transforms in space and Laplace transforms in time. However, the exact inversion of the transforms would be virtually impossible. It is, however, possible to use asymptotic methods (*e.g.*, Carrier *et al.*, 1966) to advantage. Such methods give valid results for times greater than the time needed for establishing modes in the meridional direction. For very short times, the continuous spectrum may contribute (Farrell, 1982b). The application of the asymptotic approach to plasma instabilities is developed by Briggs (1964) who usefully introduces the concept of absolute versus convective instability. For the latter, a growing disturbance travels away from the region of initial perturbation so fast that after a finite time only a decaying disturbance is found in this region; for the former an original perturbation gives rise to a growing disturbance in the originally perturbed region. Gaster and Davey (1968) introduced this approach to ordinary hydrodynamic shear instabilities, while Merkine (1977) and Merkine and Shafranik (1980) applied the approach to baroclinic instability in a two layer model. Farrell (1982a,c) approached the Charney (1947) problem in this way and it is this last approach which will be followed here. Details are given in Farrell's papers.

The approach is greatly facilitated if one has analytic expressions for normal mode instabilities relating complex frequency to complex horizontal wavenumber. The development of such relations for the Charney problem by Lindzen and Rosenthal (1981) made Farrell's (1982a,c) studies possible. In an accompanying paper (Lindzen *et al.*, 1983), we show that a wide range of barotropic profiles can be approximated by broken line profiles which are mathematically equivalent to the Charney problem, thus facilitating the use of pulse asymptotics here as well. It is shown in Lindzen *et al.* that the limitations of the broken line profile are not severe.

In Section 2 of this paper we display monthly mean zonal winds from the Indian Ocean Expedition (Ramage and Raman, 1972) for June–August. Capitalizing on earlier results (Goswami *et al.*, 1981) which show:

1) For zonal winds over the Indian region the dominant instability is barotropic instability.

2) The characteristics of the instability are determined by the barotropic instability of the flow at a given level (the depth through which this flow extends determines the depth of the instability).

We shall concentrate on the mean flows at 200 mb (which are typical of the layer 250–100 mb) and at 500 mb (which are typical of the lower troposphere).[1] In all cases, it is easy to approximate the profiles with broken line profiles for which the normal mode instabilities are already described by Lindzen *et al.* (1982).

In Section 3 we investigate the pulse aymptotics for the relevant basic states. For the June profiles the instabilities associated with 200 mb dominate. However, for July and August the pulse asymptotics in the neighborhood of India are dominated by the instability of the lower troposphere. Dominant horizontal wavelengths are ~2600 km and phase speeds are westward ~ 1.5 m s^{-1} much as observed (Fig. 6).

This paper focuses on the wave dynamics of monsoon depressions without a detailed discussion of the most important meteorological manifestation: namely, rain. Earlier in this section, we dismissed the role of latent heat in the wave instability. However, in Section 4 we will discuss a possible role for latent heat as the source of the initial perturbation.

2. Observed zonal winds

The zonal winds, whose instability we will examine, are taken from the Atlas of the Indian Ocean Expedition (Ramage and Raman, 1972). Meridional sections of wind for each month are given for longitudes 30, 73, 100 and 140°E. The middle two bound the Bay of Bengal. Since winds seem to change smoothly from section to section, we averaged zonal winds from sections 73 and 100°E to obtain profiles representative of the Bay of Bengal. In Fig. 1, we show the variation of zonal wind with latitude at 200 mb for June, July and August. In Fig. 2, the winds at 500 mb are shown.

In terms of Lindzen *et al.* (1982), all profiles of Fig. 1 are easterly "semi-jets." While there are modest shifts in the 200 mb profiles from month to month, all of the profiles are very similar and can be approximated by the broken line profile in Fig. 3a. The variations at 500 mb are more significant. The profile for June is again an easterly semi-jet approximated by the broken line profile shown in Fig. 3b. The profile for July is essentially a symmetric easterly jet approximated by the broken line profile in Fig. 3c. The profile for August is similar to that for July, although there is a tendency for the profile to return to the semi-jet configuration. Nevertheless, our stability cal-

[1] More accurately, the winds at 500 mb represent the amount of barotropic instability available to the lower troposphere.

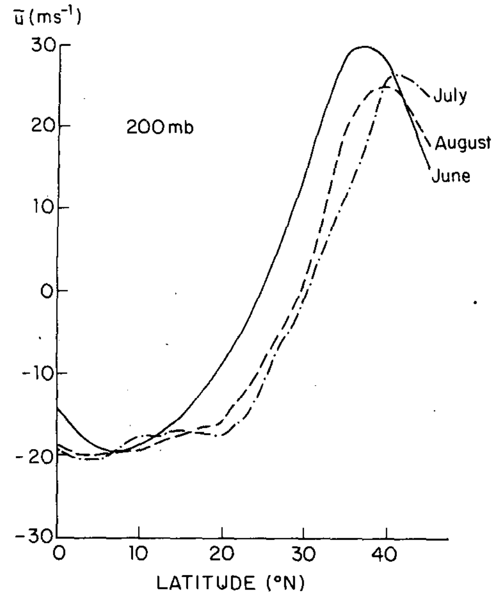

FIG. 1. Zonal wind at 200 mb for the months of June, July and August. Values are averages of the 73 and 100°E sections and are representative of the Bay of Bengal.

culations show that the August profile behaves more nearly like a full jet (*i.e.*, there are steering latitudes on both sides of the jet vertex). Profiles for September (not shown) are more nearly of the semi-jet type.

As noted by Lindzen *et al.* (1982), the shift from a semi-jet to a symmetric jet leads to almost doubling of maximum unstable growth rates. As a result, the normal mode growth rates associated with the profile in Fig. 3c are more than half those associated with the profile in Fig. 3a despite the fact that shears in

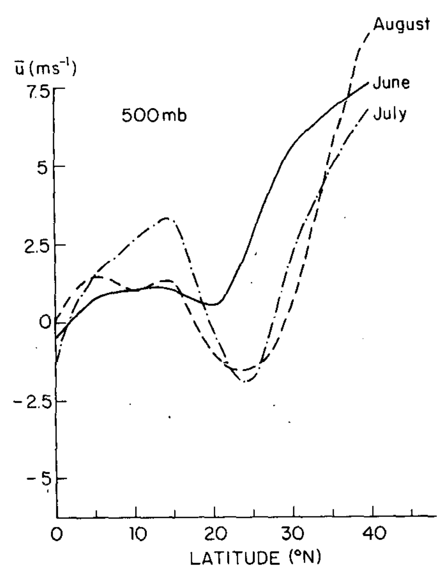

FIG. 2. Zonal wind at 500 mb for the months of June, July and August at the longitude of the Bay of Bengal.

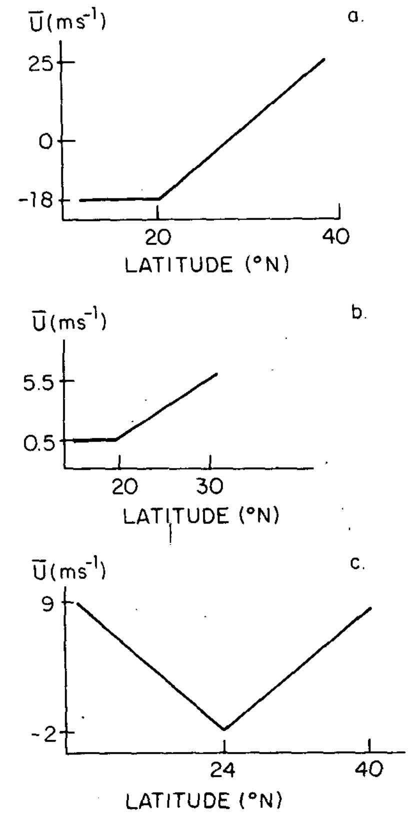

FIG. 3. Broken line profiles approximating observed zonal winds at the longitude of the Bay of Bengal: (a) 200 mb for the months of July and August; (b) 500 mb for the month of July; (c) 500 mb for June.

Fig. 3c (and at 500 mb) are only about ¼ those in Fig. 3a (and at 200 mb). Properties of the most unstable modes on profiles in Fig. 3a, b and c, taken directly from Lindzen *et al.* (1982) are shown in Table 1.[2] Clearly the 200 mb disturbances grow fastest. However, as noted in Section 1, the most unstable disturbance has a much larger wavelength and travels to the west much faster than observed monsoon depressions. There can be little doubt that these tropospheric disturbances are unrelated to the depressions.

---

[2] Referring to Fig. 3, we may write $\bar{u} = u_0 + m(y - y_0)$ for $y > y_0$ (y_0 = position of easterly jet vertex). The stability properties are determined by Lindzen *et al.* (1982) in terms of non-dimensional wavenumbers α and phase speeds $\tilde{c}$. Dimensional quantities are obtained by means of the relations

$$k = \frac{\beta}{m}\alpha, \quad (1)$$

$$c - u_0 = \frac{m^2}{\beta}\tilde{c}. \quad (2)$$

m is easily obtained from Fig. 3. $\beta = a^{-1}2\Omega\cos\phi$ (where $\Omega = 2\pi$ day^{-1}, $a = 6400$ km) $\approx 2.28 \times 10^{-11}$ m^{-1} s^{-1} for $\phi \approx 18°$.

TABLE 1. Properties of the most unstable normal mode for profiles shown in Figs. 3a, b and c.

Profile		a	b	c
Wavelength	[km]	6060	1430	2260
e-folding	[days]	3.8	16.1	6.5
$c_r + u_0$	[m s^{-1}]	−15.1	0.64	−1.35

The tropospheric instabilities for the June 500 mb flow grow too slowly (in all likelihood) to overcome existing dissipation. Moreover, their wavelength is too small and they travel eastward albeit slowly.

Interestingly, the most rapidly growing normal modes associated with the July/August 500 mb flow not only grow reasonably fast but also have horizontal wavelengths and phase speeds fairly close to those of observed monsoon depressions. This was already noted by Yanai and Nitta (1968). As we shall see in the next section, the slow travel speed of instabilities on the July 500 mb flow will cause them to dominate the response in the neighborhood of the Bay of Bengal.

In connection with travel speed for disturbances, the value of u_0 in Fig. 3 proves important. Small easterly values of u_0 are especially conducive to absolute instability.

3. Pulse asymptotics

The evaluation and interpretation of pulse asymptotics[3] is discussed at length by Farrell (1982a,c) and, for the most part, will not be repeated here. Briefly, for the special case where $u_0 = 0$, pulse asymptotics are dominated at any particular choice of x/t (where x is the eastward distance and t the time) by the complex values of ω and k for which

$$\frac{x}{t} = \frac{\partial \omega}{\partial k}, \tag{3}$$

where ω is frequency and k is zonal wavenumber. The solution of (3) leads to a specific value of k, k_s, and $\omega(k_s)$ at each x/t.[4] The quantity

$$\nu_i = \omega_i(k_s) - \mathrm{Im}(k_s)\frac{x}{t} \tag{4}$$

gives the local growth rate at any x/t. Here $\nu_i(x/t)$ may also be interpreted as the logarithm of the pulse amplitude envelope.

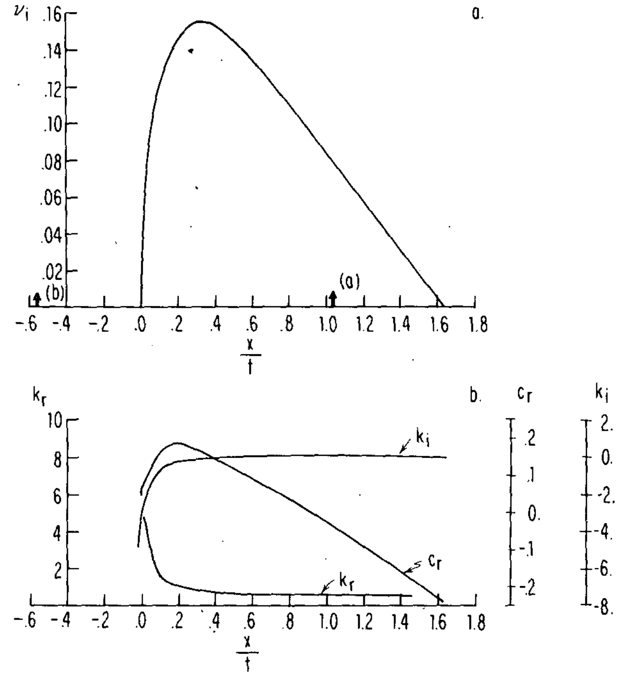

FIG. 4. Asymptotic pulse for the semi-jet, (a) contains the local growth rate ν_i as a function of x/t and (b) the rest frame phase speed c_r with k the complex wavenumber.

Figs. 4 and 5 show ν_i, $\mathrm{Im}(k_s) = k_i$, $c_r = \omega_r/k_r$ (real phase speed associated with k_s relative to u_0) and $\mathrm{Re}(k_s) = k_r$ as functions of x/t for the semi-jet and symmetric jet, respectively. The results differ mainly in magnitudes. The results in these figures are nondimensional. As noted in Section 2, lengths are scaled by $m\beta^{-1}$ and times are scaled by m^{-1}. Similarly, speeds are scaled by $m^2\beta^{-1}$.

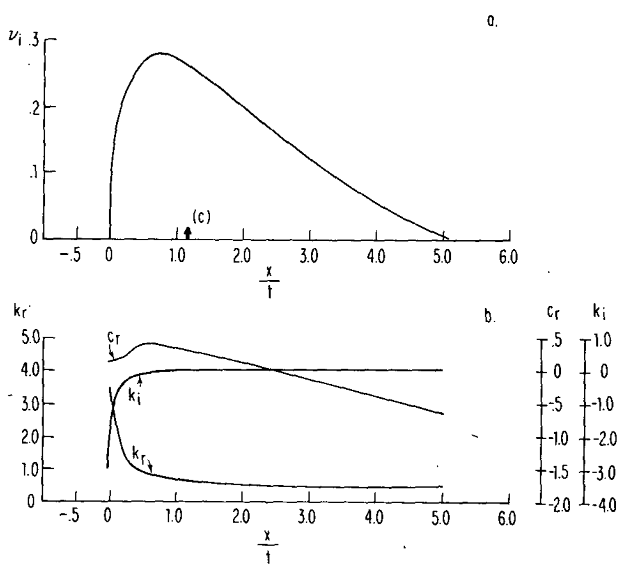

FIG. 5. Asymptotic pulse for the symmetric jet as in Fig. 4.

[3] Pulse asymptotics refers to the asymptotic response to initial perturbations in the form of regional pulses in contrast to normal modes.

[4] As noted by Lindzen *et al.* (1982), barotropic instability, like baroclinic instability, can have both Charney and Burger modes. Separate solutions to (3) exist for each of these modes. However, the solutions associated with Burger modes, for the present parameters, tend to have unrealistically large scales and/or small growth rates. We, therefore, only consider Charney modes in this paper.

TABLE 2. Values of parameters for profiles in Figs. 3a, b and c.

Profile		a	b	c
m	[s^{-1}]	19.7×10^{-6}	4.63×10^{-6}	6.37×10^{-6}
m^2/β	[$m\ s^{-1}$]	17.0	0.94	1.78
u_0	[$m\ s^{-1}$]	−18.0	0.51	−2.06
$-u_0(m^2/\beta)$	[$m\ s^{-1}$]	1.06	−0.55	1.16

TABLE 3. Properties of the absolute instabilities for profiles in Figs. 3a and c.

Profile		a	c
Wavelength	[km]	9290	2620
e-folding	[days]	7.5	6.8
$c_r + u_0$	[$m\ s^{-1}$]	−18.8	−1.5

In order to extend the above results to the case where $u_0 \neq 0$ we merely replace

$$\left(\frac{x}{t}\right)_{\text{non dim}} \quad \text{with} \quad \left(\frac{x}{t}\right)_{\text{non dim}} - \frac{\beta}{m^2} u_0, \tag{5a}$$

or replace

$$\left(\frac{x}{t}\right)_{\text{dim}} \quad \text{with} \quad \left(\frac{x}{t}\right)_{\text{dim}} - u_0. \tag{5b}$$

The values of m and m^2/β appropriate to the profiles in Figs. 3a, b and c are shown in Table 2. The relevant values of u_0 are shown in Fig. 3. On Figs. 4 and 5 we mark the displaced origins given by (5a). Note that the displaced origin for the June tropospheric basic state is to the left of the region of positive ν_i. This means that *there is no absolute instability for this profile.* Such a situation is typical when u_0 is more than slightly westerly (positive). The displaced origin for 200 mb flows is well to the right of the maximum of ν_i. This is typical of situations where u_0 is strong and easterly. The growth rates of upper tropospheric instabilities in the neighborhood of India will be much smaller than the maximum growth rates possible in the upper troposphere. The largest growth rates are associated with disturbances which rapidly travel away to the west of India and do not affect the Indian region. Finally, note that the displaced origin for the July lower troposphere profile almost coincides with the maximum in ν_i. This can occur when u_0 is easterly and relatively small.

Various properties of the above absolute instabilities are given in Table 3. We see that in the Indian region the July lower tropospheric growth rate is, in fact, larger than that associated with 200 mb winds. The dominant horizontal wavelength ~2600 km is very nearly what is observed, and the phase speed $\sim -1.5\ m\ s^{-1}$ is sufficiently slow to be consistent with the notion of regional development.[5] Note that the existence of a finite westward phase speed is consistent with the disturbance having almost zero group velocity. The fact that disturbances with zero group velocity and relatively small wavelengths exist is itself important. Such modes do not "see" regions to the east or west and hence do not depend on the assumption that the mean zonal flow is uniform in x. The existence of such modes thus makes the local instability of a barotropic flow meaningful.

Note, finally, that Fig. 5 shows a large perturbed region to the east of the Bay of Bengal. The group velocity of these disturbances is eastward even though their phase velocity is westward. Fig. 6 from Saha *et al.* (1981), shows phase paths of monsoon depressions showing propagation of phase from southeast Asia.[6] The present work shows that this is consistent with an origin in the Bay of Bengal.

4. Concluding remarks

In this brief paper we attempt to make use of recent results which relate barotropic instability to baroclinic instability (Lindzen *et al.*, 1982), which provide analytic solutions for such stability problems (Lindzen and Rosenthal, 1981), and which use these results to calculate the asymptotic response to local perturbations (*i.e.*, pulse asymptotics; Farrell, 1982a,c). The results demonstrate that the local barotropically unstable response to regional perturbations in the Bay of Bengal during July and August will be dominated by the lower troposphere; the disturbance will have a realistic scale (wavelength ≈ 2600 km) and will be slowly moving. This contrasts with normal mode results which imply a dominance of the upper troposphere and disturbances of very large scale (wavelength ≈ 6000 km) traveling rapidly westward (~15 $m\ s^{-1}$).

Moreover, the analysis clearly identifies those features of the mean flow which lead to monsoon depressions in July. These are:

1) The development of an easterly jet as opposed to semi-jet structure in the mean flow.

2) The development of a modest easterly flow at the jet center as opposed to westerly flow.

In view of the discussion in the introduction which argued that both baroclinic instability and wave CISK were incapable of producing the wave instabilities associated with monsoon depressions, it is reassuring

---

[5] It may be of interest that the associated period, $2\pi/(\alpha_r c_r)$, is O(20 days), which Sikka (1977) cites as a characteristic period for monsoon depressions.

[6] We assume that tracks shown in Saha *et al.* (1981) represent phase paths—rather than packet paths. Their data was used to select depressions at each station which yielded phase continuity with other stations. The depressions were sometimes weak and difficult to identify.

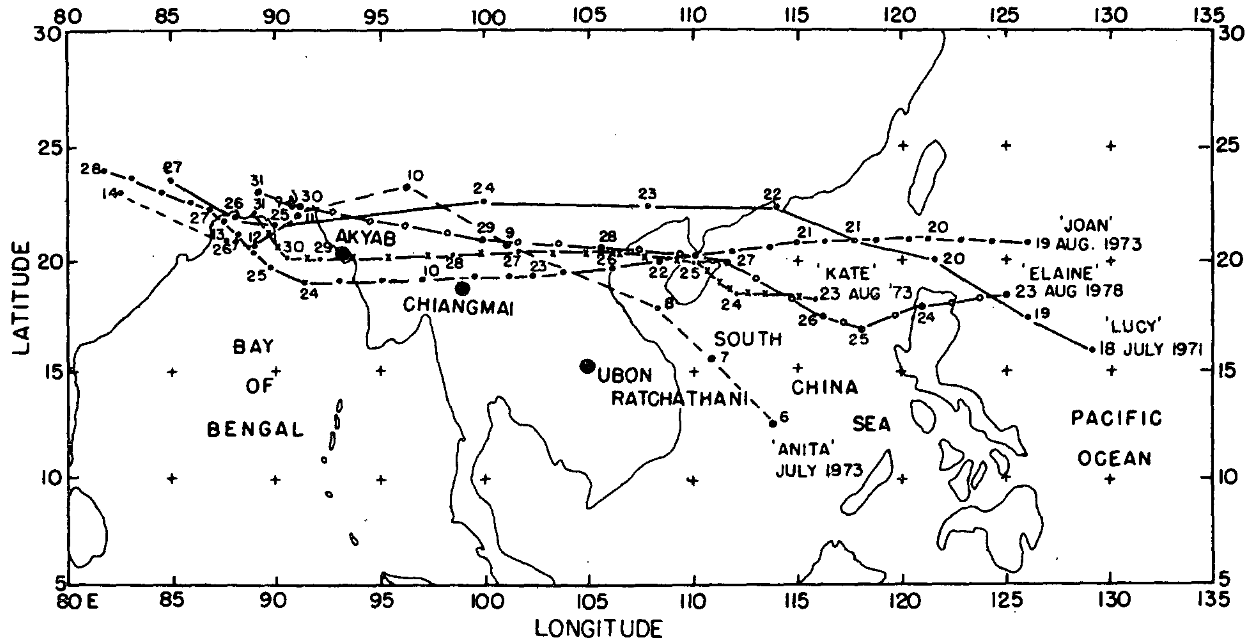

FIG. 6. Tracks of five tropical storms which appeared to have direct associations with lows and depressions in the Bay of Bengal. After Saha *et al.*, 1981.

to see that barotropic instability is, after all, consistent with observations.

The analysis of this paper suggests, moreover, that disturbances to the east of the Bay of Bengal whose phase is propagating rapidly to the west may nonetheless have their origin in the Bay of Bengal.

However, the present results are by no means complete. Two remaining questions are particularly serious:

1) Why do initial perturbations occur in the Bay of Bengal?

2) Why do monsoon depressions appear to develop more rapidly than the *e*-folding times in Table 3 would suggest?

We believe the answer to both questions resides in the role of rainfall. As noted by Stevens and Lindzen (1978) and Lindzen (1981), waves do not cause rain in the tropics; they merely organize precipitation which would otherwise occur in response to evaporation (and is perhaps already redistributed by larger scale low-level convergence). Our tentative impression is that the warming of the Bay of Bengal leads to a concentration of precipitation in that region. The latent heat release in turn serves as the initiating perturbation for the barotropic instability. In this concept, the rainfall in the Bay of Bengal region *precedes* the development of the wave disturbance which subsequently reorganizes the distribution of rainfall. The time scale for initial onset of rain may, therefore, be substantially shorter than the time scale for development of the wave. It should also be noted that the detailed morphology of depressions will, in some regions, depend on the extreme topographic features. Clearly, the confirmation of the above is beyond the scope of the present paper.

Acknowledgments. The authors wish to acknowledge the support of the National Science Foundation under Grant ATM-78-23330 at Harvard and Grant ATM-802-3523 at MIT and the National Aeronautics and Space Administration under Grant NGL-22-007-228. Discussions with J. Shukla and B. N. Goswami are also acknowledged.

REFERENCES

Briggs, R. J., 1964: *Electron-Stream Interactions with Plasmas.* MIT Press, 8–46.

Carrier, G. F., M. Krook and C. E. Pearson, 1966: *Functions of a Complex Variable.* McGraw Hill, 438 pp.

Charney, J. G., 1947: The dynamics of long waves in a baroclinic westerly current. *J. Meteor.*, **4,** 135–162.

Farrell, B. F., 1982a: Pulse asymptotics of the Charney baroclinic instability problem. *J. Atmos. Sci.*, **39,** 507–517.

——, 1982b: The initial growth of disturbances in a baroclinic flow. *J. Atmos. Sci.*, **39,** 1663–1686.

——, 1982c: Three-dimensional baroclinic pulses. *J. Atmos. Sci.*, (in press).

Gaster, M., and A. Davey, 1968: The development of three-dimensional wave-packets in unbounded parallel flows. *J. Fluid Mech.*, **32,** 801–808.

Goswami, B. N., V. Satyan and R. N. Keshavamurty, 1981: Growth of monsoon disturbances over western India. *Monsoon Dynamics,* J. Lighthill and R. P. Pearce, Eds., Cambridge University Press, 415–428.

Krishnamurti, T. N., 1972: Transient disturbances in the tropics:

Energy sources. *Dynamics of the Tropical Atmosphere,* NCAR Summer Colloquium Notes, Boulder, 67–73.
——, J. Molinari, Hua-Lu Pan and V. Wong, 1977: Downstream amplification and formation of monsoon disturbances. *Mon. Wea. Rev.,* **105,** 1281–1297.
Lindzen, R. S., 1981: Some remarks on cumulus parameterization. *Clouds in Climate: Modeling and Satellite Observational Studies,* NASA Workshop Report, pp. 42–51.
——, and A. J. Rosenthal, 1981: A WKB asymptotic analysis of baroclinic instability. *J. Atmos. Sci.,* **38,** 619–629.
——, B. Farrell and K. K. Tung, 1980: The concept of wave overreflection and its application to baroclinic instability. *J. Atmos. Sci.,* **37,** 44–63.
——, A. J. Rosenthal and B. Farrell, 1982: Charney's problem of baroclinic instability applied to barotropic instability. *J. Atmos. Sci.,* **40,** 1029–1034.
Merkine, L., 1977: Convective and absolute instability of baroclinic eddies. *Geophys. Astrophys. Fluid Dyn.,* **9,** 129–157.
——, and M. Shafranik, 1980: The spatial and temporal evolution of localized unstable baroclinic disturbances. *Geophys. Astrophys. Fluid Dyn.,* **16,** 175–206.
Pearce, R. P., 1981: Downstream development of baroclinic waves in the upper-tropospheric monsoon easterlies suggested by a simple model experiment. *Monsoon Dynamics,* J. Lighthill and R. P. Pearce, Eds. Cambridge University Press, 381–402.
Ramage, C. S., and C. R. V. Raman, 1972: *Meteorological Atlas of the International Indian Ocean Expedition,* Vol. 2, *Upper Air.* National Science Foundation, Washington, DC, 223 pp.
Saha, K., F. Sanders and J. Shukla, 1981: Westward propagating predecessors of monsoon depressions. *Mon. Wea. Rev.,* **109,** 330–343.
Shukla, J., 1977: Barotropic-baroclinic instability of mean zonal wind during summer monsoon. *Pure Appl. Geophys.,* **115,** 1449–1462.
——, 1978: CISK-barotropic-baroclinic instability and the growth of monsoon depressions. *J. Atmos. Sci.,* **35,** 495–508.
Sikka, D. R., 1977: Some aspects of the lift history, structure and movement of monsoon depressions. *Pure Appl. Geophys.,* **115,** 1501–1529.
Stevens, D. E., and R. S. Lindzen, 1978: Tropical wave-CISK with a moisture budget and cumulus friction. *J. Atmos. Sci.,* **35,** 940–961.
Yanai, M., and T. Nitta, 1968: Finite difference approximations for the barotropic instability problem. *J. Meteor. Soc. Japan,* **46,** 389–403.