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GRAVITY WAVES IN THE MESOSPHERE

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ABSTRACT

The simple theory for the effect of gravity wave breaking on eddy diffusion and on mean stress is reviewed. Attempts are made to identify the magnitudes of known mechanisms for generating gravity waves. The relation between observed periods and unobserved horizontal wavenumbers is discussed. Finally the role of group velocity, wave packets and wave inhomogeneity is discussed.

1. INTRODUCTION

The recognition that gravity waves play a major role in mesospheric dynamics is relatively recent. For many years, the most famous picture of upper atmosphere wind was the meteor trail observation by Liller and Whipple (1954). However, as can be seen from reading the Proceedings of the International Symposium on Fluid Mechanics in the Ionosphere, Cornell University, July, 1959 (Bolgiano, 1959), the consensus opinion was that the wind irregularities were due to turbulence. It was Hines (1960) who forcefully argued that the irregularities were due to internal gravity waves--although the specific origin of the gravity wave was unclear. Lindzen (1967) showed that the Liller-Whipple observation was consistent with the first propagating diurnal tidal mode.

Much work was done on internal gravity waves over the next decade. An excellent summary of this work and thought appears in the proceedings of a meeting held in Boulder, Colorado in July of 1968 (Georges, 1968). It is striking to see how much current work was anticipated in these proceedings.

The potential role of breaking waves in generating turbulence was recognized (see also Hodges, 1969; Lindzen, 1968, 1971). In the wake of

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the pioneering work of Booker and Bretherton (1967), the fact that gravity waves could modify the mean flow was recognized (Lindzen, 1968a). Also, the need for friction in producing the reversal of meridional temperature gradient at the mesopause was noted (Leovy, 1964; Lindzen, 1968b)--as was the possible role of gravity waves in producing this friction. There was, moreover, by the late 1960's data available which, although coarse, could have been used to quantitatively assess these interactions (Theon et al., 1967). Nevertheless, it was not until Lindzen (1981) that the first crude attempt to parameterize the role of gravity wave breaking in generating turbulent diffusion and organized stress was made. What was specific to Lindzen (1981) was the recognition that the wave stress followed from the turbulent breaking of gravity waves--but was nonetheless distinct. Also Lindzen (1981) recognized that the origin of mesospheric gravity waves was primarily in the troposphere. This plus the filtering properties of the intervening stratospheric mean flow permitted fairly tight estimates of gravity wave phase speeds in the mesosphere. Such estimates are essential components of quantitative parameterizations.

The results of Lindzen (1981) are reviewed in other papers at this symposium and are, by now, well known. Thus, in section 2 only the most perfunctory review of the parameterization will be given--including a description of some improvement found in Holton (1982) and of some recent thoughts on turbulence due to sub-breaking waves (Lindzen and Forbes, 1983).

The remainder of this paper will deal briefly with some shortcomings in our present descriptions of gravity waves. In section 3 we will consider the explicit generation of internal gravity waves by mountains and by shear collapse--attempting to set bounds on the possible wave fluxes associated with various phase speeds. In section 4 we will examine the relation between wave phase speeds, wave numbers and observed periods in order to better understand some observations. In section 5 we will consider the horizontal as well as vertical propagation of gravity wave packets in order to develop some intuition about the origin of mesospheric gravity waves. In section 5 we will also remark on the horizontal inhomogeneity of gravity waves and its implications.

2. PARAMETERIZATION OF TURBULENCE AND STRESS

The NASA rocket probings of the mesosphere in the 1960's (Theon et al., 1967) revealed intense wave activity in winter above 50 km. Wave amplitudes were characteristically such as to marginally produce unstable lapse rates. In summer such "breaking" amplitudes did not occur until greater heights $\sim$ 70 km. Recent radar data (Balsley, 1983) confirms this seasonal variation.

Lindzen (1981) introduced the simplest model capable of describing the effects of breaking gravity waves. He considered zonally travelling gravity waves which were standing waves in the meridional direction, i.e., waves with the following form

$$e^{ik(x-ct)} \cos (\ell y + \phi) \tag{1}$$

where

x = eastward distance
y = northward distance
t = time
k = eastward wavenumber
c = eastward phase speed
ℓ = northward wavenumber
ϕ = arbitrary phase constant

In a slowly varying medium (only inhomogeneities in the z (vertical) direction were considered), Lindzen (1981) showed, using the WKB approximation,

$$\delta T \simeq A\Gamma^{1/2} T^{-1/2} \lambda^{1/2} e^{i\int\lambda dz} e^{z/2H} e^{ik(x-ct)} \cdot \cos (\ell y + \phi) \tag{2}$$

where

A = Amplitude factor

Γ = Static stability = $\frac{dT}{dz} + \frac{g}{c_p}$

T = Basic temperature
δT = Perturbation temperature
λ = Vertical wavenumber

$$\approx |N/(\bar{u} - c)| \ (1 + \ell^2/k^2)^{1/2} \tag{3}$$

$\bar{u}$ = Mean zonal flow

$N^2 = \frac{g}{T} \left(\frac{dT}{dz} + \frac{g}{c_p}\right)$

(2) applies beneath the breaking level.
Consistent with the WKB approximation

$$\frac{d\delta T}{dz} \approx iA\, \Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{i\int\lambda dz} e^{z/2H} e^{ik(x-ct)} \cdot \cos (\ell y + \phi) \tag{4}$$

and breaking occurs when

$$\left|\frac{d\delta T}{dz}\right| = \Gamma$$

or, using (4) when

$$A\, \Gamma^{1/2} T^{-1/2} \lambda^{3/2} e^{z/2H} \approx \Gamma \tag{5}$$

If the breaking height, z_{break}, is observed then (5) determines A.

Above z_{break}, it is assumed that sufficient turbulence is generated to prevent $\frac{d\delta T}{dz}$ from growing further.

Lindzen (1981) shows that, in the absence of damping, $\frac{d\delta T}{dz}$ would grow exponentially with a local exponent given by

$$\frac{1}{2H} - \frac{3}{2}\frac{1}{\bar{u}-c}\frac{d\bar{u}}{dz} \qquad (6)$$

Now damping gives rise to an imaginary part of c, c_i, which, in turn, produces an imaginary contribution to λ (viz equ. (3)). The damping is then chosen so that λ_i is exactly equal to the growth exponent given by (6); i.e., so that the growth is exactly cancelled. This degree of damping is, in fact, sufficiently small to permit Lindzen (1981) to relate c_i to eddy diffusivity, D_{eddy}, with the following relation

$$kc_i \approx \lambda^2 D_{eddy}, \qquad (7)$$

and the requirement that growth be cancelled then leads to

$$D_{eddy} \approx \frac{k\ |u-c|^4}{N^3(1+\ell^2/k^2)^{3/2}}\left|\frac{1}{2H} - \frac{3}{2}\left(\frac{1}{\bar{u}-c}\frac{d\bar{u}}{dz}\right)\right| \qquad (8)$$

above z_{break}.

Lindzen (1981) also shows that the condition for breaking (5) implies that at the level of breaking, z_{break}, we have

$$\overline{w'u'} \approx \frac{k}{2}\frac{N^2}{\lambda^3} \quad \text{at } z = z_{break}. \qquad (9)$$

where

w' = perturbation vertical velocity
u' = perturbation zonal velocity

Now, for plane waves of the form (1) in the absence of damping, the Eliassen-Palm theorem requires

$$\frac{d}{dz}\left(\rho_o\ \overline{u'w'}\right) = 0, \qquad (10)$$

implying no acceleration of the mean flow. However, in the presence of damping due to wave breaking (10) is replaced by

$$\rho_o\overline{u'w'} = \rho_o(z_{break})\ \frac{k}{z}\frac{N^2}{\lambda^3}\Bigg|_{z_{break}} e^{-2\int_{z_{break}}^{z}\lambda i dz} \qquad (11)$$

from which it can be shown from (6) that the flow acceleration, F_x, is given by

$$F_x = -\frac{1}{\rho_o}\frac{d}{dz}\left(\rho_o\,\overline{u'w'}\right)$$

$$= \overline{w'u'}\left(\frac{1}{H} - \frac{3\frac{d\bar{u}}{dz}}{\bar{u} - c}\right) \qquad (12)$$

Lindzen (1981) considered the case where the second term in (12) was negligible.

There is little point in reproducing the discussion of relations (8) and (12) given in Lindzen (1981). However, it should be stressed that, in the context of this simplified model, even given observations of z_{break} we must still know c, k and ℓ in order to evaluate D_{eddy} and F_x. We shall discuss these matters further later. For the moment let us note that Lindzen (1981), taking account of the filtering properties of tropospheric and stratospheric winds concluded that for winter $c \approx 0$ while for summer $c \approx 20$ m/s. k was chosen to be the smallest value consistent with

$$k\,(\bar{u} - c) \gtrsim f \qquad (13)$$ [1]

where

$$f = 2\Omega \sin\phi$$
$$\phi = \text{latitude}$$
$$\Omega = 2\pi/\text{day}$$

(13) being necessary for vertical propagation in the presence of rotation. It should be noted that the present analysis otherwise ignores rotation. For $(k/\ell)^2$ Lindzen (1981) chose 1.17 as the value which best duplicated observed vertical wavelengths. Certainly the selection of k and ℓ was casual at best. Nevertheless, with these choices, equ (8) predicted the magnitude and the vertical distribution of D_{eddy} (a peak near 70 km and a minimum near 90 km) called for by totally independent chemical measurements and provided an immediate explanation of anomalous D-region absorption during sudden warmings. In addition equ (12) predicted mesopause level accelerations 0 (100 m/s/day). To be consistent with steady flows these accelerations would have to be balanced by meridional circulations 0 (8 m/s) and such meridional circulations are approximately what are needed to produce the observed reversal of pole to pole temperature gradient at the mesopause. These matters are discussed in more detail in Lindzen (1981). In view of these successes one can be reasonably confident that observed gravity waves do play a major role in the general circulation of the middle atmosphere. Indeed it is commonly stated that the choice of k, above, is too small; larger values would lead to larger values of D_{eddy} and F_x.

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[1] This led to assumed horizontal wavelengths $\sim$ 1600 km.

Nevertheless, it is equally obvious that the above model for gravity waves is naive, inadequate, and uncertain on many counts. Indeed, there are so many potential improvements to be made on the present model (which after all was chosen to be the simplest model capable of describing the relevant physics) that ultimately we will need observational guidance in our choice.

The remainder of this paper will be devoted to discussing a few of the uncertainties and inadequacies. In general, these may be divided into three categories:

i) Technical: Difficulties in actually using (8) and (12) in numerical models.
ii) Parameter choice: How to best determine c, k, ℓ, and A (as functions of latitude).
iii) Basic assumptions: How to best deal with the fact that gravity waves are unlikely to be described by (1), i.e., they are not likely to be steady, homogeneous in longitude, etc.

There is one technical matter (which may have some basic ramification) which we will discuss immediately. Equs (5), (8), and (12) imply a sudden onset of D_{eddy} and F_x at z_{break}. This, in turn, is generally hard on numerical schemes. Lindzen and Forbes (1983) have recently considered the possibility that stable gravity waves (by equ (5)) might cascade energy to smaller scales which were unstable, thus generating eddy diffusion. Lindzen and Forbes (1983) determined an upper bound for such a cascade and found that the resulting diffusion only modestly affected the original wave until the breaking level was reached. Thus, regardless of the validity of the cascade mechanism, Lindzen and Forbes (1983) provide a harmless way to objectively smooth the onset of D_{eddy} and F_x.

3. HOW MUCH GRAVITY WAVE IS THERE?

The parameterization described in section 2 assumes that the breaking observed by Theon et al. (1967) at a single station (Wallops Island) is characteristic of the whole hemisphere (at least around a latitude circle). It is also presumed to be associated with specific phase speeds. Holton (1982) takes even greater freedom with his choice of fluxes and phase speeds. Similar assumptions underlie the work of Matsuno (1982). The question addressed in this section is whether one can plausibly estimate and bound the generation of gravity waves in the troposphere.

All proposed mechanisms for gravity wave generation fall into 3 broad categories:

i) Mountain waves forced by flow over topography. These waves are associated with phase speeds ~ 0; they need not be exactly zero since the flows are unsteady and can generate phase speeds other than zero.

ii) Unstable shear zones can radiate internal gravity waves with phase speeds equal to the flow speeds in the shear zones (Lindzen, 1974). We shall refer to this mechanism as shear collapse for reasons that will be made clearer.

iii) Geostrophic adjustment in principle leads to gravity wave generation (Blumen, 1972); however, it is unclear how this mechanism operates on a day-to-day basis or whether it is, in fact, distinguishable from the first two mechanisms. We shall not discuss geostrophic adjustment further.

For the first two mechanisms we will attempt to crudely estimate gravity wave generation in order to see whether it is capable of generating the breaking levels used in section 2.

3a. Mountain waves

Mountain waves are forced by flow over topography. The forcing appears in the lower boundary condition where

$$w'(o) \approx U_o \frac{\partial}{\partial x} h\ (x) \text{ at } z = 0 \tag{14}$$

where

h = surface elevation
U_o = surface mean wind

We shall assume surface elevation of the form

$$h\ (x) = h\ e^{ikx} \cos\ (\ell y + \phi)$$

Then (14) becomes

$$w'(o) \approx ikU_o h\ e^{ikx} \cos\ (\ell y + \phi) \tag{15}$$

From Lindzen (1981) we have

$$w'(z) \approx A\ e^{z/2H}\ \lambda^{-1/2}\ e^{i\int_o^z \lambda dz}\ e^{ikx} \cos(\ell y + \phi) \tag{16}$$

where, for c = o,

$$\lambda^2 \approx \frac{N^2}{U_o^2}(1 + \ell^2/k^2)$$

From (15) we have

$$A = i\ k\ U_o\ h\ \lambda_o^{1/2} \tag{17}$$

We also have from Lindzen (1981)

$$u' \approx \frac{\lambda}{k\left(1 + \frac{\ell^2}{k^2}\right)}\ w' \tag{18}$$

From (16) and (18) we get by averaging over x,

$$\overline{w'u'} \approx \frac{k\, U_o\, h^2\, N_o}{2\left(1 + \frac{\ell^2}{k^2}\right)^{1/2}}\; e^{z/H} \tag{19}$$

and from (9) we have

$$\overline{w'u'}\,\Big|_{z_{break}} = \frac{k}{2}\,\frac{U^3\left(z_{break}\right)}{N\left(z_{break}\right)\left(1 + \frac{\ell^2}{k^2}\right)^{3/2}} \tag{20}$$

Equating (20) gives us a simple expression for z_{break}:

$$e^{z_{break}/H} \approx \frac{U^3\left(z_{break}\right)}{N_o\, N\left(z_{break}\right) h^2\, U_o \left(1 + \frac{\ell^2}{k^2}\right)} \tag{21}$$

For simplicity we will take

$$N \approx N_o \approx \frac{2\pi}{300s}$$

We will also take

$$U_o \sim 10 \text{ m/s}$$

which probably is excessive.

From Sankar-Rao (1965) we estimate that for $k \geq \frac{2\pi}{1600 \text{ km}}$, $h \sim 20$ m is probably conservative as well (recall that it is a global value). Finally following Lindzen (1981), we take $\left(1 + \frac{\ell^2}{k^2}\right)^{1/2} \approx 1.5$.

Rather than including the functional form of U(z) in evaluating z_{break}, we will anticipate that breaking will occur between 50 km and 70 km (in winter) and evaluate the r.h.s. of (21) for U = 30 m/s, and 50 m/s--values characteristic of this height range. For U = 30 m/s we find $\frac{z_{brk}}{H} \approx 8.83$ ($z_{brk} \approx 62$ km assuming $z \approx 7$ km) and for U = 50 m/s, $\frac{z_{brk}}{H} \approx 10.4$ ($z_{brk} \approx 73$ km). Given that U = 50 m/s is characteristic of z = 50 km while z = 30 km is characteristic of 62 km, we may conclude that mountain wave forcing on a global basis can account for breaking levels no lower than 62 km--especially since we have ignored damping. This suggests that the observed breaking at 50 km over Wallops Island (Theon et al., 1967) is irregular, regional or both. We will return to this later. Note, however, that this result is independent of the choice for k.

3b. Shear collapse

In attempting to estimate gravity wave production due to shear collapse we must deal with a problem for which no solutions exist so far. Of necessity our approach will be even coarser and less direct than our approach to mountain waves. We will begin with the assumption that the general circulation of the troposphere is trying to produce shear layers which we will idealize as Helmholtz discontinuities of magnitude ΔU. Following Lindzen (1974) we will further assume that such unstable shear zones are stabilized (smoothed until the Richardson number equals 1/4) by the emission of gravity waves, half of which are travelling upward. Thus we assume that half the difference in energy between the Helmholtz profile and the neutral profile goes into gravity waves which may (if the intervening profile of U is favorable) penetrate the middle atmosphere. From Lindzen (1974) we have

$$\Delta E \sim \frac{\rho_o \left(z_{shear}\right) \left(\frac{\Delta U}{2}\right)^3}{3N} = \frac{\rho_o \left(z_{shear}\right) (\Delta U)^3}{24N} \qquad (22)$$

This energy is produced over a time, τ, over which the general circulation is assumed to be trying to maintain the ΔU jump. We will take

$$\tau \sim 10\ f^{-1}$$

where

$$f = 2\Omega \sin\phi .$$

Then

$$\overline{p'w'} \underset{\uparrow}{=} \rho_o \frac{\Delta U}{2} \overline{u'w'} \lesssim \frac{\Delta E/2}{\tau} \sim \frac{\rho_o (\Delta U)^3}{48N\tau} \sim \frac{\rho_o (\Delta U)^3 f}{48N \times 10}$$

by the Eliassen-Palm theorems

and

$$\overline{u'w'}\Big|_{z_{shear}} \lesssim \frac{(\Delta U)^2 f}{240\ N} . \qquad (23)$$

Above z_{shear}, the constancy of $\rho_o \overline{u'w'}$ implies

$$\overline{u'w'} \lesssim \frac{(\Delta U)^2 f}{240\ N}\ e^{\left(\frac{z - z_{shear}}{H}\right)} \qquad (24)$$

Again, from section 2 we have that at z_{break}

$$\overline{u'w'} \approx \frac{k}{2}\frac{N^2}{\lambda^3} \approx \frac{k}{2}\frac{(U-c)^3}{N\left(1+\frac{\ell^2}{k^2}\right)^{3/2}} \qquad (25)$$

Combining (24) and (25) we then get

$$\frac{(\Delta U)^2 f}{240\ N}\, e^{\left(\frac{z_{break} - z_{shear}}{H}\right)} \gtrsim \frac{k}{2} \frac{(U - c)^3}{N\left(1 + \frac{\ell^2}{k^2}\right)^{3/2}} \qquad (26)$$

For purposes of estimation we will take

$$\frac{f}{N} \sim 5 \times 10^{-3}$$

$$N \sim \frac{2\pi}{300\ \text{sec}}$$

$$\left(1 + \frac{\ell^2}{k^2}\right)^{1/2} \sim 1.5$$

(26) then becomes

$$e^{\left(\frac{z_{break} - z_{shear}}{H}\right)} \gtrsim 3.4 \times 10^5 \quad \frac{k(U - c)^3}{(\Delta U)^2} \qquad (27)$$

Following Lindzen (1981) the smallest value we may take for k is

$$k \sim 3.7 \times 10^{-6}\ m^{-1}$$

For ΔU we shall somewhat arbitrarily take

$$\Delta U \sim 5\ m/s,$$

and for $|U - c|$ we will anticipate $|U - c| \sim 50$ m/s. (27) then becomes

$$e^{\left(\frac{z_{break} - z_{shear}}{H}\right)} \gtrsim 6.3 \times 10^3$$

or

$$\frac{z_{break} - z_{shear}}{H} \gtrsim 8.75$$

or

$$z_{break} - z_{shear} \gtrsim 61\ \text{km}.$$

Clear air turbulence is generally associated with the upper troposphere so that $z_{shear} \sim 10$ km and

$$z_{break} \gtrsim 71\ \text{km}$$

This is at least compatible with summer observations, but again uncertainties and the neglect of damping should be kept in mind. It should also be kept in mind that larger values of k will lead to higher breaking levels.

As a final remark we should note that this is probably the most efficient mechanism for generating gravity waves with phase speeds greater than zero. Thus, in invoking phase speeds greater than average tropospheric wind speeds, we must consider what could possibly be trying to maintain an unstable shear layer at that speed.

4. WAVENUMBER AND OBSERVED PERIODS

In the previous sections we noted that there is substantial uncertainty over horizontal wavelengths--largely because our data is restricted to single station records. Coherence studies over short distances are not likely to be convincing since phase fronts in nature are unlikely to be flat. Nevertheless, the value of k plays an important role in determing D_{eddy} and F_x. In this section we will look at the relation between wavelength and period--hopefully to gain some insight into the question of wavelength.

Historically we have considered the generation of waves to be associated with specific periods--the period being thought to be a signature of physical mechanism. This is certainly the case with tides. Unfortunately, all known mechanisms for generating tides are incapable of maintaining breaking in middle and high latitudes on a global scale. The waves discussed in section 3, on the other hand, are characterized by phase speed and not period.[2]

To the extent that periods, τ, are observed, they are related to phase speed by the simple relation

$$\tau = \frac{2\pi}{kc} = \frac{\text{horizontal wavelength}}{\text{phase speed}} \qquad (28)$$

Anticipating phase speeds $\sim$ 20 m/s in summer, we see from (28) that a horizontal wavelength of 1000 km will be associated with a ground measured period, τ, of 14 hours. It would be easy to confuse such a period with a tidal period. A clue to the primacy of period or phase speed is given by equ (3) where we see a tendency for all waves with the same phase speed to have the same vertical wavelength independent of k and hence period. This is in line with currently available data.

5. WAVE PACKETS AND GROUP VELOCITY

The preceding sections were concerned with a zonally homogeneous--or at least, zonally averaged picture of gravity waves. However, there

[2] In this connection it is interesting to consider the period, 3 hrs, commonly associated with gravity waves. To the best of my knowledge, this is not based on observation, but was introduced on an ad hoc basis by Hines (1960). Presumably this was a period between the Coriolis period and the Brunt period--at least in the absence of crucial doppler effects.

is no question that in reality gravity waves are excited regionally and even episodically. Ideally, the theorist should be dealing with the statistics of wave packets rather than with plane waves. Indeed a start in this direction was made by Jones (1968). However, the difficulty of the packet approach and the absence of an adequate data base for such an approach make it uninviting for the moment.

Nevertheless some crude analysis is helpful, at least for the interpretation of data. Let us for a moment consider a gravity wave packet travelling in the x-z plane. Locally we have

$$\sigma \approx N \frac{k}{m} \tag{29}$$

where σ = frequency observed in moving frame following mean flow = $k\ (c-U)$

k = wavenumber in x direction
m = wavenumber in z direction
N = Brunt-Vaisala frequency

The group velocity in the x-direction is given by

$$C_{gx} = U + \frac{\partial\sigma}{\partial k} = U + \frac{N}{m} = U + \frac{\sigma}{k} = U + (c-U) = c \tag{30}$$

In the z-direction we have

$$C_{gz} = \frac{\partial\sigma}{\partial m} = -N\frac{k}{m^2} = -\frac{\sigma}{m} = -\frac{k\ (c-U)}{m} \approx -(c-U)^2\frac{k}{N} \tag{31}$$

As a rule a packet will travel at the group velocity, and from (30) and (31) we have

$$\frac{C_{gx}}{C_{gz}} = -\frac{m}{k}\frac{c}{c-U} \tag{32}$$

Now for vertical wavelengths of 10 km and horizontal wavelengths of 1000 km, m/k = 100. For stationary waves where $c \equiv 0$, waves will still be travelling vertically; however, when $c \neq 0$ packet travel will become almost horizontal, and packets originating in the troposphere will, in travelling vertically 60 km, also travel several thousand km horizontally. Thus, observations at a single location (especially in summer when $c \neq 0$) will frequently be unable to trace the origin of upper atmosphere gravity waves.

In addition, as long as the large-scale wind distribution permits meridional propagation, breaking waves in the upper atmosphere at a particular latitude can originate in the troposphere at very different latitudes. The presence of mountains or unstable jets directly below may be neither necessary nor relevant.

We finally note from equ (31) that variations in U, the large-scale wind, markedly affect the vertical group velocity. In general there is a tendency for wave packets to be deflected into regions of large group velocity. It is of some use to consider the fact that the environment into which gravity waves propagate is not zonally homogeneous. Stationary waves (generally of wave numbers 1-3) will produce large regions of enhanced or depressed zonal velocity. This will affect gravity waves and their mean flow interactions in a number of ways:

i) There will be a tendency for waves to be focused into the regions where the stationary wave enhances the magnitude of $(\bar{u}-c)$. This may not seem too important for a very localized wave packet which might presumably only "see" the local phase of the stationary wave. However, from equ (32) we see that the ray paths in regions of low $|\bar{u}-c|$ will become more horizontal, almost certainly allowing focussing to act before the packet reaches the mesosphere. More commonly, gravity waves will be excited over a broad range of longitudes and this question does not arise.

ii) The focussing of waves into regions of larger $|\bar{u}-c|$ should lead to greater amplitudes and lower breaking heights in these regions. However, as can be seen from equ (4), large $|\bar{u}-c|$ leads to smaller λ's and smaller values for $\left|\frac{d\delta T}{dz}\right|$ which in turn should elevate the breaking height. Finally, damping is less important when $|\bar{u}-c|$ is larger--again suggesting larger amplitudes.[3] It is not clear, without further calculations, which effect will dominate and under what conditions.

iii) Despite the ambiguity in item (ii) above, it seems clear that gravity wave breaking can occur at different heights at different locations due to inhomogeneities in excitation, focussing by planetary scale stationary waves, etc. This is especially important in view of the suggestion in section 3 that mountain waves are inadequate to maintain the winter breaking levels observed at Wallops Island on a global basis. Such inhomogeneities can give rise to interactions between gravity waves and stationary waves. In regions of large $|\bar{u}-c|$, F_x due to gravity waves will act to attenuate the stationary wave; however, in regions of small $|\bar{u}-c|$, F_x will act to amplify the stationary wave. Again, it is difficult to anticipate the net effect.

iv) Finally, zonal variations in breaking height will act to vertically spread the zonal averaged distributions of D_{eddy} and F_x compared to the distributions derived from section 2.

It is amply clear that the correct calculation of the effects described in this section will be difficult--though important. However, it is almost as clear that the qualitative expectations are general and not too dependent on specific details.

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[3] Damping (presumably from infrared cooling) complicates matters in other ways. It permits, for example, the deposition of gravity wave momentum fluxes without breaking and without the generation of D_{eddy}.

6. REMARKS

The present paper follows closely the contents of the lecture the author presented at the U.S.-Japan Symposium--except for the omission (for the sake of brevity) of material on tides, which can, however, be found in Lindzen (1981). In this paper I have reviewed the parameterization of Lindzen (1981) wherein the effects of gravity wave breaking on the generation of eddy diffusion and on the deposition of wave momentum flux can be seen in the simplest (and rather idealized) context. This context consists in zonally and vertically propagating waves whose properties are independent of longitude propagating through an axially symmetric zonal flow. Even in this context certain questions arise. In section 3 we inquire whether known sources for gravity waves can account for the breaking levels observed at Wallops Island. We note that known forcing may be inadequate. A possible solution to the resulting problem is that wave amplitudes at Wallops Island may not be characteristic of other longitudes. This matter is discussed further in section 5.

In section 4 we deal with the question of the horizontal wavelength of gravity waves. This is a very important variable in determining D_{eddy} and F_x due to breaking waves. It is also almost impossible to measure directly at isolated stations. However, if one can otherwise determine phase speed, then single station observations of period can yield horizontal wavelength. It is noted that the periods noted by Balsley at this symposium (12-36 hrs) are compatible with horizontal wavelengths $\gtrsim$ 1000 km.

In section 5 the group velocity of gravity wave packets is considered. We noted that the origin of gravity waves observed in the mesosphere is likely to be far removed horizontally. We also discuss the fact that gravity waves in fact propagate through significant planetary scale stationary waves.

The above hardly begins to deal with the possible extensions of the simple parameterization of gravity wave breaking. In this connection mention should be made of the recent work of Schoeberl and Strobel (1983) on the effects of infrared damping and of Dunkerton (1982) on the possible intermittancy of gravity waves.

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