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CO_2 Feedbacks and the 100 K Year Cycle in Climate

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With 5 Figures

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Summary

In an earlier paper (Lindzen, 1986), it was shown that allowing CO_2 to vary with snow/sea ice position could lead to a greatly enhanced response in glaciation to 100 K year orbital forcing – even when 20 K year forcing was much stronger. In that model, snow/sea ice position (SSIP) and glaciation were different: the former was the forcing for the latter. However, SSIP and glaciation were not decorrelated. Observations (Berner et al., 1979; Lorius et al., 1985; Neftel et al., 1982) suggest that CO_2 may be independently related to both SSIP and glaciation. In the present paper, we allow (in a highly simplified manner) such independent dependence, and show how it alters the earlier results. Briefly, the dependence of CO_2 on glaciation can contribute to and even cause a highly enhanced response to the 100 K year component of the forcing. However, the CO_2 dependence on SSIP is, on the whole, more effective in this regard. Thus, we expect time series of CO_2 to show variation on the faster time scales than does glaciation.

1. Introduction

In Lindzen (1986), a simple climate model was presented where orbital forcing concentrated at 20 K year periods – but including a small 100 K year component – could readily produce an essentially 100 K year response in glaciation. Crucial to this model were thresholds to climate transitions which could be produced by CO_2 dependence on snowcover/sea ice position (SSIP). We will briefly review this model in section 2 of the present paper. In section 2 of this paper we also include the extension of the earlier model to include explicit dependence of the thresholds (or relatedly CO_2) on glaciation, itself, as well as on SSIP. In section 3, we will present some representative results. It is shown that threshold dependence on glaciation can lead to enhanced 100 K year response, but not as easily as dependence on SSIP. The two dependences can act jointly in producing enhanced sensitivity to 100 K year forcing. Certain suggestions emerge from these calculations for useful paleoclimatic measurements. In particular, high resolution (O(10 K year)) time series of CO_2 might prove particularly illuminating. This is discussed further in the concluding section 4.

2. Mathematical Formulation

The model in Lindzen (1986) originated in the simple energy balance models of Budyko (1969) and Sellers (1969), which related the SSIP to some measure of solar heating. These models parameterized meridional heat transport in terms of either a simple linear law (Budyko, 1969) or diffusion (Sellers, 1969). The shortcomings of both approaches were discussed in Lindzen and Farrell

(1977, 1980) and Lindzen et al. (1982). In Lindzen and Farrell (1980) the simple heat transport laws were replaced by a baroclinic adjustment which depended on low level static stability. This permitted us to take account of the fact that static stability is enhanced over ice surfaces. One feature of this model was that exceedingly small changes in solar heating were sufficient to move the SSIP between the pole and about 53° latitude. Figure 1 shows SSIP as a function of Q/Q_0 for this model (Q is nominally the solar constant, while Q_0 is its present value; however, as will be noted later, Q is actually a measure of the overall radiative configuration). To move the SSIP beyond 53°, however, took very large changes in Q/Q_0. Thus, the climate as represented by SSIP was essentially neutrally stable between the pole and 53°. In such a model, with the solar constant near its present value, changes in irradiation associated with orbital variations would move the SSIP back and forth between the pole and 53°. It is suggested in Lindzen (1986) that the main condition for the existence of the 100 K year cycle in glaciation is that Q be at or very near the neutrally stable present value. If, now, the presence of the SSIP at the pole led to an increase in CO_2 while the presence of the SSIP at 53° led to a decrease, then if the SSIP were at either of these positions, the variations in irradiation would have to exceed some threshold in order for the SSIP to move to the other position. This threshold effect played a major role in Lindzen (1986) in accounting for the enhanced response to 100 K year forcing. Figure 1 schematically illustrates this threshold effect; Lindzen (1986) discusses processes other than a dependence of CO_2 on SSIP that could also lead to thresholds for transitions in SSIP.

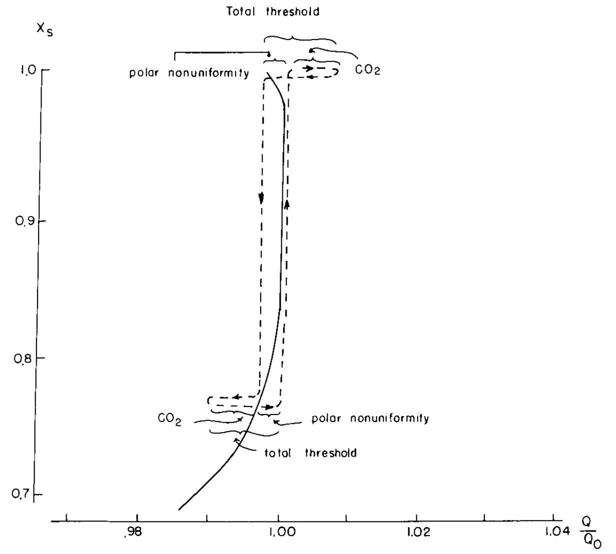


Fig. 1. The SSIP v. solar constant as calculated by Lindzen and Farrell (1980). Also shown are the paths for climate transitions between $x_s = 1$ and $x_s = 0.8$ ($x_s = \sin \varphi_s$ where φ_s = latitude of SSIP) including threshold effects associated with both polar non-uniformity and CO_2 effects. See Lindzen and Farrell (1980) for details. This figure is taken from Lindzen (1986)

In what follows, we will sketch the simple mathematical model used in Lindzen (1986). As noted in Lindzen and Farrell (1977), the solar constant, Q, in energy balance models is actually an overall measure of the radiative budget insofar as changes in the infrared cooling are equivalent to changes in Q. Thus, both changes in irradiation due to orbital variations and changes in infared cooling of the surface due to changes in CO_2 can be expressed in terms of Q (increases in CO_2 lead to decreased cooling or equivalently increased Q). The orbital variations in Q are given by

$$Q/Q_0 - 1 = f(t) \tag{1}$$

where Q_0 is the present value of Q. For $f(t)$, the following expression was used

$$\begin{aligned} f(t) = {} & \varepsilon\,(a_2 \sin(2\pi t/100\,\mathrm{K}) + \\ & + a_5 \sin(2\pi t/40\,K) + a_{10} \sin(2\pi t/20\,K)) \end{aligned} \tag{2}$$

$$f(t) = \varepsilon g(t) \tag{2'}$$

where ε is some small number which represents the translation of the orbital forcing into a perturbation in Q/Q_0. a_{10} represents the contribution due to the precession of the axis of rotation, a_5 represents the contributions due to obliquity variations, and a_2 represents the contributions due to changes in eccentricity (associated with the 100 K year periodicities). Allowing arbitrary phases to be associated with the various components of forcing did not lead to different conclusions; we therefore fixed the phases in (2).

The SSIP is represented by a variable, $S(t)$, where $S = 1$ corresponds to the SSIP at its furthest metastable excursion (nominally 53°) and $S = -1$ corresponds to the SSIP near the pole. The thresh-

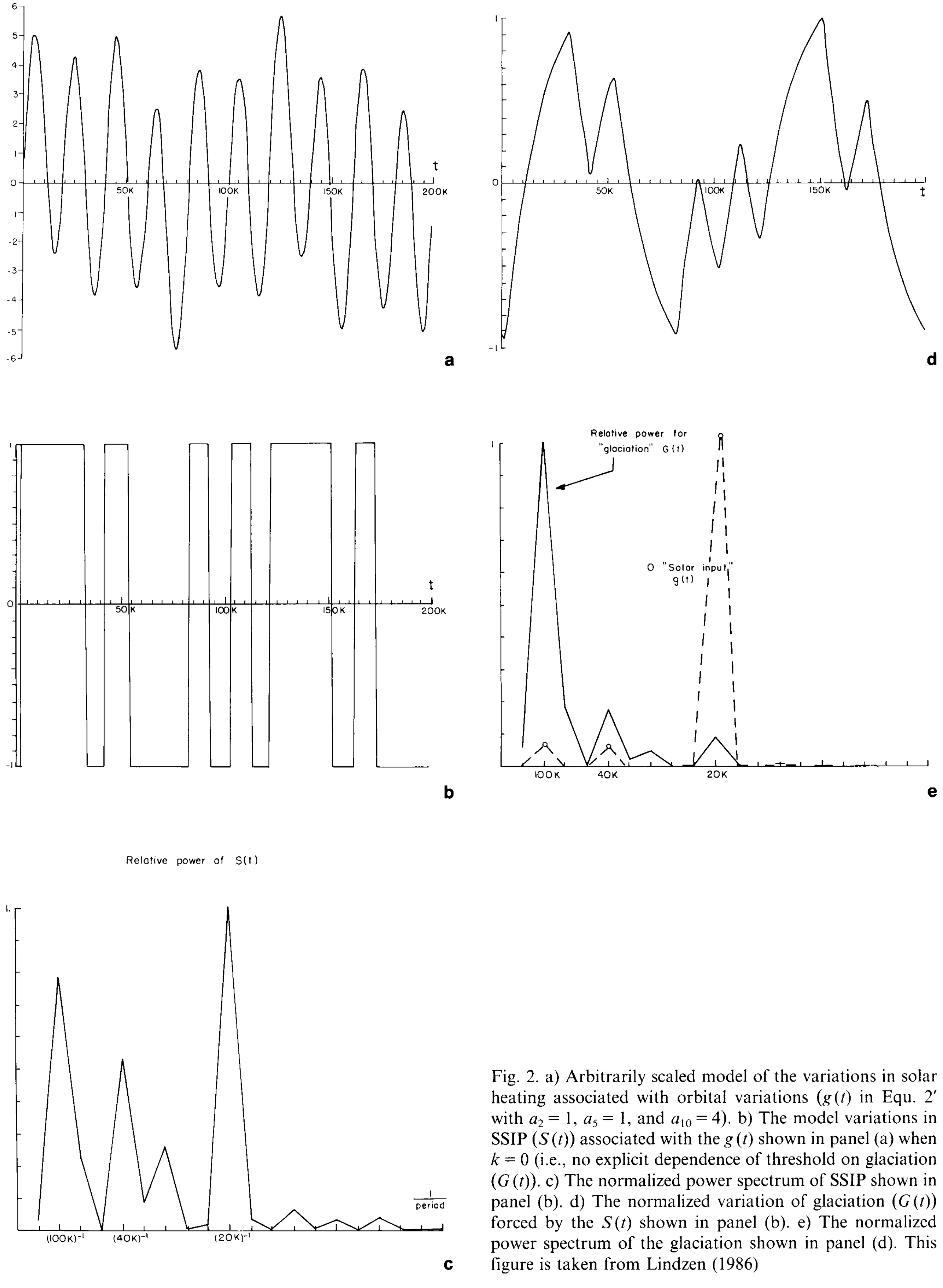
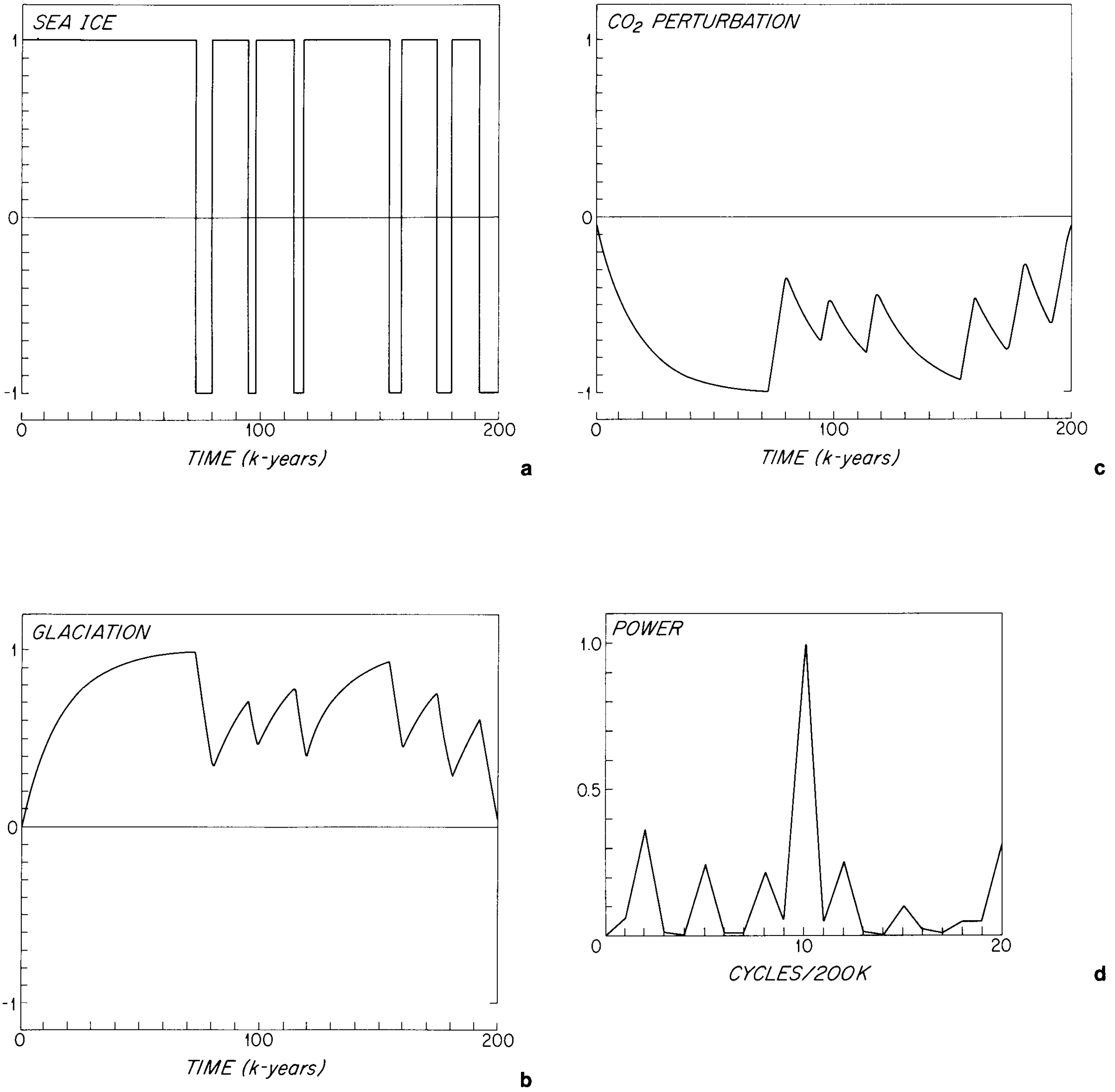


Fig. 2. a) Arbitrarily scaled model of the variations in solar heating associated with orbital variations ($g(t)$ in Equ. 2′ with $a_2 = 1$, $a_5 = 1$, and $a_{10} = 4$). b) The model variations in SSIP ($S(t)$) associated with the $g(t)$ shown in panel (a) when $k = 0$ (i.e., no explicit dependence of threshold on glaciation ($G(t)$)). c) The normalized power spectrum of SSIP shown in panel (b). d) The normalized variation of glaciation ($G(t)$) forced by the $S(t)$ shown in panel (b). e) The normalized power spectrum of the glaciation shown in panel (d). This figure is taken from Lindzen (1986)



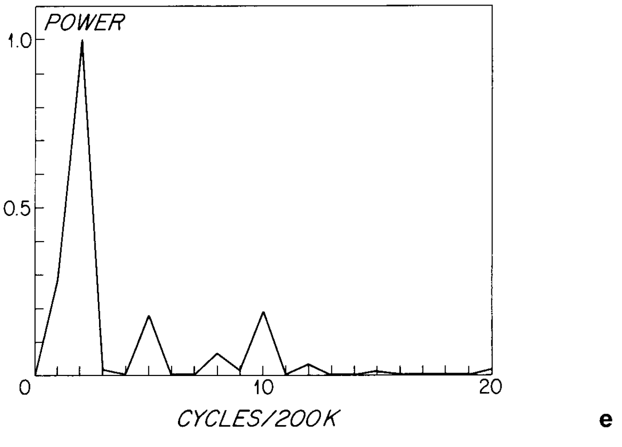


Fig. 3. Results for $g_1 = g_2 = 0.0$ and $k = 0.29$. a) $S(t)$. b) Normalized variation of glaciation ($G(t)$). c) Normalized variation in CO$_2$. d) Normalized power spectrum of $S(t)$. e) Normalized power spectrum of $G(t)$

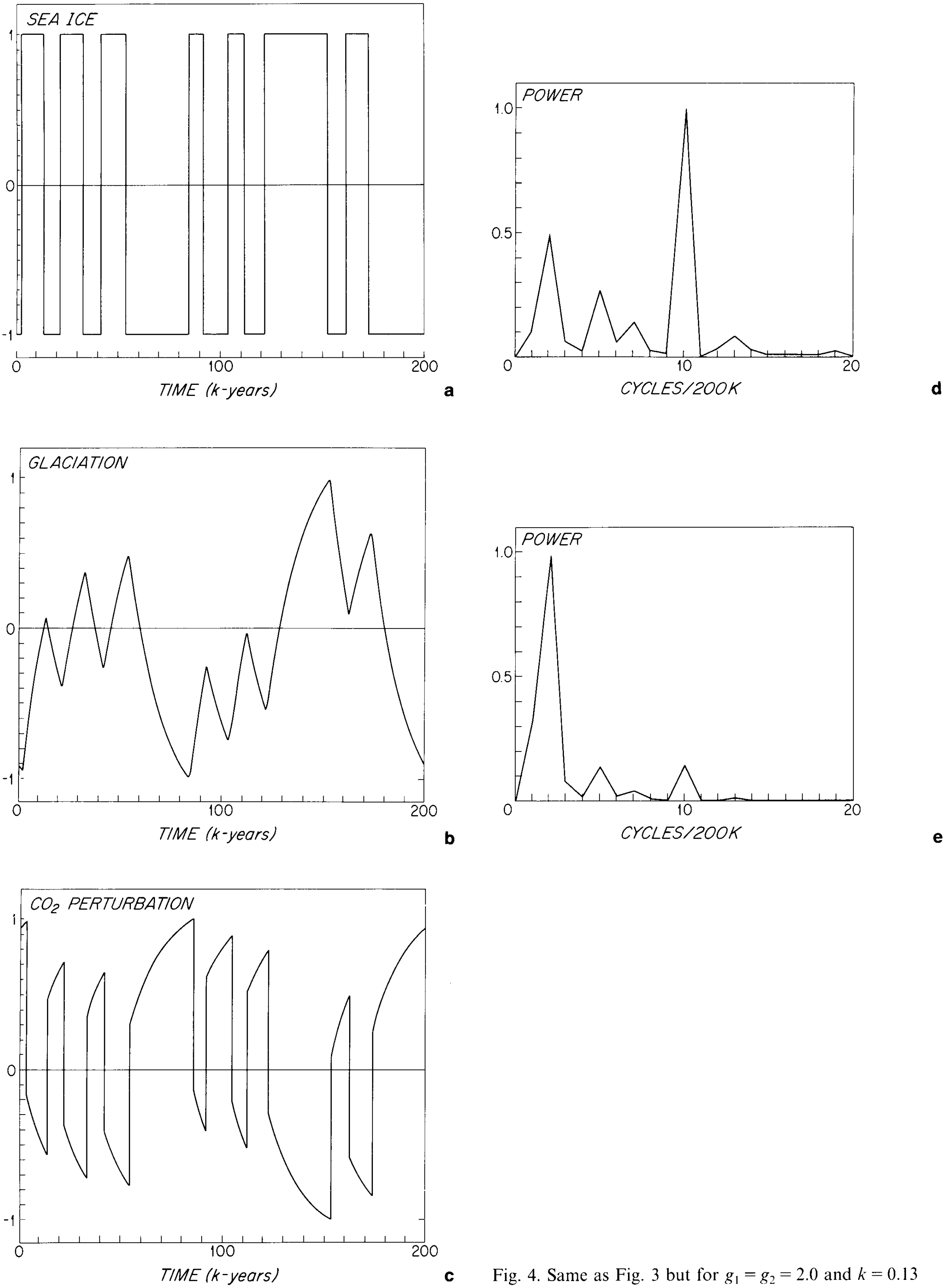
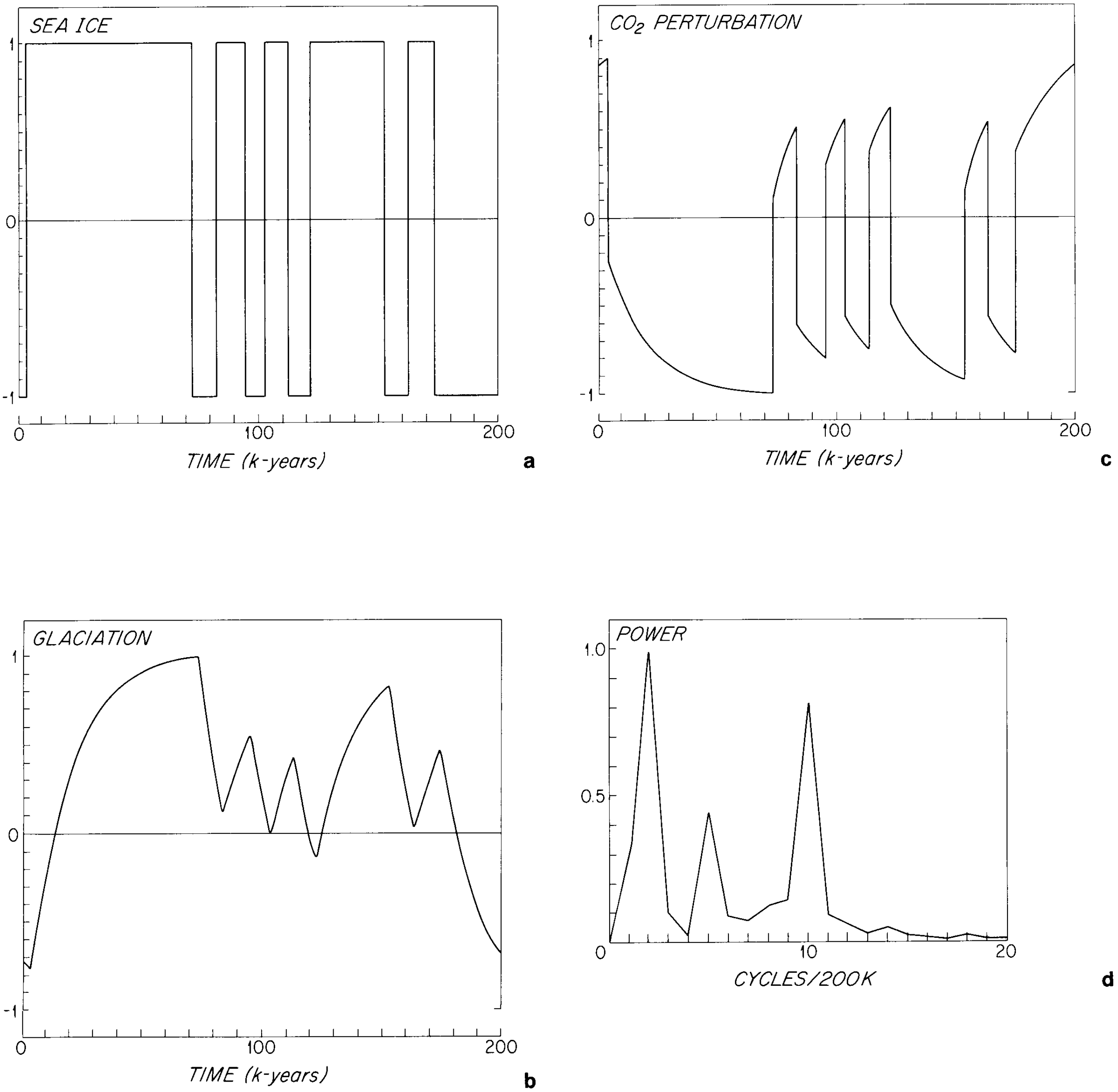


Fig. 4. Same as Fig. 3 but for $g_1 = g_2 = 2.0$ and $k = 0.13$



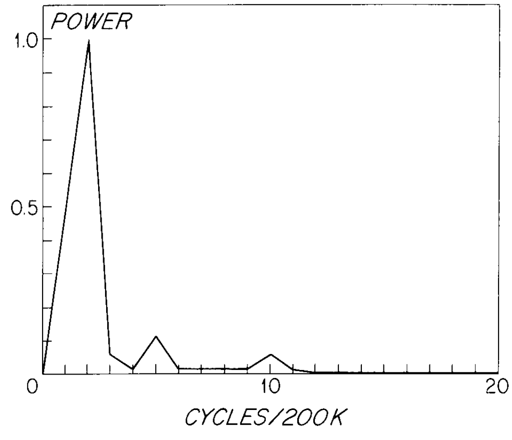


Fig. 5. Same as Fig. 3 but for $g_1 = g_2 = 2.5$ and $k = 0.12$

old parameterization (in Lindzen, 1986) corresponds to requiring that if $S = 1$, then $g(t)$ must fall below some threshold, $-g_1$, before S changes to -1, and if $S = -1$, then $g(t)$ must rise above some other threshold, g_2, before S moves back to $+1$. While most of the calculations in Lindzen (1986) were for $g_1 = g_2$, the effects of asymmetric thresholds were also investigated. (Note that for zero thresholds, S changes between $+1$ and -1 as $g(t)$ changes sign.)

Finally, Lindzen (1986) assumed that the glaciation (as opposed to SSIP) is forced by S; i.e.,

$$\frac{\mathrm{d}G}{\mathrm{d}t} = S - aG \tag{3}$$

where G is a measure of glaciation and a is some relaxation rate for glaciation estimated to be about 1/17 K (Weertman, 1964; Imbrie and Imbrie, 1980). (3) is readily integrated, yielding

$$G = G(0) + \mathrm{e}^{-at}\int_0^t \mathrm{e}^{at'} S(t')\,\mathrm{d}t'. \tag{4}$$

Note that (3) implies that G's response to S will vary inversely with frequency; this too contributes to the enhanced response to 100 K year forcing. $G(0)$ in (4) is chosen so as to make $G(t)$ periodic over 200 K years. Note that the periods chosen for precession and obliquity variation in Eq. 2 are not exactly right. However, for the choices made, $g(t)$ is periodic over 200 K years, which simplifies the presentation of results.

It was found in Lindzen (1986) that for virtually any choices of a_2, a_5, and a_{10}, wherein a_{10} was dominant, it was possible for wide ranges of choices for the thresholds, g_1 and g_2, to obtain $G(t)$'s dominated by 100 K year and even 200 K year cycles. Examples of such results for $a_2 = 1$, $a_5 = 1$, $a_{10} = 4$, and $g_1 = g_2 = 3$, are shown in Fig. 2. As already mentioned, the above approach to thresholds was tantamount to assuming that CO_2 depended only on $S(t)$; indeed, Fig. 2b for $S(t)$, suitably rescaled, would also give the time fluctuations of CO_2. Thus far, we have no detailed time series for CO_2. What Berner et al. (1979) found was that CO_2 varied significantly between 18 000 YBP and now. This is not inconsistent with the above picture, since $S(t)$ is significantly correlated with $G(t)$; however, the possibility that CO_2 (and hence, the thresholds g_1 and g_2) depends on $G(t)$ as well as $S(t)$ must be considered. To do so is the purpose of the present paper.

In the context of the present simple model, all that needs to be done in order to allow CO_2 to vary with G as well as S is to replace g_1 with $g_1 + kG$ and g_2 with $g_2 - kG$. The above system, so modified, is still easily integrated. However, at each time step, the modified thresholds must be evaluated in order to ascertain whether a transition in S will occur. Since we do not, at the moment, have an objective basis for choosing k, we will examine a range of choices. The results are presented in section 3. The purpose, as in Lindzen (1986), is not to precisely simulate glaciation cycles, but rather to see whether any values of k will still allow the long period emphasis in $G(t)$.

3. Sample Numerical Results

In this section we will examine the effects of various choices for g_1, g_2, and k. As in Lindzen (1986), we will confine ourselves to presenting results for $a_2 = 1.$, $a_5 = 1.$, and $a_{10} = 4$. Qualitatively similar results are obtained for any other choices (provided that a_{10} is dominant), and qualitative results are about all we can hope for at this stage.

As noted in Lindzen (1986), when $k = 0$, a characteristic 100 K year cycle in glaciation of the sort shown in Fig. 2 is obtained when $g_1 = g_2 = T$, for $2.49 < T < 3.7$. For $3.7 < T < 5.7$, one tends to get either 100 K or 200 K year oscillations in glaciation with unrealistically large suppression of 40 K and 20 K year components. For $T > 5.7$, glaciation was stationary. Lindzen (1986) also examined asymmetric thresholds where $g_1 = T - \Delta$ and $g_2 = T + \Delta$; Δ was associated with a secular drift in Q. In the present paper we will ignore Δ.

We began our analyses by taking $T = 0$, seeing whether we might obtain a 100 K year response in glaciation simply due to the presence of k. For $k \leqslant 0.28$, we found no domination of long periods in glaciation. Only for $k \sim 0.29$ did we obtain such dominance; for $k > 0.29$, glaciation tended to be stationary. Results for $k = 0.29$ are shown in Fig. 3. Clearly, with CO_2 dependent only on glaciation, it is difficult to obtain a 100 K year response, and the response that is obtained tends to look unrealistic (similar to results obtained with $k = 0$ and $T > 3.7$). Recall that in the present model, the threshold i staken to be a relative measure of CO_2. Thus, when $k = 0$, CO_2 behaves like $-S$, while when $T = 0$, CO_2 goes as $-G$. We next proceded to increase T. For $T = 1$, we again found a 100 K year response in G only for a narrow range of k:

this time around $k \sim 0.23$. The results for $T = 1$ and $k = 0.23$ are almost identical to those shown in Fig. 3. However, as T approached 2, things began to change. For example, for $T = 2$, a choice of $k = 0.13$ leads to results very similar to what is obtained for $2.49 \leqslant T < 3.7$ when $k = 0$. This particular case is illustrated in Fig. 4. We see that for values of T which are a little short of what is needed for 100 K year dominance in glaciation, some amount of k can help; $k > 0.14$, however, again leads to stationary glaciation. We already know that $T = 2.5$ and $k = 0$ lead to a perfectly acceptable 100 K year response in glaciation (see Fig. 2); the presence of $k < 0.11$ barely alters this response. However, for $0.11 < k < 0.14$, there is a marked enhancement in the response at 200 K year and a stronger suppression of the response at 20 K years and 40 K years. Results for $k = 0.12$ are shown in Fig. 5. For $k \geqslant 0.14$ we again get stationary glaciation. The situation is similar at $T = 3.0$; however, now k must be < 0.07 for the response to be essentially unchanged. Results almost identical to those in Fig. 5 now occur for $k = 0.09$, and stationary glaciation occurs for $k > 0.11$. Not surprisingly, as T increases, smaller and smaller values of k suffice to produce stationary glaciation.

4. Concluding Remarks

The above results suggest that CO_2 decreases tied to glaciation are less effective than those tied to SSIP in producing a reasonable 100 K year response in glaciation. However, some tie to glaciation tends to decrease the tie to SSIP needed to produce a reasonable response. To the extent that the above model is correct, it suggests that although CO_2 will be highly correlated with $-G$, it will depend primarily on S, and hence will display larger high frequency fluctuations than does G (viz Fig. 4). This is a point which should not be impossible to check.

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