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Some Remarks on Cumulus Parameterization

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Abstract—A simple parameterization of cumulonimbus convective heating is presented. The model is primarily based on preserving a moisture budget and on detraining cloud air at levels corresponding to the neutral buoyancy of the air converged at low levels. Results are compared with data from the western Pacific and GATE. Agreement is good. Suggestions are offered for improving the model and extending it to other regions.

Key words: cumulus parameterization, cumulonimbus convection, tropical meteorology, cumulus modeling, convective parameterization.

1. Introduction

The present note is based on LINDZEN (1981) (subsequently referred to as L). L describes a simple and effective approach to cumulus parameterization. Unfortunately, L appears in a NASA report which is not widely available. A paper by GELEYN *et al.* (1982) describing the implementation of this parameterization at the European Centre for Medium Range Forecasting is available, but it was felt, in view of continuing interest, that some purpose might still be served by the wider dissemination of the original work. This paper differs from L primarily in the inclusion of some modifications based on a recent paper by LINDZEN and NIGAM (1987) (subsequently referred to as LN). LN show the importance of the trade cumulus layer being well mixed and allowing a finite time for cumulus mass flux to respond to the moisture budget. References have also been updated.

The total precipitation (or equivalently the atmospheric heating) associated with cumulus convection is generally accurately represented by a simple moisture budget:

$$\text{Precipitation} = \text{Evaporation} + \text{Convergence of Moisture} \tag{1}$$

The relevant convergence appears to be that which occurs in the lower troposphere—

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below the trade inversion where such a feature exists. Such a budget has been observed in the western Pacific (OGURA and CHO, 1973)[1] and during all phases of GATE (BURPEE and REED, 1982). The total cumulus heating is simply the latent heat of condensation, L, times the precipitation.

We shall discuss the above budget later in this note. Clearly, such a budget cannot be true always; otherwise there would be no way for humidity in the lower troposphere to change. For the moment, however, let us ignore this problem. Equation (1) also fails to give us the vertical distribution of heat release. Some additional closure conditions are needed to obtain such a distribution. The question arises as to whether we need a detailed specification of the vertical distribution of cumulus heating. The answer to this question is not entirely clear. STEVENS *et al.* (1977) concluded that the vertical distribution was not too important for fully developed easterly waves because of the important role of cumulus friction. The same insensitivity may not hold for weaker developing waves (STEVENS and LINDZEN, 1978). Furthermore, the ability of latent heat release in the tropics to force stationary waves in middle latitudes intuitively ought to depend on whether the latent heat is deposited in the lower troposphere, where we have prevailing easterlies, or at upper levels where prevailing westerlies may exist (CHARNEY and DRAZIN, 1961). Perhaps more important is the fact that the feedback between latent heat and moisture convergence (as in CISK) depends crucially on how much of the heat is deposited below the trade inversion. At least in some circumstances, therefore, we *do* expect the vertical distribution of latent heat release to be important.

Among the various closures suggested for obtaining this distribution is that due to ARAKAWA and SCHUBERT (1974). This particular approach is difficult (if not impossible) to solve for, and its physical basis is not at all clear. An attempt to apply the Arakawa-Schubert scheme to GATE date has been made by LORD (1982). Lord was unable to obtain a solution to the Arakawa–Schubert scheme. However, using simplex methods, Lord obtained answers close (in some sense) to being solutions. A comparison between the predicted distribution of heating (using Lord's approach) and observations is shown in Figure 1. The agreement is quite poor. Figure 2 shows the implementation of various cumulus parameterizations in the European Centre model (GELEYN *et al.*, 1982). The problems of the Arakawa-Schubert scheme are again evident—as are problems with simple convective adjustment models. The Arakawa–Schubert scheme predicts excessive cloud heating near the surface; observations from GATE show fairly broad heating between the 800 mb and 300 mb. In

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[1] OGURA and CHO (1973) seek a proportionality constant between cumulus mass flux and convergence in the subcloud layer ($0 < z < 0.5$ km). The coefficient is about 3–4. However, it is evident from their data that this coefficient is unity for convergence below the trade inversion (or in their case, the minimum in moist enthalpy).

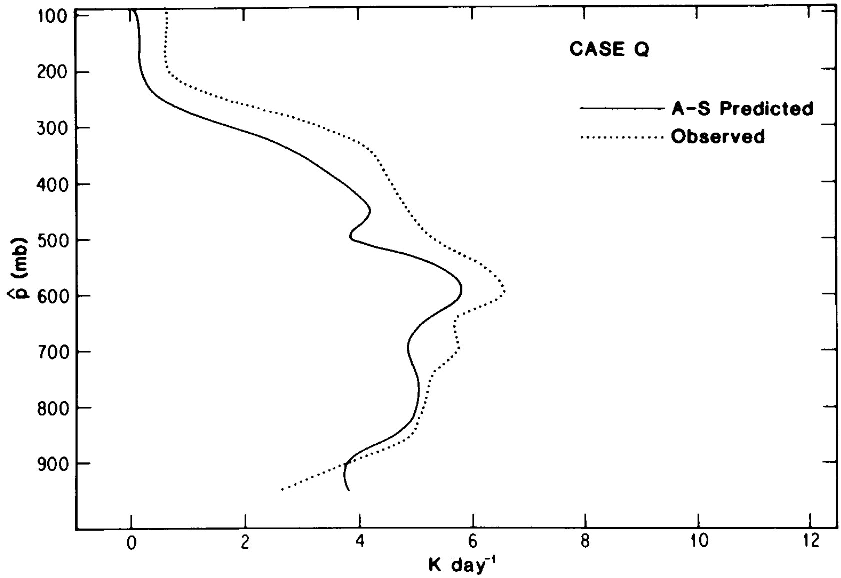

Figure 1
Time-averaged calculated (on the basis of the Arakawa–Schubert parameterization) and observed cumulus heating for Phase III of GATE. From LORD (1982).

view of our earlier remarks, we might anticipate serious problems arising from this misrepresentation.

We may next ask whether this failure implies that a more effective parameterization will prove still more complicated than the Arakawa–Schubert scheme. I doubt it. I will attempt to show, in this note, that an almost trivial but plausible scheme provides a rather good simulation of the observed heating.

2. *Simple Model*

The basic approach is that described in Appendix 1 of STEVENS and LINDZEN (1978). As shown by OOYAMA (1971) and ARAKAWA and SCHUBERT (1974), cumulus heating can be expressed

$$H = M_c \frac{\partial s}{\partial z} = \rho Q \tag{2}$$

where

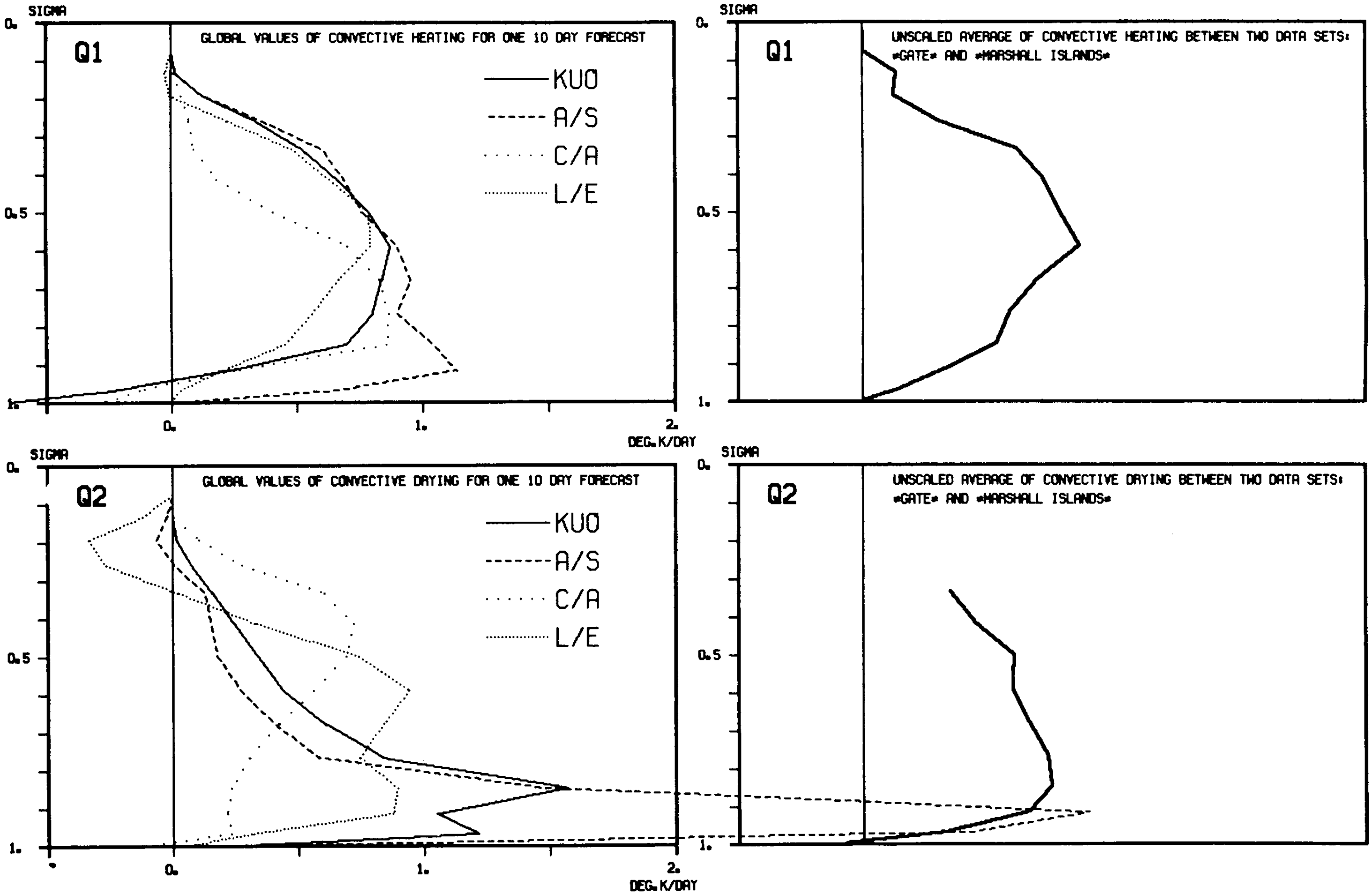

Figure 2

Heating (Q_1) and drying (Q_2) vertical profiles, globally averaged over one ten-day forecast at the European Centre for Medium-Range Weather Forecasting, for four convection schemes (Kuo, Arakawa–Schubert, convective adjustment, and Lindzen—ECMWF). On the right are the unscaled averages of the same profiles obtained from the GATE and Marshall Islands experiments by THOMPSON *et al.* (1979). From GELEYN *et al.* (1982).

Q = Htg. in deg/day
M_c = mass flux in cumulus clouds
$\frac{\partial s}{\partial z}$ = dry static energy gradient in air outside clouds ($s = c_pT + gz$)[2]

The specification of the vertical distribution of heating becomes equivalent to the specification of M_c's distribution. Certain aspects of M_c's distribution are fairly clear: for example, M_c should not extend beyond a height, z_T, where

$$c_pT(0) + Lq(0) = c_pT(z_T) + gz_T + Lq(z_T) \quad (3)$$

where

$$c_pT(0) + Lq(0) = h(0) = \text{moist enthalpy } (s + Lq) \text{ at the ground}$$

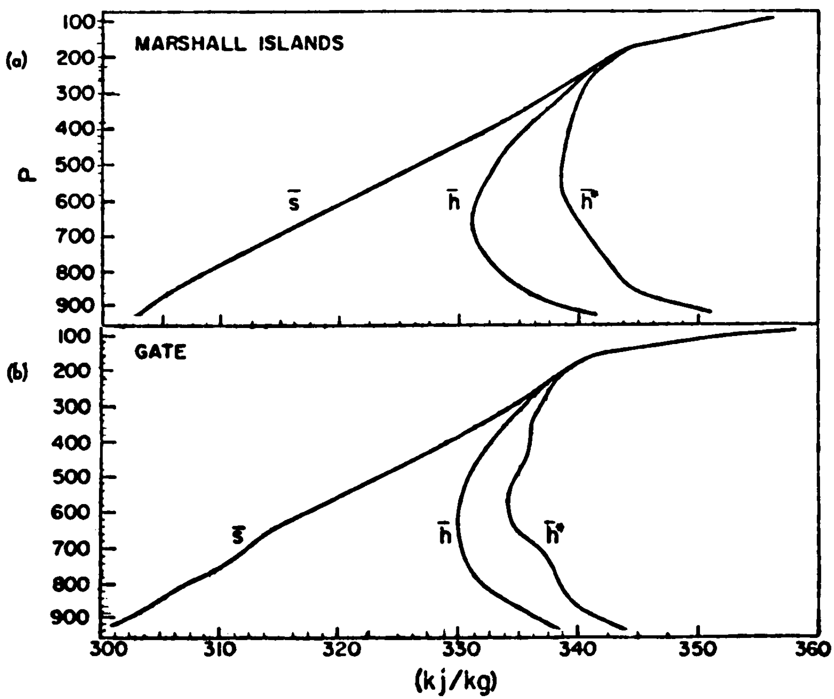

Figure 3
Dry static energy, $\bar{s}$, moist static energy, $\bar{h}$, and saturation moist static energy, h^*, in units of Kj Kg^{-1} vs. height (mb). From Figure 4.7 in LORD (1978).

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[2] Equation (2) is commonly interpreted as implying that heating is due to compensating subsidence. This is not strictly correct. Equation (2) results from the fact that that portion of upward flow, w, which is due to cumulus mass flux, M_c, is not subject to dry adiabatic cooling (because of the heating due to condensation of water vapor), and, hence, represents an effective heating.

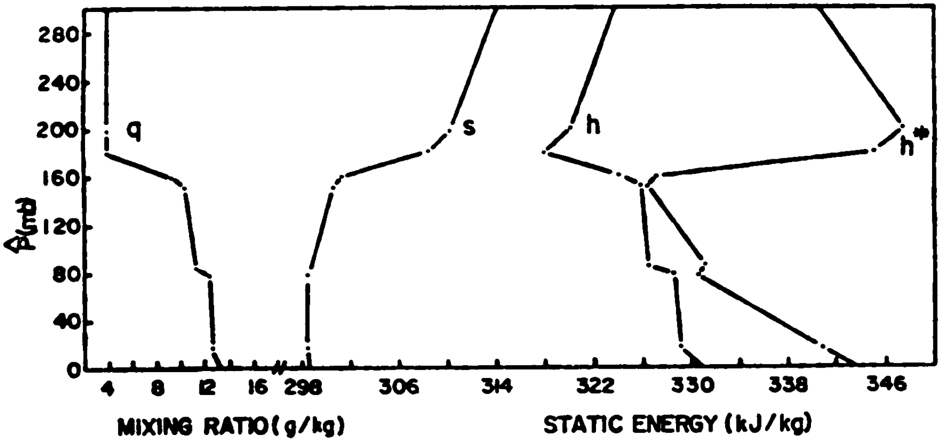

Figure 4
Vertical profiles of water vapor mixing ratio, q, dry static energy, S, moist static energy, h, and saturation static energy, h^*, for the ship *Planet*, February, 7–12, 1969. From AUGSTEIN *et al.* (1974).

and

$$c_pT(z_T) + gz_T + Lq(z_T) = h(z_T) = \text{moist static energy at } z_T.^3$$

Equation (3) defines z_T. Equation (1) gives us that

$$\int_0^{z_T} M_c \frac{\partial \theta}{\partial z} dz = L(E + \text{conv. of water vapor}) \tag{4}$$

where E = rate of evaporation. The simplest choice of M_c consistent with (3) and (4) would be to make M_c a constant between $z = 0$ and $z = z_T$. Such a choice would lead, via (2), to a better fit to the observed heating during GATE than that obtained by Lord. However, it would do poorly for the Marshall Islands. Following an admittedly *ad hoc* procedure, one can do much better. In Figure 3 we show s, h and h^* (saturated moist enthalpy) *vs.* p for GATE and the western Pacific (during the Marshall Islands nuclear tests of the late 1950s). (Note that s is very nearly linear in p—rather than z. This implies that $\partial s/\partial z \propto \rho$ and hence Q will follow M_c rather than M_c/ρ.) Clearly, air above the minimum in h cannot participate significantly in cumulus convection. We shall assume (counter to the most straightforward arguments) that all air below the minimum can participate in cumulus

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[3] For z_T above 350 *mb*, the presence of $Lq(Z_T)$ makes little difference (*viz.* Figure 3); for simplicity, we have generally ignored it.

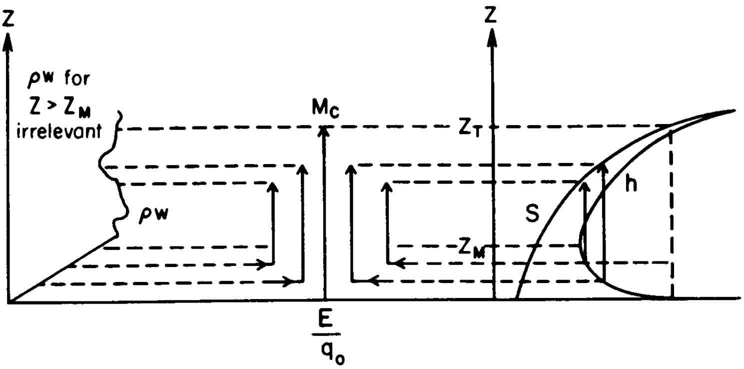

Figure 5
Schematic illustration of how M_c is related to distribution of s, h and ρw ($\sim -\omega$).

convection,[4] and that the moisture convergence referred to in Equation (1) is that which takes place below the minimum in h. We next assume that portion of M_c which arises from evaporation, $M_{co} = E/q_0$ (where E = surface evaporation and q_0 = specific humidity of surface air) detrains at z_T (viz. Equation (3)), while the portion of the air converged between some z and δz, $\delta(\rho w)$ gives rise to a $\delta M_c = \delta(\rho w)$ which detrains at that z_d where $s(z_d) = h(z)$. This procedure completely specifies the vertical distribution of M_c. The above procedure is unambiguous when low level convergence is occurring. Its application is schematically illustrated in Figure 5. Additional considerations must be applied when we have divergence. Indeed, when divergence exceeds evaporation, drying must occur. For the purposes of this brief note, divergence will be ignored.

3. *Application*

We shall use data from the Marshall Islands (Reed and Recker, 1971) and from the third phase of GATE (Lord, 1982) to calculate the vertical distribution of cumulus heating. For both GATE and the Marshall Islands it was the case that evaporation was small ($\sim$20% of convergence in trough regions), and, hence, we will ignore M_{co}. Figure 3 gives the vertical distribution of s and h for both cases. The distributions do not change markedly during the passage of an easterly wave. Hence

[4] Conventional reasoning (Holton, 1972) would hold that only air with values of h such that h^* at some greater height is less than h can become convectively buoyant. However, Augstein *et al.* (1974) show that at any instant the minimum in h arises from a precipitous drop in h from larger values below, and that all the air below can be convectively unstable (*viz.* Figure 4). The layer in which the precipitous drop occurs is identified with the trade inversion. The broad minimum in Figure 3 seems likely to be the result of averaging over a trade inversion which is moving up and down. This matter will be discussed further in Section 4.

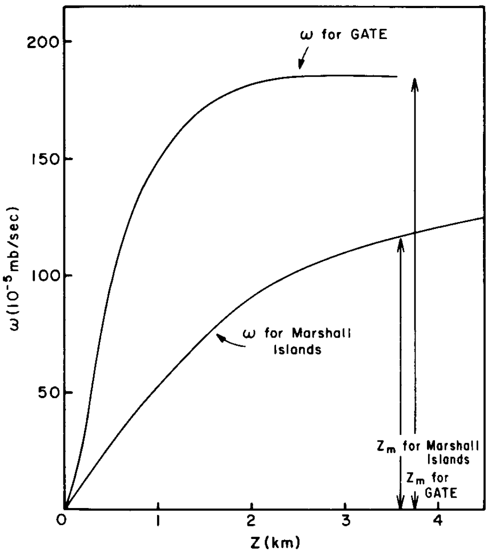

Figure 6
Vertical distribution of $-\omega$ (10^{-5} mb/sec) during convergent phase of easterly wave.

this figure can be used to determine at what height air converged below h's minimum will eventually detrain. Figure 6 shows ω (p-velocity $\sim -\rho w$) vs. p for the convergent phases of easterly waves in the Marshall Islands (taken from Fig. 8 of REED and RECKER, 1971) and during GATE (taken from Figure 5.6 of LORD, 1978). Note that during GATE, convergence (increasing ρw with height) was confined far more closely to the surface than during the Marshall Islands tests. We now calculate M_c using the approach described above and in Figure 5. The results are shown in Figures 7 and 8. Also shown is the cumulus heating given by Equation (2)–using $\partial s/\partial z$ from Figure 3. Note that the vertical distribution of heating is very different in the two cases—largely but not totally reflecting the different distributions of M_c. The differences in M_c arise from differences in the large-scale low-level convergence. For the GATE case large-scale convergence is confined to a region well below the minimum in h. The converged air, therefore, has a large value of h and does not detrain until it reaches the upper troposphere. In between there is a large region of constant M_c. For the Marshall Islands large-scale convergence occurs throughout the region below the level of minimum h, and hence detrainment begins shortly above this level. Interestingly, in both cases the predicted profiles are very similar to the observational estimates of cumulus heating (viz. Figure 14 in REED and RECKER, 1971 and Figure 1).[5] This measure of agreement in two very different cases supports the notion that

[5] Q in Figure 7 is about double Q in Figure 1. Figure 7 corresponds to a maximum in rain while Figure 1 is an average.

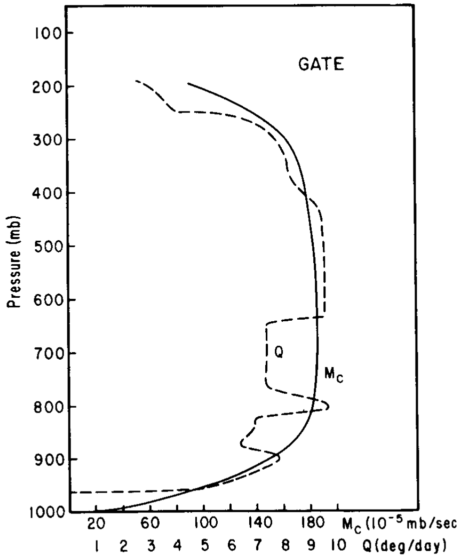

Figure 7
Predicted M_c and Q for convergent phase of easterly wave during phase III of GATE.

the vertical distribution Q depends strongly on the distribution of low-level convergence. In contrast, the Arakawa–Schubert scheme, wherein the distribution of Q depends on the distribution of ω at upper levels (STARK, 1976), yields poor agreement with observed distributions. To be sure, the results in Figures 7 and 8 are not in perfect agreement with observations—obviously these observations are not perfect either. First there is a small underestimate of total heating in both cases ($\sim$0(10–20%)), which almost certainly results from ignoring evaporation. Finally, in Figure 7, for the Marshall Islands, the peak in Q is 1–2 km lower than observed, while in Figure 7, for GATE, the low-level wiggles are somewhat more extreme than those observed. These modest discrepancies are associated with small variations in $\partial s/\partial z$ with height (*viz.* Figure 3 and Equation (2)). While such variations of $\partial s/\partial z$ are probably not of fundamental consequence (and probably not very measurable), they do effect the detailed structure Q. In Figure 9 we show the observed zonally averaged distribution of s (from OORT and RASMUSSON, 1971) for the equatorial region. The use of this profile rather than that in Figure 3 (for the Marshall Islands) does not greatly alter the distribution of M_c, but does lead to a new hearing distribution—labelled Q^* in Figure 7. Q^* peaks at an altitude in agreement with observation but has a second (unobserved) peak in the upper troposphere. There is no reason to worry about this upper level peak; the heating would rapidly alter s so as to eliminate the peak. Finally, Figure 2 shows the European Center implementation of the present parameterization. Again, it appears to be quite successful.

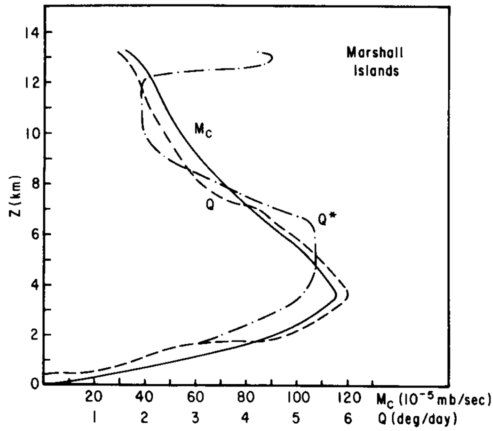

Figure 8
Predicted M_c, Q and Q^* for convergent phase of easterly wave over the Marshall Islands. See text for definition of Q^*.

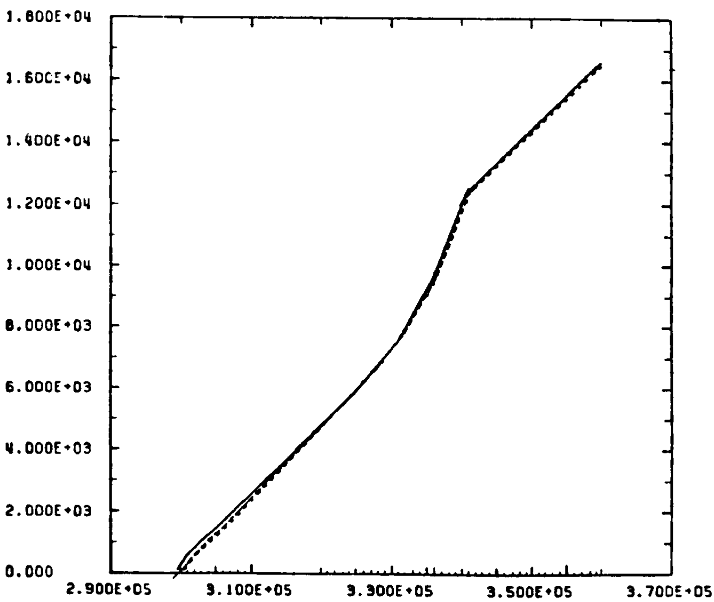

Figure 9
$s(j/kg)$ vs. $z(m)$ from OORT and RASMUSSON (1971).

4. *Conclusions and Remarks*

Our results suggest that the physically accurate parameterization of cumulus may be relatively simple. However, some difficulties remain. Equation (1), for example, by requiring that all moisture converged below z_M be released as precipitation, does not permit any changes in the moisture content of the air below z_M. Thus, for example, the present model would not allow the development of deep instabilities in previously more stable configurations. In the maritime tropics, where the thermodynamic state is fairly steady this might not be a severe problem. However, it would certainly hinder modelling the onset of monsoons. At issue is our failure to note that shallow cumulus clouds do not rain efficiently. Thus it might suffice to replace (1) with a relation.

$$\text{Precipitation} = f(z_T)\ \{\text{Evaporation} + \text{Convergence of Moisture}\} \tag{5}$$

where (3) defines z_T, and $f(z_T)$ is less than one, approaching one as z_T approaches the upper troposphere. At the moment, the determination of $f(z_T)$ is *ad hoc*, but, it should not be difficult to tune.

Two other issues have been uncovered by LN. First, our assumption that M_c is always in instantaneous balance with low level convergence, requires that all low level mass convergence be taken up by cumulus updrafts instantly; none remains to redistribute mass beneath z_M. This can lead to small errors in the low level pressure gradients which can, in turn, lead to significant errors in low level flow in the neighborhood of the equator. In reality, while clouds respond rapidly to low level convergence, they do not respond instantaneously. LN found that allowing a short relaxation time greatly improved matters. In the present context, this would amount to replacing the relation

$$\partial M_c = \delta(\rho w)$$

in Section 2 with a relation of the form

$$\tau_c \frac{\partial}{\partial t} \delta M_c + \delta M_c = \delta(\rho w) \tag{6}$$

where $\tau_c \approx 30$ minutes. The precise implementation (6) in any given model will depend on such details as the time resolution of the model. It may be sufficient in a time-dependent model to simply let M_c as calculated in previous sections lag about 1/2 hour behind the convergence distribution which gave rise to the calculated M_c.

The second issue raised by LN relates to the role of cumulonimbus heating in the thermal balance of the lower troposphere below Z_M. LN's results suggest the possibility that, at least, over the tropical Pacific, pressure gradients below Z_M are largely determined by the distribution of sea surface temperature rather than by cumulonimbus heating. The sea surface temperature distribution is mixed throughout the tradewind boundary layer by penetrative convection in the form of dry convection near the surface (mixed layer) and trade cumuli in the upper part of the layer (SARACHIK, 1985). If the above is true, then not only Arakawa's scheme (viz. Figures 1 and 2) but also observations and our own calculations are indicating excessive cumulonimbus heating below Z_M.

A possible reason for such a discrepancy in both the observation and in our scheme may be related to the differences in the profiles of s in Figures 3 and 4. It should be noted that Q is not directly observed; rather, it is inferred from large scale measurements of convergence, temperature, etc. Profiles such as those in Figure 3 are usually used. The use of the profiles in Figure 3 would largely eliminate Q below the inversion because $\partial s/\partial z$ is so small; this would also be the case for our model (*viz.* equation 2). It is clear that a proper specification of the vertical distribution of cumulonimbus heating depends not only on the method of parameterization but also on the specification of $\partial s/\partial z$ in the lower troposphere. From the work of SARACHIK (1985), and ALBRECHT *et al.* (1979), Figure 3 may often be the more appropriate picture.

Finally, it should be stated that while the above-described parameterization is readily implemented (though details may have to be altered for specific models; GELEYN *et al.*, 1982), the more important purpose of this note is simply to suggest that the development of physically based, phenomenological parameterizations of cumulonimbus convection need not be a difficult task.

Acknowledgements

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