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SOME REMARKS ON THE DYNAMICS OF THE JOVIAN ATMOSPHERES

RICHARD S. LINDZEN

Center for Meteorology and Physical Oceanography, M.I.T., USA

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This paper examines the nature and role of lunar tides on the outer planets. It is shown that lunar tidal forcing on the outer planets is much stronger than it is on Earth. It is also shown that the dynamics of large, rapidly rotating planets (i.e., Jupiter, Saturn, Uranus and Neptune) favors higher order meridional modes. We discuss how this might contribute to the banded structure of the outer planets.

KEY WORDS: Lunar tides, outer planets, zonal flows.

1. INTRODUCTION

This paper presents a set of ideas concerning the dynamics of the atmospheres of the outer (Jovian) planets. Relative to the Earth, the Jovian planets (Jupiter, Saturn, Uranus, and Neptune) are large and rapidly rotating. This is readily seen in Table 1, where physical parameters relevant to these planets' outer atmospheric layer are presented. The table is taken from Ingersoll (1990). Ingersoll, in reviewing what we have learned about these planets from recent space missions, points to the large zonal flows associated with the banded structures of the planets' atmospheres, as well as the bands themselves, as major features which remain unexplained. Ingersoll emphasizes that the observed winds on these planets are large compared to terrestrial values. Noting that solar forcing on the outer planets is very small compared to Earth, he concludes that there must be some mechanism whereby the outer planets utilize this energy with much greater efficiency than the Earth's atmosphere does.

In this paper, I propose to re-examine the problems posed by Ingersoll from a somewhat different perspective. In Section 2, I consider what, exactly, constitutes a large wind. It is quickly seen that the winds on the outer planets are small relative to the linear speed of rotation on these planets. In this sense, the winds on the outer planets are, in fact, smaller than those on Earth. In Section 3, we examine an alternative to solar forcing: namely, tidal forcing by the outer planets' moons. We will see that, in contrast to solar thermal forcing, this forcing is much larger on the outer planets than it is for the Earth (and its moon). In Section 4, we examine certain properties of tides on the outer planets, and in Section 5 we relate these properties to the problem of the mean winds and banded structure of the outer planets. Finally, in Section 6, we discuss some difficulties and possibilities involved in the complete evaluation of tides on the outer planets.

2. THE SCALING OF MEAN WINDS

By tracing identifiable features on some of the outer planets, it has been possible

Table 1 Properties of outer planets (after Ingersoll, 1990). For reference purposes the properties of the Earth are included. Also the linear speed of rotation at the equator (*rotation frequency* × *radius*) has been included.

Parameter	*Jupiter*	*Saturn*	*Uranus*	*Neptune*	*Earth*
Equatorial radius (10^3 km)	74.1	60.3	25.6	24.8	6.371
Rotation period (hours)	9.92	10.66	17.24	16.11	24
Equatorial gravity (ms^{-2})	22.9	9.1	8.8	11.1	9.8
Emission temperature (K)	124	95	59	59	255
Emission pressure (bars)	0.4	0.3	0.4	0.5	0.4
Scale height (km)	20	39	25	20	7.8
Sound speed (m/s)	810	705	560	560	410
Linear rotation speed (m/s)	13 037	9873	2592	2687	463

to estimate the magnitudes of winds. Estimates for Jupiter are on the order of 120 m/s; for Saturn are on the order of 450 m/s; for Uranus on the order of 200 m/s; and for Neptune on the order of 400 m/s. Such magnitudes seem large compared to winds in the terrestrial troposphere—although even there winds of the order of 100 m/s are found. Winds of the order of twice this may be found in the thermosphere. It should be noted that observations of winds for the outer planets are certainly not for levels near the surface—especially since for these planets there are no well defined solid surfaces. The outer planets are essentially gaseous spheres, though condensation almost certainly takes place near their centers. On Earth, only winds near the surface have to oppose friction—and are therefore limited in magnitude by energetic considerations. Above the surface, the main constraint is the availability of angular momentum (Schneider, 1977). The latter may be presumed to be the major constraint on the outer planets, and given the size and rotation rate of these planets, none of the observed winds approach the limit imposed by this constraint.

In this paper we will consider the forcing of mean winds by Reynolds stresses arising from lunar tides. Crudely speaking, such effects depend significantly on mean winds being smaller than the planet's linear speed of rotation, ωa, where ω is the planet's rotation rate, and a is the planet's radius where the winds are observed. As may readily be seen in Table 1, the winds on the outer planets are virtually negligible by this criterion.

3. LUNAR FORCING

Chapman and Lindzen (1979) derive the following expression for the lunar contribution to the geopotential on a planet,

$$\Omega_{\text{tidal}} = -\frac{3}{2}\frac{\gamma M a^2}{D^3}\left(\frac{1}{3} - \cos^2\Theta\right), \tag{1}$$

where γ is the gravitation constant and M is the planet's mass.

The relevant geometry is shown in Figure 1. A list of the relevant moons on the outer planets (as well as the Earth for comparison purposes) is given in Table 2.

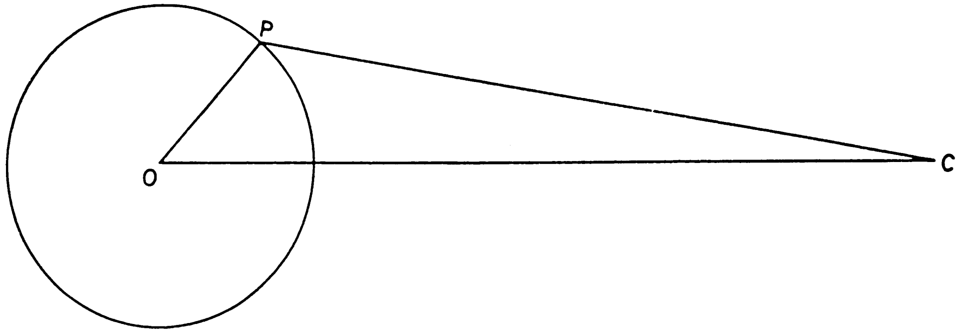

Figure 1 Geometry for calculation of tidal potentials; O is the planet's center; C is the center of the disturbing body. $D=OC$; $a=OP$; $\Theta=\angle POC$.

Table 2 Relevant properties of the moons on Earth and the outer planets (from Burns, 1977). The magnitude of the most important contribution to the tidal potential on each planet (relative to the Earth's moon) is highlighted. See text for details.

Planet	*Moon*	d/a	$d(10^6\,m)$	$m(10^{20}\,kg)$	Potential	Ratio to Earth
Earth	Moon	60.3	384.4	734.9	5.26E−04	1
Jupiter	Io	5.905	421.6	894	6.08E−02	**1.16E+02**
	Europa	9.397	670.9	480	8.10E−03	1.54E+01
	Ganymede	14.99	1070	1482	6.16E−03	1.17E+01
	Callisto	26.37	1883	1076.6	8.22E−04	1.56E+00
Saturn	Rhea	8.736	527.04	24.9	6.19E−04	1.18E+00
	Titan	20.25	1221.85	1346	2.69E−03	**5.11E+00**
	Iapetus	59.03	3561.3	18.8	1.51E−06	2.88E−03
Neptune	Triton	14	354.3	1300	1.87E−02	**3.56E+01**
Uranus	Miranda	4.95	129.8	0.71	2.23E−04	4.25E−01
	Ariel	7.3	191.2	14.4	1.41E−03	**2.69E+00**
	Umbriel	10.15	266	11.8	4.31E−04	8.19E−01
	Titania	16.64	435.8	34.3	2.84E−04	5.41E−01
	Oberon	22.4	582.6	28.7	9.82E−05	1.87E−01

For purposes of this Section, we are only interested in the factor, Ma^2/D^3 in (1), evaluated at the planetary radius. This is also listed in Table 2 (under "Potential"). We see that for each of the outer planets, there is a single moon whose contribution to the tidal potential on the relevant planet is clearly dominant. Moreover, on each of the outer planets, the tidal geopotential is clearly greater than it is for the Earth. For Jupiter and Neptune, the tidal potential is much greater than it is for the Earth. For Saturn, the tidal potential is five times greater than it is on Earth, and even on Uranus, the tidal potential due to the small moon, Ariel, is almost three times greater than the lunar potential on Earth.

4. TIDAL PROPERTIES ON THE OUTER PLANETS

At least in the near surface, stably stratified portions of the outer planets, the equations for tidal perturbations are the same as those for the Earth's atmosphere. These are given in detail in Chapman and Lindzen (1979), and will not be rederived here. Basically, the terrestrial approach is to assume that tides are linearized perturbations on a static basic state. It proves covenient to reduce the equations to an equation for a single variable,

$$y = e^{-x/2} \frac{1}{\gamma p_0} \frac{Dp}{Dt}, \tag{2}$$

where p = perturbation pressure, p_0 is the basic state pressure, $\gamma = c_p/c_v$, D/Dt is the linearized advective derivative, and x is the height in scale heights.* All other fields are readily derived from y. Solutions of the form,

$$y = y(\theta, x)\, e^{i(\sigma t + s\phi)}, \tag{3}$$

are assumed. The colatitude (θ) and height (x)-dependence of y are separable with solutions of the form,

$$y(\theta, z) = \sum_n y_n(x)\Theta_n(\theta).$$

The Θ_n are solutions of Laplace's Tidal Equation. For our present purposes we need simply note that Laplace's Tidal Equation (subject to the boundary condition that the solutions remain bounded at the poles) defines an eigenfunction–eigenvalue problem. The operator in Laplace's Tidal Equation is a second order differential operator in θ, with parametric dependence on s, and on a parameter f which is simply one-half the ratio of the oscillation frequency, σ, to the rotational frequency of the planet, ω (i.e., $f = \frac{1}{2}\sigma/\omega$). The set of eigenvalues will be designated $\{e_n\}$. In the traditional way, the eigenvalues serve as separation constants in the equation for x-dependence (known as the Vertical Structure Equation). Before turning to the Vertical Structure Equation, it is important to note that Laplace's Tidal Equation has *no explicit dependence* on the characteristics of particular planets. For example, all semidiurnal tides (i.e., $\sigma = 2\omega$) with zonal wavenumber 2 will have the same eigenfunctions (known as Hough Functions) and eigenvalues —regardless of planet. The first few symmetric semidiurnal Hough Functions are shown in Figure 2. The first twenty eigenvalues corresponding to symmetric semidiurnal modes† are given in Table 3.

The Vertical Structure Equation is simply,

$$\frac{d^2 y_n}{dx^2} + \left[\frac{\kappa H + (dH/dx)}{h_n} - \frac{1}{4}\right] y_n = 0, \tag{4}$$

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*$x = \int_{z_0}^{z} (1/H)\, dz$, where $H = RT_0/g$.

†We emphasize semidiurnal modes since the primary gravitational forcing is approximately semidiurnal.

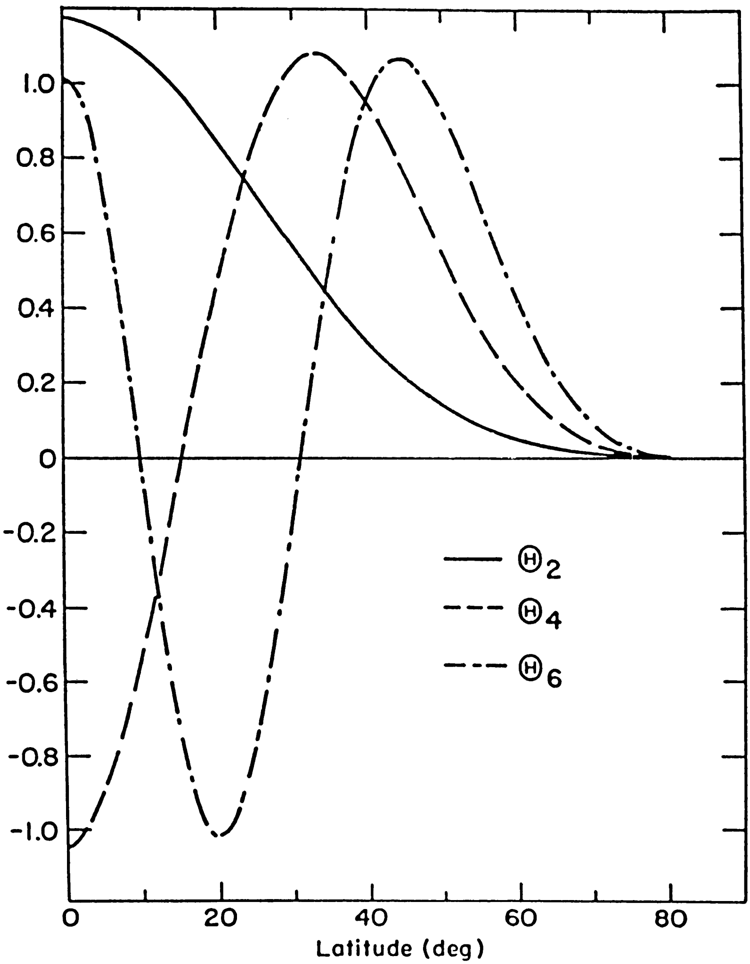

Figure 2 The first three symmetric semidiurnal Hough Functions.

where $\kappa=(\gamma-1)/\gamma$. The quantity,

$$\kappa H+\frac{dH}{dx}=\frac{RH}{g}\left(\frac{dT}{dz}+\frac{g}{c_p}\right),$$

is clearly proportional to the atmosphere's static stability. The separation constant appears in (4) as h_n (known as the Equivalent Depth), which is related to e_n as follows:

$$h_n=\frac{4\omega^2a^2}{g}e_n. \tag{5}$$

Table 3 Eigenvalues of Laplace's Tidal Equation for the first 20 symmetrical semidiurnal tidal modes. Also shown are the Equivalent Depths appropriate to the Earth and the outer planets.

n	e_n	*Equivalent Depths (km)*				
		Earth	*Jupiter*	*Saturn*	*Uranus*	*Neptune*
2	8.99E−02	7.87	2667.83	3849.99	274.35	233.76
4	2.42E−02	2.12	718.17	1036.40	73.85	62.93
6	1.10E−02	0.96	325.98	470.43	33.52	28.56
8	6.23E−03	0.55	185.05	267.05	19.03	16.21
10	4.01E−03	0.35	119.02	171.76	12.24	10.43
12	2.79E−03	0.24	82.89	119.62	8.52	7.26
14	2.06E−03	0.18	61.01	88.05	6.27	5.35
16	1.58E−03	0.14	46.76	67.48	4.81	4.10
18	1.25E−03	0.11	36.99	53.38	3.80	3.24
20	1.01E−03	0.09	29.99	43.27	3.08	2.63
22	8.35E−04	0.07	24.79	35.77	2.55	2.17
24	7.02E−04	0.06	20.84	30.07	2.14	1.83
26	5.98E−04	0.05	17.76	25.63	1.83	1.56
28	5.16E−04	0.05	15.32	22.10	1.58	1.34
30	4.50E−04	0.04	13.35	19.26	1.37	1.17
32	3.95E−04	0.03	11.73	16.93	1.21	1.03
34	3.50E−04	0.03	10.39	15.00	1.07	0.91
36	3.12E−04	0.03	9.27	13.38	0.95	0.81
38	2.80E−04	0.02	8.32	12.01	0.86	0.73
40	2.53E−04	0.02	7.51	10.84	0.77	0.66

Finally, we have included neither thermal nor gravitational forcing in (4). We are ignoring the former; the latter is concentrated below the near surface layers of the outer planets' atmospheres; on the Earth, gravitational forcing appears in the surface boundary condition. Note that the vertical structure of a given mode depends on h_n, and that, in contrast to e_n, the Equivalent Depth depends profoundly on planetary characteristics. Table 3 shows the semidiurnal values of h_n for the Earth, Jupiter, Saturn, Uranus, and Neptune.

For simplicity, we will consider atmospheres with isothermal basic states. From (4) we see that the vertical structure of a mode depends on the ratio $\kappa H/h_n$; in particular, we see that when $\kappa H/h_n$ is less than 1/4, the mode is exponentially trapped, while then $\kappa H/h_n$ is greater than 1/4, the mode propagates vertically with a wavelength that decreases as h_n decreases. Finally, reference to Table 1 shows that, in contrast to the quantity $4\omega^2 a^2/g$, the scale heights on the outer planets differ from the scale height on Earth by factors of only 2–5. The implications of this are profound. Table 4 shows the quantity $4\kappa H/h_n$ for the first 20 symmetric semidiurnal modes* on Earth, Jupiter, Saturn, Uranus, and Neptune. We see that all the modes essentially propagate on Earth, while on Uranus and Neptune, propagation doesn't begin until $n \geqq 8$ (i.e., the fourth symmetric mode), on Saturn until $n \geqq 20$ (i.e., the tenth symmetric mode), and on Jupiter until $n \geqq 24$ (i.e., the

*The model indices, n, follow the convention for spherical harmonics, i.e., for the first symmetric mode, $n=s=2$, for the second mode, $n=s+2=4$, etc.

Table 4 The quantity, $4\kappa H/h_n$, for the first 20 symmetric semidiurnal modes on the Earth and the outer planets. When this quantity is less than one, the modes are vertically trapped; when it is greater than one, the modes propagate vertically. The first mode on each planet which is able to propagate is highlighted.

n	*Earth*	*Jupiter*	*Saturn*	*Uranus*	*Neptune*
2	**1.1E+00**	8.57E−03	1.16E−02	1.04E−01	9.78E−02
4	4.21E+00	3.18E−02	4.30E−02	3.87E−01	3.63E−01
6	9.27E+00	7.01E−02	9.47E−02	8.52E−01	8.00E−01
8	1.63E+01	1.24E−01	1.67E−01	**1.5E+00**	**1.4E+00**
10	2.54E+01	1.92E−01	2.59E−01	2.33E+00	2.19E+00
12	3.64E+01	2.76E−01	3.73E−01	3.35E+00	3.15E+00
14	4.95E+01	3.75E−01	5.06E−01	4.55E+00	4.28E+00
16	6.46E+01	4.89E−01	6.61E−01	5.94E+00	5.58E+00
18	8.17E+01	6.18E−01	8.35E−01	7.51E+00	7.05E+00
20	1.01E+02	7.62E−01	**1.0E+00**	9.27E+00	8.70E+01
22	1.22E+02	9.22E−01	1.25E+00	1.12E+01	1.05E+01
24	1.45E+02	**1.1E+00**	1.48E+00	1.33E+01	1.25E+01
26	1.70E+02	1.29E+00	1.74E+00	1.56E+01	1.47E+01
28	1.97E+02	1.49E+00	2.02E+00	1.81E+01	1.70E+01
30	2.26E+02	1.71E+00	2.31E+00	2.08E+01	1.95E+01
32	2.57E+02	1.95E+00	2.63E+00	2.37E+01	2.22E+01
34	2.91E+02	2.20E+00	2.97E+00	2.67E+01	2.51E+01
36	3.26E+02	2.47E+00	3.33E+00	3.00E+01	2.81E+01
38	3.63E+02	2.75E+00	3.71E+00	3.34E+01	3.13E+01
40	4.02E+02	3.04E+00	4.11E+00	3.70E+01	3.47E+01

twelfth symmetric mode). The numerical relation between these numbers and the number of bands on each of the outer planets is extremely suggestive. The relation was first noted by Lindzen (1967), but has not, until now, been exploited.

5. TIDES AND MEAN FLOWS

The question, quite obviously arises, as to how tidal oscillations traveling around a planet with zonal wavenumber 2 could produce patterns of steady banding with associated zonal streaming. Several likely possibilities exist, and are currently being explored. For example, most of the tidal excitation is likely to go into the gravest meridional mode ($n=2$). However, according to Table 3 and (4) such a mode on Jupiter is likely to be associated with no amplitude growth with height.* On the other hand, the propagating modes (associated with larger meridional mode numbers) will grow as $\exp(x/2)$. Thus, even if these modes are more weakly excited, there will be some height where the tidal response in these modes will be as large as the gravest mode. This level might still be below the level we are observing. Quadratic terms in the tidal fields associated with both the gravest mode and the higher order modes, will have non-zero time and zonal averages

*The solution to (4) will go as $\exp(-x/2)$ (because $\kappa H/h_n \sim 0$) which will just balance the $\exp(x/2)$ growth in (2).

which in the form of Eliassen–Palm fluxes (Eliassen and Palm, 1961; Boyd, 1976; Andrews and McIntyre, 1978) will redistribute both zonal angular momentum and heat on the scale of the higher order modes (Fels and Lindzen, 1974). Such wave–mean flow interactions are facilitated by the presence of damping, and, indeed, the tidal responses, if strong enough, might contribute to such damping by the production of turbulence. A suggestive observation, in this regard, is that the planets with the strongest tidal forcing, Jupiter and Neptune (Table 2) have turbulent great spots, whereas Saturn and Uranus have smoother patterns.

6. THEORETICAL PROBLEMS AND POSSIBILITIES

The above discussion was based on classical tidal theory developed for the Earth's atmosphere, and, at least as far as we have gone, the theory appears to be appropriate. However, the nature of gravitational tidal forcing on the outer planets is very different from what it is on Earth. The Earth's atmosphere is a shallow, stably stratified gaseous envelope resting on a solid surface. As shown in Chapman and Lindzen (1979), gravitational forcing appears primarily as an inhomogeneous term in the lower boundary condition. No such boundary exists on the outer planets. On the outer planets, we must calculate the distributed effect of the tidal forcing acting throughout the mass of what are in essence gaseous spheres. Feature like self-gravitation may eventually have to be considered. More immediately, the fact that the bulk of the planet seems to consist in a well-mixed, convectively neutral gas, means that in these regions the hydrostatic approximation may be inappropriate.* In a non-hydrostatic atmosphere the response to gravitational forcing may be considerably greater than the simple equilibrium response. According to calculations in progress, there may also be enhanced response at smaller horizontal scales. All of this may lead to tidal forcing being more important than even the results in Table 2 suggest. Finally, at least in the well mixed interior of the outer planets, it is necessary to consider a fully spherical geometry rather than assume a shallow atmosphere. Fortunately, the fact that this region is well mixed (i.e., isentropic) simplifies matters. Most importantly, the separability of radial and meridional dependencies is retained. In the outer stably stratified layers, it remains still possible to assume a shallow fluid.

Finally, it must be noted that the orbital orientation and the rotation periods of the moons on the outer planets are such that the forcing period can be significantly different from semidiurnal. This is a simple technical issue which does not alter any of the present discussion.

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*The condition for the appropriateness of the hydrostatic approximation is that the oscillation period be much greater than the Brunt–Vaisala period (viz. Lindzen, 1990). For the Earth's atmosphere, the Brunt–Vaisala period is approximately 5 minutes, which is certainly shorter than tidal periods. However, in a convectively neutral atmosphere, the Brunt–Vaisala period is infinite—a limit which is hard to exceed.

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