Reprinted from JOURNAL OF THE ATMOSPHERIC SCIENCES, Vol. 50, No. 8, 15 April 1993
American Meteorological Society

# Baroclinic Neutrality and the Tropopause

RICHARD S. LINDZEN

*Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts*

25 November 1991 and 5 June 1992

ABSTRACT

It is noted that the atmosphere has a baroclinically neutral state that permits strong temperature gradients at the ground. In this state, the concentrated potential vorticity gradient at the ground is separated from the concentrated potential vorticity gradient at the tropopause by a region where the potential vorticity gradient is zero. The resulting basic state is analogous to the basic state in the Eady problem. A minimum meridional wavenumber is established by the width of the subtropical jet, and neutrality is established by moving the tropopause to a sufficient height so that the total wavenumber corresponds to the short-wave cutoff in the Eady problem. The resulting height is approximately the observed tropopause height.

## 1. Introduction

The question of the neutrality of a baroclinically unstable basic state, and the related question of baroclinic equilibration, have been important questions in dynamic meteorology for many years (Pocinki 1955; Thompson 1960; Charney and Stern 1962; Stone 1978; Lindzen and Farrell 1980; Cehelsky and Tung 1991; Stone and Branscome 1992). The overall question was whether baroclinically unstable eddies acted to bring the atmosphere to a neutral state. All of the above studies focused directly or implicitly on the condition derived by Charney and Stern (1962). This condition is simply that in order for a flow to be unstable, there must be some surface where the meridional gradient of pseudo–potential vorticity changes sign.

In the traditional Charney (1947) problem for baroclinic instability, this surface is immediately above the ground. As shown in Bretherton (1966) and in Lindzen and Tung (1978), the pseudo–potential vorticity gradient in the Charney problem is positive everyplace above the ground, but at the ground there is a delta-function contribution to the pseudo–potential vorticity that is negative. This delta-function contribution is equivalent to a layer of infinite curvature in which the shear in the zonal wind immediately above the ground is brought to zero at the ground. As noted by Charney and Stern (1962), the existence of shear (and, by the thermal wind relation, a meridional temperature gradient) at the ground is necessary to the instability of the flow. As Lindzen and Farrell (1980) noted, the simple elimination of temperature gradients at the ground is not sufficient to neutralize the flow. Rather, one must stretch out the delta function in curvature sufficiently to effect the transition from zero shear at the ground to the interior shear through a layer with zero pseudo–potential vorticity gradient. The neutral basic state clearly is different from what actually occurs in nature; however, the vertically averaged meridional temperature gradient is close to the actual vertically averaged meridional temperature gradient.

In the two-level model there is a critical shear for baroclinic instability, and Pocinki (1955) and Stone (1978) attempted to relate this shear (or equivalently, meridional temperature gradient) to observed values. A priori, this may seem a more plausible approach to baroclinic adjustment than the adjustment based on the Charney problem. However, as Lindzen (1990) notes, the zonal wind profile in the two-level problem is actually equivalent to a profile with the stated shear at the middle of the two-level model and zero shear at top and bottom. The critical shear corresponds to that value for which the curvature of the flow in the bottom layer is such as to produce a zero gradient of pseudo–potential vorticity. The neutralization is thus a special case of the neutralization in the Charney problem, where, however, the stretching of the delta-function contribution to the pseudo–potential vorticity is required to occur (rather artificially) over the thickness of the bottom layer in the two-level model.

In both the above examples, the neutral state is so different from the actual state of the atmosphere, where meridional temperature gradients at the ground are actually larger than they are in the midtroposphere, that

*Corresponding author address:* Richard S. Lindzen, Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Building 54, Room 1720, Cambridge, MA 02139.

one may reasonably question whether baroclinic neutrality is actually relevant. The main point of the present note is that a baroclinically neutral state exists that is different from the above states, and that this new neutral state, which permits temperature gradients at the surface, may, in fact, be relevant to the atmosphere's actual state. The new neutral state, moreover, determines a tropopause height that turns out to be approximately the height of the actual midlatitude tropopause.

## 2. Geometric neutrality and the Eady problem

The Eady problem (Eady 1949) deals with the baroclinic instability of a Boussinesq fluid on an $f$ plane. The basic flow consists in a simple linear shear. Since $\beta = 0$, there is no potential vorticity gradient in this flow. The instability arises from the delta-function contributions to the potential vorticity gradient at the top and bottom boundaries. The Eady problem usually considers a meridionally unbounded fluid. Just as in the Charney problem, there is no critical shear. However, in contrast to the Charney problem, there is a short-wave cutoff in the Eady problem. The expression for the short-wave cutoff is

$$k^2 + l^2 \leqslant (2.3394)^2 \frac{f^2}{N^2h^2} = \mu_c^2, \qquad (1)$$

where $f$ is the Coriolis parameter, $N$ the Brunt–Väisälä frequency, and $h$ the depth of the fluid. Note that the relevant wavenumber is the total wavenumber; that is, $(k^2 + l^2)^{1/2}$, where $k$ is the zonal wavenumber and $l$ is the meridional wavenumber. The Eady problem was considered germane to the baroclinic instability of flows in a rotating annulus (Lorenz 1962). However, in the annulus there are sidewalls separated by a distance, $L$, that imposes a minimum meridional wavenumber, $1 = \pi/L$. By shrinking $L$ until $\pi/L \geqslant \mu_c$, one can *geometrically* neutralize the fluid.

It is readily seen that the Eady condition for neutrality is also appropriate to a fluid without a rigid upper boundary, where a vertical depth exists above the lower boundary where the pseudo–potential vorticity gradient (rather than $\beta$ as in the Eady problem) is zero. The point, quite simply, is that in both problems, disturbances decay exponentially in the $q_y = 0$ region ($q$ refers to the zonally averaged pseudo–potential vorticity) with an $e$-folding rate proportional to the horizontal wavenumber. The Eady condition only implies that neutrality occurs when the minimum horizontal wavenumber is large enough so that disturbances in the upper and lower $q_y \neq 0$ regions can no longer "see" each other. That said, it must be added that with rigid boundaries, the existence of free waves associated with the delta functions in $q_y$ is guaranteed. The same is not necessarily so in the absence of a boundary. Thus, in the absence of a rigid upper boundary, the Eady condition for neutrality may be too strict. However, observationally (Sanders 1988) and theoretically (Rivest 1990; Snyder and Lindzen 1988), there is ample basis for the existence of such upper-level waves in the real atmosphere.

In the atmosphere there are also meridional constraints imposed by the finite extent of the earth and by the meridional scale of the basic-state jet. Insofar as we shall consider planetary-scale instability on the zonally averaged flow, the basic-state jet will essentially be the subtropical jet. This would also be appropriate for the *conceptual* problem of a zonally symmetric earth, where baroclinic instability takes place on a symmetric Hadley circulation whose angular momentum flux maintains the subtropical jet (Schneider and Lindzen 1977; Schneider 1977; Hou and Lindzen 1992). To be sure, regional instability exists for storm paths associated with high-latitude jets, but the very regional nature of these instabilities suggests a limited role for them in the general circulation. The constraint imposed by the jet is more important than that imposed by the scale of the earth. In the following section we will review the result of Ioannou and Lindzen (1986) wherein a minimum meridional wavenumber is established by the width of the basic subtropical jet. Neutrality in the atmosphere can be established by eliminating gradients of pseudo–potential vorticity over a region of depth, $h$, such that the minimum meridional wavenumber corresponds to $\mu_c$. As we will see in the following section, this depth corresponds closely to the tropopause height at 30°.

## 3. Minimum meridional wavenumber

Ioannou and Lindzen (1986) [see also Ioannou and Lindzen (1990), for correction of typographic error in the former] considered the effect of the meridional jet structure on baroclinic instability. For the Eady problem, they considered jets of the form

$$u = mz/\left(1 + \frac{y^2}{L^2}\right)^{1/2} \qquad (2)$$

where $m$ is shear at $y = 0$, $y$ is meridional distance, $z$ height, and $L$ a measure of the jet width. For a characteristic winter subtropical jet, $L = 0.15r$, where $r$ is the radius of the earth. From Eq. (3.7) in Ioannou and Lindzen (1986), we have that the jet imposes a meridional wavenumber, $1 = 1/L$, on baroclinic instabilities so that Eq. (1) above becomes

$$k^2 + \frac{1}{L^2} = (2.3394)^2 \frac{f^2}{N^2h^2} = \mu_c^2 \qquad (3)$$

for instability. We will take

$$k = \frac{s}{r \cos\phi}, \qquad (4)$$

where $s$ is the zonal wavenumber. The latitude at the jet core, $\phi$, is taken to be 30°, and (4) becomes

$$k^2 = \frac{4s^2}{3r^2} \qquad (5)$$

and

$$k^2 + l^2 = (1.333 \cdots s^2 + 43.77)\frac{1}{r^2}. \quad (6)$$

Note that the contribution of the meridional confinement is comparable to wavenumber 6.

Given the meridional confinement imposed by the observed jet structure, the only disposable parameter available in (3) for establishing neutrality is $h$. Instability now requires

$$h \leqslant \frac{r}{\sqrt{1.333 \cdots s^2 + 43.77}} \times 2.3994 \frac{f}{N}. \quad (7)$$

The largest neutral value for $h$ will correspond to $s = 1$ (recall that growth rate is equal to $kc_i$, where $c_i$ is the imaginary part of the phase speed, and that the growth rate must, therefore, be zero for $k = s = 0$). Taking $N = 1.05 \times 10^{-2}\ \mathrm{s}^{-1}$, one gets $h = 15.9$ km for $s = 1$. Given the strong forcing at other wavenumbers, one might conceivably consider other values of $s$. For $s = 3$, $h = 14.3$ km; for $s = 4$, $h = 13.2$ km; and for $s = 5$, $h = 12.15$ km. Given the simplicity of the Eady model, these values can only be regarded as approximate. In particular, they assume a dissipationless atmosphere. Presumably, lower values will be appropriate when dissipation is included. The values, however, are clearly in the neighborhood of the actual tropopause height. This not only offers support to the notion that the atmosphere *tends* toward baroclinic neutrality, but also offers an explanation of what accounts for the extratropical tropopause. The traditional radiative–convective approach is only applicable within the Hadley domain where vertical cumulonimbus convection is a dominant process (Schneider 1977; Sarachik 1985). Outside the tropics, the basis for the radiative–convective approach is absent. The boundary between these regimes appears in the almost discontinuous jump in the tropopause that occurs as one passes 30°. Held (1982) also suggested that the midlatitude tropopause was determined by quasigeostrophic dynamics. Of necessity, the dimensional form of his relation for the tropopause height was the same as obtained here.

## 4. Remarks

The preceding sections describe a baroclinically neutral state consisting in a broad region without pseudo-potential vorticity gradients, bounded above by the tropopause with concentrated gradients of pseudo-potential vorticity and below by the surface, immediately above which are also concentrated gradients of pseudo-potential vorticity. The question remains as to whether the atmosphere actually attains such a state. It has been noted for some time that there are concentrations of such gradients at the tropopause and near the surface with much weaker gradients in-between (Hoskins et al. 1985; Hoskins 1991). Current studies continue to confirm this picture (Morgan, personal communication; Hoskins, personal communication), although isolated regional exceptions exist as storm paths. As noted by Lindzen and Wirth (1990), it is not easy to calculate small potential vorticity gradients from observations. Small modifications of zonal wind—within observational uncertainty—can make significant differences in calculated potential vorticity gradients. The small interior potential vorticity gradients are compatible with the possibility of zero gradients. To be sure, even should baroclinic neutralization prove relevant to the general circulation, since the agency of potential neutralization is baroclinic instability, some degree of instability must remain (Schoeberl and Lindzen 1984).

Another question of some importance is why, if the neutral states based on the Charney–Stern theorem are inappropriate, do the results based on such a state produce nearly the observed temperature distribution. Here one must note that the agreement with observations obtained by both Lindzen and Farrell (1980) and Stone (1978) was hardly precise. In each case, moreover, a certain amount of adjustment was possible. However, more important is that the neutralized states in the earlier studies also involved eliminating potential vorticity gradients over deep layers: over half the atmosphere in the case of Stone (1978) and commonly over almost the entire troposphere in the case of Lindzen and Farrell (1980). Thus, one may expect a measure of similarity with the neutral states described in this note. It is worth noting, in this regard, that Schoeberl and Lindzen (1984), Nielsen and Schoeberl (1984), and Schoeberl and Nielsen (1986), having noted that the Charney problem was mathematically homomorphic to the barotropic instability of a triangular jet (Lindzen et al. 1983), investigated the nonlinear equilibration of such a jet and found that the equilibration used by Lindzen and Farrell (1980) was appropriate. However, the barotropic problem is intrinsically two-dimensional and does not allow a counterpart to meridional confinement.

Finally, it should be noted that the present work suggests that baroclinic instabilities with planetary-scale zonal wavenumbers may play a major role in climate since meridional confinement by the subtropical jet already produces a wavenumber close to that associated with maximum growth rate. Relatedly, the present work suggests that regional cyclogenesis may well be associated with regional depressions of the tropopause height.

In the present note, both the observed width of the subtropical jet and a characteristic static stability were used in order to determine a tropopause height consistent with baroclinic neutrality. In a more complete treatment, presumably, these quantities must all be internally determined.

*Acknowledgments.* This work has been supported by NASA Grant NAGW-525 and NSF Grant 8520354-

ATM. Useful suggestions by L. Branscome, M. Mak, and I. Held are gratefully acknowledged.

## REFERENCES

Bretherton, F. P., 1966: Critical layer instability in baroclinic flows. *Quart. J. Roy. Meteor. Soc.*, **95**, 213–243.

Cehelsky, P., and K.-K. Tung, 1991: Nonlinear baroclinic adjustment. *J. Atmos. Sci.*, **48**, 1930–1947.

Charney, J. G., 1947: The dynamics of long waves in a baroclinic westerly current. *J. Meteor.*, **4**, 135–163.

——, and M. E. Stern, 1962: On the instability of internal baroclinic jets in a rotating atmosphere. *J. Atmos. Sci.*, **19**, 159–172.

Eady, E. T., 1949: Long waves and cyclone waves. *Tellus*, **1**, 33–52.

Held, I. M., 1982: On the height of the tropopause and the static stability of the troposphere. *J. Atmos. Sci.*, **39**, 412–417.

Hoskins, B. J., 1991: Towards a PV-$\theta$ view of the general circulation. *Tellus*, **43AB**, 27–35.

——, M. E. McIntyre, and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.*, **111**, 877–946.

Hou, A. Y., and R. S. Lindzen, 1992: Intensification of the Hadley circulation due to concentrated heating. *J. Atmos. Sci.*, in press.

Ioannou, P., and R. S. Lindzen, 1986: Baroclinic instability in the presence of barotropic jets. *J. Atmos. Sci.*, **43**, 2999–3014.

——, and ——, 1990: W.K.B.J. approximation of the stability of a frontal mean state. *J. Atmos. Sci.*, **47**, 2825–2831.

Lindzen, R. S., 1990: *Dynamics in Atmospheric Physics*, Cambridge University Press, 310 pp.

——, and K. K. Tung, 1978: Wave overreflection and shear instability. *J. Atmos. Sci.*, **35**, 1626–1632.

——, and B. Farrell, 1980: The role of polar regions in global climate, and the parameterization of global heat transport. *Mon. Wea. Rev.*, **108**, 2064–2079.

——, and V. Wirth, 1990: Zero potential vorticity gradient basic states in the neighborhood of the equator. In *Proc. 14th Annual Climate Diagnostics Workshop*. Springfield, NTIS, U.S. Dept. of Commerce, 256–259.

——, A. J. Rosenthal, and B. Farrell, 1983: Charney's problem for baroclinic instability applied to barotropic instability. *J. Atmos. Sci.*, **40**, 1029–1034.

Lorenz, E., 1962: Simplified dynamic equations applied to the rotating-basin equations. *J. Atmos. Sci.*, **19**, 39–51.

Nielsen, J. E., and M. R. Schoeberl, 1984: A numerical simulation of barotropic instability. Part II: Wave–wave interaction. *J. Atmos. Sci.*, **41**, 2869–2881.

Pocinki, L., 1955: Stability of a simple baroclinic flow with horizontal shear. Air Force Cambridge Research Center, Res. Paper, No. 38, 78 pp.

Rivest, C., 1990: Upper level waves of synoptic scale. D.Sc. Thesis, Massachusetts Institute of Technology, 166 pp. [Available from MIT libraries.]

Sanders, F., 1988: Life history of mobile troughs in the upper westerlies. *Mon. Wea. Rev.*, **116**, 2629–2648.

Sarachik, E. S., 1985: A simple theory for the vertical structure of the tropical atmosphere. *Pure Appl. Geophys.*, **123**, 261–271.

Schneider, E. K., 1977: Axially symmetric steady-state models of the basic state for instability and climate studies. Part II: Nonlinear calculations. *J. Atmos. Sci.*, **34**, 280–296.

——, and R. S. Lindzen, 1977: Axially symmetric steady state models of the basic state of instability and climate studies. Part I: Linearized calculations. *J. Atmos. Sci.*, **34**, 253–279.

Schoeberl, M. R., and R. S. Lindzen, 1984: A numerical simulation of barotropic instability including wave–mean flow interaction. *J. Atmos. Sci.*, **41**, 1368–1379.

——, and J. E. Nielsen, 1986: A numerical simulation of barotropic instability. Part III: Wave–wave interaction in the presence of dissipation. *J. Atmos. Sci.*, **43**, 1045–1050.

Snyder, C., and R. S. Lindzen, 1988: Upper-level baroclinic instability. *J. Atmos. Sci.*, **45**, 2446–2459.

Stone, P. H., 1978: Baroclinic adjustment. *J. Atmos. Sci.*, **35**, 561–571.

——, and L. Branscome, 1992: Diabatically forced nearly inviscid eddy regimes. *J. Atmos. Sci.*, **49**, 355–367.

Thompson, P. D., 1960: *Numerical Weather Analysis and Prediction*, Macmillan, 170 pp.