Wave Structure of the Tropical Stratosphere over the Marshall Islands Area during 1 April–1 July 1958

R. S. Lindzen and C.-Y. Tsay

Center for Earth and Planetary Physics, Harvard University, Cambridge, Mass. 02138
(Manuscript received 27 August 1974, in revised form 20 June 1975)

ABSTRACT

Stratospheric wind data for the Marshall Islands region during 1 April–1 July, 1958, are analyzed for contributions in the 4–6 day period range. It is shown that, excluding waves with vertical wavelengths <2 km from the data, 4–6 day power in the equatorial stratosphere during this period must be due to some combination of Kelvin, mixed gravity-Rossby, and n=1 Rossby waves with zonal wavenumbers <7–10. It is further shown that a theoretical model wherein each of the above three wave types is associated with a zonal wavenumber of either 3 or 4 is consistent with the data. The resulting observationally calibrated model is used to calculate the acceleration of the mean flow by wave absorption, which is then compared with the observed acceleration. In general, the waves satisfactorily account for accelerations above 23 km. Below 23 km there is a need for an additional source of easterly momentum with a specific vertical distribution which we show could be provided by an n=1 easterly gravity wave whose vertical wavelength, however, would be too short for the wave to be seen in radiosonde data. We also show that if a mean flow together with 4–6 day waves is spectral analyzed, there will be power at periods >4–6 days due to the acceleration of the mean flow by waves, and there may also be power at periods <4–6 days due to the modification of the waves by the changing mean flow. We finally examine what the theory suggests is happening at levels where wave absorption is altering the mean flow, and show some of the difficulties in relating such behavior to data averaged over a three-month period.

1. Introduction

Since the data analyses of Yanai and Maruyama (1966), Maruyama (1967), and Wallace and Kousky (1968), it has been a common belief that during the descending westerly phase of the quasi-biennial cycle the equatorial lower stratosphere is dominated by Kelvin waves of 12–20 day period while during the descending easterly phase, easterly mixed gravity-Rossby waves of 4–6 day period are dominant [viz. Wallace (1973) for a review]. Temperature and zonal velocity are antisymmetric about the equator for the latter wave. From the work of Maruyama (1967), however, it is clear that the mixed gravity-Rossby wave is not the only 4–6 day wave present during the descending easterly phase of the quasi-biennial cycle. Maruyama found that the power in the u field for 4–6 day periods was as strong at Kapingamarangi as at other stations. Now Kapingamarangi (01°04'N) is essentially at the equator and for the mixed gravity-Rossby wave, u=0 at the equator. This point was discussed by Lindzen (1972); the question is what other waves are present in the 4–6 day period range. A priori, we expect other waves to be present in view of the believed origin for the mixed gravity-Rossby wave, namely, the ubiquitous 4–6 day oscillations in rainfall in the tropical troposphere. Such forcing in the troposphere has neither the symmetry nor the shape of the stratospheric mixed gravity-Rossby wave, and hence, such forcing must excite a variety of both symmetric and antisymmetric waves.

In examining the data it is convenient to separate oscillations which are antisymmetric about the equator from those which are symmetric. Theoretically, such a separation is desirable since the lowest order symmetric and antisymmetric modes have markedly different dispersive properties (Lindzen and Matsuno, 1968). There are two obvious ways in which waves with different symmetries may be separated:

1) One may consider pairs of stations at approximately the same longitude and equidistant (north and south) from the equator. Symmetric and antisymmetric modes may, in principle, be separated by considering sums and differences of fields at each of the station pairs. This approach has been used by Yanai and Murakami (1970). However, it has several drawbacks associated with the fact that suitable station pairs are 7°–10° away from the equator. First, as we shall show, in Section 3, the relevant waves should, at various levels, be more closely confined to the equator than this. Second, although modal symmetries should be meaningful near the equator (Holton 1972) shows that the effects of horizontal shear are not

1 Present affiliation: National Taiwan University.
profound near the equator, they will be less so at greater distances. We will, therefore, adopt a second approach.

2) In this case we will restrict ourselves to stations very near the equator where, to a reasonable degree of approximation, the \( u \) field (zonal velocity) is associated with symmetric modes while the \( v \) field (southerly velocity) is associated with antisymmetric modes.\(^2\) (Such associations would be exact if the stations were precisely on the equator.) Data from the Marshall Islands test period (1 April to 1 July, 1958) includes four near-equatorial stations (Kapingamarangi, Nauru, Tarawa and Canton; see Fig. 7). At each of these stations, wind data are generally available to at least 25 mb. Data from stations away from the equator will be used in order to examine meridional structure.

Sections 2 and 3 deal, respectively, with data processing and theoretical interpretation, although in practice the two are not readily separated since each provides part of the rationale for the other. Our first step is to use equatorial data to determine monthly mean zonal winds (as well as the mean zonal wind over the three-month period) and power spectra for the \( u \) and \( v \) fields at each level. For the \( v \) field clear maxima in the power spectra exist for periods of 4-6 days. For the \( u \) field maxima of comparable magnitude exist for 4-6 day periods, but they do not stand out as clearly because of the large amount of lower frequency power. We will, in this paper, concentrate on the 4-6 day period range. In identifying the nature of power in this period range, theory immediately restricts us very severely. The following points are crucial to our discussion:

(i) For 4-6 day periods and observed magnitudes, all relevant motions must approximately satisfy linearized wave equations and the subsequent dispersion relation.

(ii) Assuming a tropospheric source for 4-6 day energy, and given the mean zonal flow, zonal wavenumbers for which critical levels exist between the troposphere and the observing level should not exist. This typically eliminates zonal wavenumbers \( |s| \geq 7 \) above 50 mb, though higher wavenumbers might be permitted below.

(iii) Due to the limited vertical resolution of radiosonde observations, the observed fields have, in effect, been subjected to a filter which suppresses motions with vertical wavelengths \( \leq 2 \) km (a model for this

\(^2\) In wave parlance (viz. Matsumo, 1966) antisymmetric modes are given even meridional indices \( (n=0, 2, \ldots) \) while symmetric modes are given odd meridional indices \( (n=-1, 1, 3, \ldots) \); \( n=-1 \) refers to Kelvin waves, while \( n=0 \) refers to mixed gravity-Rossby waves. The higher indices may be associated with westerly gravity waves, easterly gravity waves, and easterly Rossby waves. The words "easterly" and "westerly" are used as relative to the mean flow.

filter is presented in Appendix C).\(^3\) The exclusion of wave types with vertical wavelengths less than 2 km leads to the following additional restrictions:

1) For the 4-6 day period, the observed mean zonal flow, and the zonal wavenumber restrictions cited in item (ii) above, only one type of antisymmetric mode, \( n=0 \) mixed gravity-Rossby waves, can have observably large vertical wavelengths; and only two types of symmetric modes, \( n=-1 \) westerly Kelvin waves and \( n=1 \) easterly Rossby waves can have sufficiently large vertical wavelengths.

2) At levels where item (ii) allows large zonal wavenumbers, the modes allowed in 1) would have vertical wavelengths less than 2 km for \( |s| \geq 10-15 \).

The theory of tropical waves (Lindzen, 1971, 1972) therefore tells us that if there is 4-6 day power in radiosonde \( v \) fields at the equator it must be due to mixed gravity-Rossby waves, and if there is power in radiosonde \( u \) fields at the equator it must be due to some combination of westerly Kelvin waves and easterly \( n=1 \) Rossby waves. Thus, the data are not called upon to prove the existence of mixed gravity-Rossby waves—little else could account for 4-6 day power in equatorial \( v \) fields; the situation for power in the \( u \) fields is only slightly more ambiguous.

Our next step is to attempt to make a simple model for 4-6 day power in the tropical stratosphere. For the \( v \) field at the equator we wish to see whether the data suggest the predominance of a particular zonal wavenumber among the very limited number of possibilities, and whether the theoretically calculated structure of the resultant mixed gravity-Rossby wave for the observed distribution of mean wind [calculations are based on the results of Lindzen (1971, 1972)] is compatible with observed structures. For the \( u \) field at the equator, we have the more difficult problem of determining whether any simple combination of westerly Kelvin and \( n=1 \) easterly Rossby waves can reasonably account for observed structures. The determination of particular wavenumbers among the limited possibilities simplifies the model, but is not otherwise crucial; a range of wave

\(^3\) It follows that such a filter must be applied to theoretical results before they are compared with data.

\(^4\) In using modal terminology one is not presuming the existence of pure, monochromatic modes. The theory of atmospheric wave modes simply says that for given time and zonal scales (characterized by Doppler-shifted frequency and zonal wavenumber), dynamically consistent motion systems can be characterized by a variety of meridional structures (characterized by meridional indices \( n \)) each of which is associated with a particular vertical structure (local vertical wavelength or exponential decay rate). Since these relations (dispersion relations) are mathematically continuous and the observed amplitudes are not such as to suggest that nonlinear interaction between different scales dominated the hydrodynamics, the relations may be used for oscillatory motions which are not steady or monochromatic.
numbers (and frequencies for that matter) have very similar behavior especially as concerns interactions with the mean flow. However, the partition of 4–6 day power in the $u$ field between easterly and westerly waves is more important.

In studying 4–6 day power, we have applied a bandpass filter to the data and restrict ourselves to power in the 4–6 day period range (vis. Appendices A and B). We first examine the amplitudes at each observational level for the four equatorial stations as well as the phase differences between the stations at each level. The southerly velocity (antisymmetric component) is generally characterized by easterly phase propagation at all levels, while the westerly velocity is characterized by westerly phase propagation below 50 mb and a mix of both westerly and easterly phase propagation above 50 mb. All the above-mentioned phase propagation is found to be characterized by zonal wavenumbers 3–4. This is most suggestive of the origin of such waves since just as tropical rainfall is characterized in significant measure by a 4–6 day periodicity, its spatial distribution in the tropics is frequently dominated by wavenumbers 3, 4, and, sometimes, 1 and 2 as well—the same wavenumbers which dominate tropical topography [spatial spectra for rainfall and topography are given in Webster (1971)]. Indeed, in view of this, it could be argued that the observed wavenumbers were to be expected. The above is discussed in more detail and with due concern for uncertainties and coherences in Section 2.

The theoretical model consisting in a mixed gravity-Rossby wave, a Kelvin wave, and an $n=1$ Rossby wave based on the above-described data analysis is developed in Section 3. Theory determines the three-dimensional structure of each mode (given radiative damping rates and mean flow distributions); amplitudes and phase constants are estimated from the data. We show that the resulting observationally calibrated theoretical model is consistent with observed vertical structures and with the observed meridional structure of 4–6 day period waves as obtained from stations away from the equator. The observed vertical structures suggest upward energy flux.

At some levels such comparisons are questionable due to the low coherences in the observed disturbances. In Section 5 it is shown that these are usually levels where waves are strongly interacting with the mean flow and where, in consequence of this, the waves cannot have a steady coherent form over the three-month observation period. Lack of coherence in this instance probably constitutes an observation of the nature of the motion at such a level and, as such, is in agreement with the theoretical prediction of incoherence. In general, it seems to us that incoherence as data could be exploited more fully than it has since theories of waves interacting with mean flows predict incoherence. Incoherence in the data can also arise when waves whose vertical wavelength is only marginally suppressed by inadequate vertical resolution are present with substantial amplitude (see Section 4). Such situations are also theoretically predictable.

In Section 4, the convergence of the vertical flux of zonal momentum due to the observationally calibrated wave model consisting in $n=-1$, $n=0$ and $n=1$ (Rossby) modes is calculated. The acceleration of the mean flow due to this convergence is compared with the observed acceleration of the mean flow, and above 23 km over the equator there is good agreement suggesting that above 23 km 4–6 day waves with vertical wavelengths in excess of 2 km provide the momentum source for the observed acceleration. There is, however, a need for an additional source of easterly momentum immediately below 23 km with the deficit diminishing to zero by 19 km. Some possibilities for this additional source have been considered. In the power spectrum for $u$ there is appreciable power at periods longer than 6 days which $a$ priori might be associated with the needed momentum. However, at the heights in question, it is shown that power at the lowest frequencies may, in fact, be largely due to the changes in mean flow associated with the quasi-biennial cycle. Considerable power remains at periods near 12 days which most probably is associated with Kelvin waves (since no corresponding power exists in the $v$ field) and hence can only produce westerly acceleration. While stationary waves might produce easterly acceleration, there is no reason for such stationary waves to produce the particular vertical distribution of acceleration needed at the equator. One last possibility is considered: namely that the same source which generated the other modes in the 4–6 day period range with wavenumbers 3–4 also generated an $n=1$ easterly gravity wave. It would be difficult to understand why such a mode would not be generated. However, such a mode's vertical wavelength is (except marginally at 19 km) too short to appear in the present data. We show that such a wave, given a modest amplitude ($1 \text{ m s}^{-1}$ in zonal velocity at 100 mb) would provide the magnitude and distribution of acceleration needed. This mode seems to us the most likely possibility for accounting for the deficit.

2. Data analysis at the equator

The upper air data over the Marshall Islands area and its vicinity during 1 April–1 July, 1958, are used for our study. The map of stations is shown in Fig. 7a. Analyses are made at constant pressure levels of 100, 80, 60, 50, 40, 30 and 25 mb. The vertical distribution of mean zonal wind over the equatorial stations is shown in Fig. 1; the period of observation is one of prevailing westerly flow with the easterly phase of
the quasi-biennial cycle descending. Also shown are monthly means of zonal wind, from which we estimate mean zonal accelerations.

Twice-daily data for 96 days were subjected to spectral analysis using Bartlett’s smoothing method (see Appendix A). For reasons mentioned in the Introduction (i.e., to permit disentanglement of symmetric and antisymmetric waves), we will initially concentrate on equatorial data. Power spectra averaged over four equatorial stations are shown in Figs. 2a and 2b for the meridional and zonal wind components respectively. We see that the meridional wind component has a pronounced spectral peak for periods of 4–6 days; the zonal wind has peaks of similar magnitude in this period range. The peaks in zonal wind do not stand out so prominently because of the large spectral amplitudes for longer periods.

Although we will discuss the above spectra somewhat more in subsequent sections, our detailed analysis will concern itself only with the power in the 4–6 day period range. All our analysis beyond this point will involve time series subjected to a simple bandpass filter (see Appendix B) similar to that used by Maruyama (1967). The filtering enables us to better resolve 4 6 day oscillations. As discussed in Section 1, meridional wind oscillations at the four equatorial stations (Kapingamarangi, Nauru, Tarawa, Canton) are associated with antisymmetric waves, while zonal wind oscillations are associated with symmetric waves.

It is clear from Fig. 1 that variations in the mean zonal wind contribute little to long-period power in $u$ at 80 mb; however, at 30 mb this is no longer the case. We estimate that the variations in zonal wind shown in Fig. 1 contribute at 30 mb, 80.6 (m s$^{-1}$)$^2$ day at 1/24 cycle day$^{-1}$, 20.2 (m s$^{-1}$)$^2$ day at 2/24 cycle day$^{-1}$, 9.1 (m s$^{-1}$)$^2$ day at 3/24 cycle day$^{-1}$, and negligible power at higher frequencies to the $u$ power spectrum. If this power were removed from the spectrum shown in Fig. 2b then the $u$ power at 1/24 cycle day$^{-1}$ would be comparable to the $v$ power; a significant peak [O(115 m$^2$ s$^{-3}$ day$^{-1}$)] would remain in the neighborhood of 2/24 cycle day$^{-1}$ followed by another peak in the 4–6 day period range.

Fig. 3a shows the amplitude of 4–6 day meridional wind oscillations at various levels for each of our equatorial stations. Fig. 3b shows the phase differences between station pairs and coherence squares. Relative phases are shown for station pairs with maximum coherence. Also shown are the 95% confidence limits for phase appropriate to the degrees of freedom (approximately 29.3 in the present analysis—\textit{viz.} Appendix A) and coherences squared. Coherence squared is shown in Figs. 3 and 4; for squared coherence greater than 0.15, phase uncertainty is less than 36° which is small enough for many of our purposes. Most of our coherences squared are much larger than 0.15. Although phase differences are, of course, ambiguous to integral multiples of 360°, only the smallest phase differences are possible for reasons given in the Introduction. Positive phase differences imply the station to the west leads the station to the east, while negative phase differences imply the station to the east leads. We see that negative phase differences dominate at levels 100–40 mb. There is
In addition to the above, we have examined the vertical variation of phase. The results are shown in Table 1, where phase differences between levels at the four equatorial stations are given for both the $u$ and $v$ fields. Positive phase differences imply upward phase propagation, while negative phase differences imply downward phase propagation. Also shown are the squared coherences. For the $u$ field, results are shown only for 100-50 mb. Little coherence is expected for higher levels because of the influence of the easterly symmetric wave. In general, one sees downward phase progression with roughly similar results at all stations. An exception to this are the results for the phase variations between 80 and 60 mb in the $u$ field; phase differences vary substantially from station to station at these levels and coherences are generally low.

3. Observationally calibrated theoretical model

As explained in the Introduction, 4–6 day power in radiosonde data for the equatorial stratosphere (given the mean zonal wind) must be due to mixed gravity-Rossby waves, Kelvin waves, and $a=1$ Rossby waves, and a very restricted range of zonal wavenumbers (typically $|s|<7$). In this section we investigate whether a still simpler structure in which each of the three possible wave types is associated with a single zonal wavenumber is still compatible with observations. It should be recalled that for each choice of period and zonal wavenumber, theory yields a complete three-dimensional structure for each mode pro-

![Image](image-url)
Table 1. Observed phase differences (ΔΦ, deg) and squared coherences (K²) between levels at the four equatorial stations and also the phase differences at the equator calculated for the corresponding waves.*

<table>
<thead>
<tr>
<th>Stations</th>
<th>100–80</th>
<th>80–60</th>
<th>60–50</th>
<th>50–40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kapingamarangi</td>
<td>−85/0.112</td>
<td>−114/0.353</td>
<td>−74/0.229</td>
<td>−69/0.514</td>
</tr>
<tr>
<td>Nauru</td>
<td>−37/0.647</td>
<td>−190/0.044</td>
<td>−89/0.379</td>
<td>−90/0.267</td>
</tr>
<tr>
<td>Tarawa</td>
<td>−39/0.598</td>
<td>−124/0.121</td>
<td>−79/0.114</td>
<td>−88/0.058</td>
</tr>
<tr>
<td>Canton</td>
<td>−45/0.291</td>
<td>−184/0.042</td>
<td>−44/0.095</td>
<td>−140/0.206</td>
</tr>
</tbody>
</table>

Calculation

s = 3, period = 4.4 days

<table>
<thead>
<tr>
<th>Level (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−70</td>
</tr>
<tr>
<td>−69</td>
</tr>
<tr>
<td>−41</td>
</tr>
<tr>
<td>−76</td>
</tr>
</tbody>
</table>

(b) Zonal wind component

<table>
<thead>
<tr>
<th>Stations</th>
<th>100–80</th>
<th>80–60</th>
<th>60–50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kapingamarangi</td>
<td>−51/0.444</td>
<td>−134/0.016</td>
<td>−20/0.096</td>
</tr>
<tr>
<td>Nauru</td>
<td>−23/0.027</td>
<td>−37/0.040</td>
<td>−76/0.512</td>
</tr>
<tr>
<td>Tarawa</td>
<td>−70/0.257</td>
<td>−56/0.117</td>
<td>−79/0.190</td>
</tr>
<tr>
<td>Canton</td>
<td>−32/0.235</td>
<td>6/0.120</td>
<td>−35/0.580</td>
</tr>
</tbody>
</table>

Calculation

s = −3, period = 5 days

<table>
<thead>
<tr>
<th>s = −3, period = 5 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>−72</td>
</tr>
<tr>
<td>−115</td>
</tr>
<tr>
<td>−80</td>
</tr>
<tr>
<td>−103</td>
</tr>
<tr>
<td>−183</td>
</tr>
<tr>
<td>−133</td>
</tr>
</tbody>
</table>

* Tabulated values are given in the form ΔΦ/K².

vided that the mean flow and the time scale for Newtonian cooling are known. Thus, where the analysis of Section 2 suggested a zonal wavenumber of either 3 or 4, we made the precise choice on the basis of a best fit to the observed vertical structure of amplitude and phase. The periods of the model modes were taken to be those of the peaks in the power spectra of the filtered time series at those levels where the particular mode appeared most clearly. In addition, the time scale for Newtonian cooling is not precisely known; hence, we have also permitted this time scale to be an adjustable parameter. However, the sensitivity of our model to this parameter was rather weak (time scales from 7–14 days provided comparably good fits); a value of 10 days was used for all calculations.

For theoretical calculations of the Kelvin and mixed gravity-Rossby waves, the results of Lindzen (1971) are used; Eqs. (47)–(59) of that paper are used for the Kelvin wave, while Eqs. (60)–(66) are used for the mixed gravity-Rossby wave. We use 1/16.2 km for the stability parameter s and 6.1 km for the scale height H. These correspond to the observed mean temperature of −6°C and lapse rate of −3°C km⁻¹. Dissipation due to Newtonian cooling is computed using Eqs. (90) and (93) of Lindzen (1972). The observational calibration of these modes was relatively straightforward. From Section 2 we found that the 4–6 day u field over the equator is characterized by westerly phase propagation between 100 and 50 mb implying the dominance of Kelvin waves. Comparisons with vertical phase variation for the u field lead to a choice of s = 3 for the Kelvin wave since s = 4 tends to substantially underestimate vertical wavelength (refer to Table 1b). Although a choice of 5 days was made for the period of the Kelvin wave, a somewhat shorter period would have lead to a significantly better fit. The analysis of Section 2 shows the 4–6 day v field over the equator to be characterized by easterly phase propagation at all levels (certainly between 100 and 40 mb) consistent with a mixed gravity-Rossby wave. Vertical phase variations for the v field are consistent with a choice of s = 3 for the gravity-Rossby wave (see Table 1a); a period of 4.4 days was used. The amplitude factors for the three-dimensional Kelvin and mixed gravity-Rossby modes were chosen for best fit to observed amplitudes. The theoretical amplitudes cannot, of course, be directly compared with observations; first, they must be subjected to smoothing comparable to that provided by radiosonde observing techniques. Actual and smoothed amplitudes for the equator are shown in Fig. 5. Observed values are indicated by crosses (for the meridional component) and circles (for the zonal component); circles are not shown above 21 km since it is evident from Fig. 4b that above that height Kelvin waves no longer predominate. The vertical wavelengths and meridional scales l₀
(viz. Lindzen, 1971 and 1972) for these waves are shown in Fig. 6. It should be noted that given the wavelength of the gravity-Rossby wave in the region 25–26 km, the calculated amplitude shown in Fig. 5 for these heights would, in fact, lead to unstable Richardson numbers and hence turbulence, accounting perhaps for the low coherences at 25 mb in Fig. 3b. As we shall see later, there are other reasons for low coherence as well. The theoretical peaking in amplitude for the mixed gravity-Rossby wave near 25 km is due to the competing effects of dissipation and the presence of an “almost” critical level near 29 km. The absence of such a peak in magnitude in the observed amplitude is explained in Section 5 where we show that the theoretical peaks, calculated here on the basis of a constant mean zonal wind, must be smoothed when account is taken of the variations of $u$ over the three-month observation period.

Using a dissipative time scale of 10 days we have calculated vertical distributions for the amplitude of Rossby waves with various values of zonal wave-number, using Eqs. (24), (30) and (96)–(101) of Lindzen (1972). A period of 5 days was chosen on the basis of spectral results. It was found that a choice of $s=4$, associated with a critical level near 26 km, leads to a vertical distribution particularly consonant with observations (i.e., significant amplitudes in the neighborhood of 30 mb, but relatively weak amplitudes between 100 and 50 mb as indicated by the absence of easterly phase propagation in Fig. 4). We are thus led to a more positive identification of the symmetric easterly wave as a wavenumber $4$, $n=1$, 5-day Rossby wave. The amplitude of the Rossby wave is determined by requiring that the sum of the Kelvin and mixed gravity-Rossby wave, already determined, plus a wavenumber $4$ Rossby wave provide a best fit to the observed distribution of amplitudes and phases at 30 mb. Again the calculated amplitudes have been smoothed before comparing with data. The calculated and smoothed amplitudes for the Rossby wave are shown in Fig. 5. The vertical wavelengths and meridional scales for the Rossby wave are shown in Fig. 6. Note that the actual and smoothed amplitudes for the Rossby wave are relatively close because of the Rossby wave’s relatively large vertical wavelength. The adequacy of the above three-wave model for describing the 4–6 day component of the observed motion can be further checked by comparing calculated and observed horizontal wind distributions. As might be expected from the vertical distributions of amplitudes and meridional scales, the combination of the Kelvin wave and the mixed gravity-Rossby wave should account for the gross 4–6 day horizontal wind distribution at levels 100–40 mb, while the combination of the Rossby and mixed gravity-Rossby waves should account for the gross 4–6 day horizontal wind distribution at 30 mb. In general, this picture is confirmed by the data. As an example of the dominance of the Kelvin and the mixed gravity-Rossby waves, the semi-empirically calculated and the observed wind distributions at 100 mb are shown in Figs. 7a and 7b respectively. The observed horizontal wind structure at each level is derived from the observed amplitudes and relative phases of the $u$ and $v$ spectra at the various stations. The phases of the $u$ and $v$ spectra at each station at each level are computed relative to spectra at Kapingamarangi using observed cross spectra. Phases at each level at Kapingamarangi are relative to 100 mb. Phase constants

---

A finite effect of the mixed gravity-Rossby wave is present, despite its antisymmetry, because the mean latitude of the “equatorial” stations is actually $1.44^\circ$N.
for each model wave mode are chosen to best fit the observed structure. Both figures display the characteristic wind distribution of the mixed gravity-Rossby wave \textit{as well as} the zonal wind oscillation near the equator. Figs. 8a and 8b show the calculated and observed horizontal wind distributions at 30 mb. As was seen in Fig. 6, the meridional scale of the Rossby wave is about 10°–11° latitude for the choice of zonal wavenumber 4, while that of the mixed gravity-Rossby wave is about 4°–5° latitude. Both Figs. 8a and 8b show the characteristic wind distribution of the mixed gravity-Rossby wave near the equator and that of the Rossby wave away from the equator. A mixed gravity-Rossby wave alone would yield little amplitude north of 5°. The observed relative phases between the meridional and zonal components of the wind also supports the existence of both the Rossby and the mixed gravity-Rossby waves at 30 mb; the agreement would be significantly worse without the Rossby wave. The agreement of the calculated and observed meridional scales for the Rossby wave at 30 mb provides further support for the choice of zonal wavenumber 4. Other zonal wavenumbers lead to discernably different scales.

In Section 1 we showed that 4–6 day energy in the equatorial stratosphere must be due to some combination of Kelvin, mixed gravity-Rossby, and $n=1$ Rossby waves with zonal wavenumbers typically less than 7. In this section we have shown that the data are compatible with a model for which each of the above wave types has a single zonal wavenumber, generally 3 or 4. That 4–6 day waves should be dominated by wavenumbers 3–4 is consistent with their being excited by latent heat release in tropical cumulus convection since not only is such rainfall associated with 4–6 day periodicities but also (especially during summer) with zonal wavenumber 3 and 4 (Webster, 1971). The resulting observationally calibrated, theoretical model has the virtue of providing continuous descriptions of all fields at all altitudes and latitudes. As we shall show in the next section, the theoretical formulas can be used to calculate the acceleration of the mean flow by waves whereas the direct use of observations of the same purpose would be misleading at best.

4. Acceleration of the mean flow by waves

The acceleration of the “mean” flow in the quasi-biennial cycle has been attributed for some time to the absorption of equatorial waves of the type described here (Lindzen and Holton, 1968; Holton and Lindzen, 1972). We will, in this section, quantitatively examine this effect. Rather than use the data directly to calculate momentum flux divergence we will use the observationally calibrated theoretical model developed in Section 3. There are several immediate advantages to this procedure: (i) results are physically consistent, (ii) flux divergences can be calculated at all altitudes and latitudes, and (iii) problems arising from radiosonde smoothing are avoided. Additional advantages will also be discussed.

To evaluate the vertical flux of momentum we use Eq. (85) of Lindzen (1972) for each wave mode and add the results. Omitted are cross correlations between different modes; these are associated with flux circulations in the meridional plane (Fels and Lindzen, 1974), and in the present circumstances, their contributions to the acceleration of the mean flow tend to be relatively small. The extrication of these effects when
where $g_0(z)$ is the inverse scale for dissipation given by Eq. (93) in Lindzen (1972). Again, the use of data alone would have permitted the separation of the two contributions to flux divergence only if there were far greater resolution of meridional structure than is presently available. Fig. 10 shows the meridional distribution of accelerating force at different levels. We see that the accelerating force turns from westerly to easterly between 40 and 30 mb. Fig. 10, because of its limited vertical resolution, tends to be somewhat misleading. For example, there is a pronounced maximum in accelerating force at the equator between 25 and 20 mb which does not appear in Fig. 10. Figs. 11a and 11b compare the vertical distribution of wave acceleration (accelerating force divided by density) with the observed distribution of average zonal wind acceleration over the three-month observing period at stations clustered around average latitudes of $1.44^\circ$N and $6.71^\circ$N respectively. Although both the wave acceleration and the actual acceleration have easterly maxima at upper levels, there are some noteworthy differences:

1) At “$1.44^\circ$N,” although the average wave acceleration between 23 and 27 km is about the same as the average observed acceleration over the same height.
interval, the wave acceleration is much more sharply peaked.

2) At "6.71°N" the average wave acceleration between 23 and 27 km is smaller than the observed acceleration, and also much more sharply peaked.

3) Below 23 km the observed acceleration is easterly, decreasing to small values at lower levels, while the wave acceleration is westerly at these heights.

The problem of the wave accelerations above 23 km being much more sharply peaked than the observed accelerations is easily explained. The wave acceleration was calculated for a fixed mean zonal flow when of course the actual mean zonal flow varied significantly over the three-month observing period. As a result, we recalculated the wave acceleration for three separate monthly mean flows (as in Fig. 1), assuming the wave amplitudes to remain constant at 100 mb, and averaged these results. The resulting wave acceleration is shown in Figs. 11a and 11b. The wave accelerations are now substantially smoothed; moreover, it is fairly clear that complete smoothing would result from taking account of the changing mean zonal flow in a continuous manner.

The fact that the easterly wave acceleration at "6.71°N" is, on the whole, less than the observed acceleration may not be significant since Lindzen and Holton (1968) found that zonally symmetric meridional circulations play a significant role in the zonal momentum budget away from the equator. However, the need for an additional easterly accelerating force below 23 km over the equator ("1.44°N") is clear. An obvious possibility is that this acceleration is provided by waves with periods outside the 4–6 day range. As discussed in Section 1, there are difficulties with this possibility. There is, however, an alternative: namely, that the additional accelerating force is due to an easterly $n=1$ gravity wave which, because of its short vertical wavelength, would not be observed in radiosonde data. In order to check this possibility we have calculated the easterly accelerating force due to such waves for the mean flow in Fig. 1 using formulas from Lindzen (1972) and assuming a period of 5 days. We found that for an amplitude in zonal velocity of 1 m s$^{-1}$ at 100 mb over the equator and zonal wavenumbers between 3 and 5, an $n=1$ easterly gravity wave could indeed account for the needed easterly acceleration. The easterly acceleration at the equator due to an $n=1$ easterly gravity wave with zonal wavenumber 4 is shown in Fig. 12. Also shown are the meridional distributions of wave acceleration at 19.6 and 23.2 km. Clearly the wave acceleration due to this mode is tightly confined to the equatorial region. The ability of such a gravity wave to account for the needed easterly acceleration (to provide the observed acceleration and cancel the acceleration due to the Kelvin wave) is satisfying since it is difficult
to believe that those processes which generated the other 4–6 day waves with \( r = 3–4 \) would fail to excite a similar \( n = 1 \) easterly gravity wave as well.

It should also be noted that such a gravity wave has a sufficiently long wavelength in the neighborhood of 19 km (see Fig. 6) to be marginally observable. This might account for the low coherences and anomalous phase behavior at 80 mb (see Fig. 4) and the low coherences between 80 and 60 mb (Table 1b).

5. Implications of wave-mean flow interactions for interactions for data analysis

From the results of Section 4, we see that not only does wave absorption alter the mean flow, but that the changing mean flow alters the vertical structure of the waves. Under these circumstances it is reasonable to inquire as to what meaning one may attach to the analysis of a three-month data record for mean wave structures. In particular, regions of small Doppler-shifted frequencies are associated with small vertical wavelengths, and large magnitudes at the equator. It is possible for an observing level to be so situated that a level of small Doppler-shifted frequency will initially be above the observing level, but move through and below the observing level during a three-month period. All the waves we have discussed are affected by this, though the effects are most pronounced for the mixed gravity-Rossby mode. For purposes of discussion we will focus on this mode. In Section 3 we calculated a theoretical structure for this mode assuming a constant mean flow. In Figs. 13 and 14 we reproduce the amplitudes (both actual and smoothed) and phases so calculated. In addition, assuming fixed amplitude and phase at 100 mb, we have also calculated the distribution of amplitudes and phases for individual monthly mean zonal flows. The latter (which are also shown in Figs. 13 and 14) are more nearly identifiable with instantaneous wave structures. Our question, at this point, is how well would the mean of these individual realizations compare with the calculation based on the mean zonal flow. We also qualitatively discuss what sort of effects on observational analyses will result from mean flow-induced wave changes.

We see from Figs. 13 and 14 that below 22–23 km calculations based on the three-month mean zonal wind yield results which are close to the mean of results based on monthly mean zonal winds. Similarly, we expect that in this region our observational analysis will be adequate for delineating the actual wave. Above 23 km we see from Fig. 13 that the smoothed amplitude based on the three-month zonal flow is a rather erratic indicator of the mean of smoothed results based on monthly mean zonal winds. However, the discrepancies are small compared to the distortions produced by vertical smoothing. From Fig. 14 we see that there is probably no meaningful to a phase above 23 km. This may account for the low coherences at 30 and 25 mb in Fig. 3b. In addition, it seems plausible that rapid phase variations above 23 km due to variable mean flow may give rise in power spectra.
to the appearance of high-frequency noise. This is consistent with the greater contribution of high-frequency components of the spectrum at 30 mb than at 80 mb (vis. Figs. 2a and 2b). The above effects may also be due to the turbulent breakdown of mixed gravity-Rossby waves, as suggested in Section 3.

It is important to treat the above statements with caution. Our calculated wave solutions assume that the changes in the mean flow occur slowly compared with wave and dissipation time scales. This is no longer correct in the regions where vertical group velocities are small; in such regions there will not be sufficient time for wave fields to fully adjust to changing zonal flows. Nevertheless, we expect the above effects to be qualitatively present. We are thus led to the conclusion that if only 4–6 day waves were present in the tropical stratosphere, power spectra of wind data would indicate low-frequency power in the $u$ field due to the acceleration of the mean flow by the waves, and high-frequency power in both fields due to the effect on the waves of the changing zonal flow. The contribution of the changing mean zonal flow to low-frequency power has been estimated in Section 2; unfortunately, no comparably simple estimate is currently possible for the contribution to high-frequency power.

Finally, we note that the considerations of this section do not significantly alter the results in Section 3 which are primarily based on observations from levels where means are well-defined.

6. Summary and conclusions

We have reexamined tropical stratospheric wind data from the Marshall Islands area for the period 1 April to 1 July 1958, a period during which the easterly phase of the quasi-biennial cycle was descending. These data were chosen since they provided better coverage near the equator (four stations) than has been available since. Equatorial data for westerly and southerly wind allowed us to separate contributions to the 4–6 day waves from symmetric and antisymmetric modes. However, as shown in Appendix C, motions with vertical wavelengths <2 km are effectively filtered out of radiosonde data. When short wavelengths are excluded, and when the effect of the mean zonal flow is considered, it is shown that 4–6 day power in the equatorial $v$ field must be due to mixed gravity-Rossby waves while 4–6 day power in the equatorial $u$ field must be due to some combination of Kelvin and $n=1$ Rossby waves. Moreover, zonal wavenumbers must be less than 7–10. In Sections 2 and 3 we show that the data are in fact compatible with a model in which the above three wave types are associated only with wavenumbers 3 or 4. The resulting observationally calibrated model provides a self-consistent three-dimensional description of 4–6 day fields in the tropical stratosphere. Indeed the data in some ways are inadequate to determine by themselves detailed wave structure; at the same time the data are adequate to check the consistency of a theoretical model and to distinguish among various theoretical models. An important point of this paper is to make as much use as possible of the constraints imposed by our present theoretical knowledge. Such constraints significantly relax our demands on the data.

We have used our model to calculate the acceleration of the mean flow by waves. We find that 4–6 day waves are able to account for the observed acceleration at upper levels, but immediately below 23 km we need an additional source of easterly momentum. Various possibilities are considered; we show that both the magnitude and the vertical distribution of the required easterly momentum can be provided by an $n=1$ easterly gravity wave of zonal wavenumber about 4 and a period of about 5.0 days with an amplitude of 1 m s$^{-1}$ for the $u$ field at 100 mb over the equator.

It is, in fact, to be expected that a source which generates $n=-1, 0$ and $n=1$ (Rossby) waves with periods 4–6 days and zonal wavenumbers 3–4 will also excite $n=1$ gravity waves with the same periods and wavenumbers. The absence of an $n=1$ easterly gravity wave in our data is consistent with the theoretical expectation that this mode will have vertical wavelengths <2 km except near 80 mb (its wavelength there is about 2 km) where, as mentioned in Section 4, there is some hint of its presence in our data.

We have finally considered in detail what the theory of tropical waves in the presence of zonally averaged mean shear predicts should be present at levels where the waves are producing significant changes in the zonally averaged mean flow. We show that even for our conceptually simple three-wave model, erratic and incoherent results are theoretically expected at these levels in data obtained from three-month long records. Similarly, spectral averages over three months are shown to be unlikely to be good indicators of either instantaneous or average wave behavior at levels of marked changes in the mean zonal flow. Perhaps the most essential point of this part of our study is to make clear that even simple wave models do not predict simple coherent results at certain predictable levels, and conversely the absence of simple coherent patterns at such levels does not rule out simple wave models.

Acknowledgments. This work was supported by the Atmospheric Science Section of the National Science Foundation under Grant GA33990-X, and by the National Aeronautics and Space Administration under Grant NGR 22-007-275. We are also grateful to Mr. Roy Jenne at the National Center for Atmospheric Research for providing the data in the present study.
APPENDIX A  

Spectral Computation

Bartlett's spectral smoothing method is employed in this study. Time series data of length \( N \) are split up into \( k \) series of equal length \( M \), and a spectrum is evaluated for each sub-series. The smoothed spectral estimate is then computed by averaging these \( k \) spectra. In the present study, 192 pieces of data are divided into four series after the linear trend is removed from the data. Each spectrum for sub-series is evaluated by the direct Fourier transform method, not by the covariance method.

Power spectra in Figs. 2a and 2b are computed by the above-mentioned Bartlett's method before averaging over four equatorial stations. In the later discussion of disturbances in the 4–6 day period range, smoothed power and cross spectra are further averaged over three frequency bands with periods of 4.0, 4.8 and 6.0 days. The degrees of freedom of such further smoothed spectra as in Figs. 3a, 3b, 4a and 4b can be estimated. Since the Bartlett's spectral window is

\[
W_B(f) = M \left( \frac{\sin fM}{\pi fM} \right)^2,
\]

where \( f \) is frequency (Jenkins and Watts, 1969), the effective spectral window of the further smoothed spectra will be

\[
W_{\text{eff}}(f) = \frac{M}{3} \left[ \left( \frac{\sin \left( \frac{f-1}{M} \right)}{\left( \frac{f-1}{M} \right)} \right) + \left( \frac{\sin fM}{fM} \right)^2 \right] + \left( \frac{\sin \left( \frac{f+1}{M} \right)}{\left( \frac{f+1}{M} \right)} \right)^2.
\]

The effective degrees of freedom can then be estimated by

\[
\nu = 2N \int_{-\infty}^{\infty} W_{\text{eff}}^2(f) df \approx 29.3.
\]

The variances of power and cross spectra decrease with increasing degrees of freedom (or increasing width of spectral smoothing). The 95% confidence interval for the power spectrum \( Q \) is 0.63Q–1.77Q for degrees of freedom of 29.3 and 0.42Q–4.8Q for degrees of freedom of 6. The 95% confidence limits for the phases corresponding to the coherence squares for the degree of freedom of 29.3 are shown in Figs. 3b and 4b (after Jenkins and Watts, 1969).

APPENDIX B  

Bandpass Filter

To resolve disturbances with periods of 4–6 days, twice-daily observations are subjected to a simple bandpass filter. Let \( X_t \) represent the observation in a time series at time \( t \), and \( \bar{X}_t \) and \( \bar{X}_t \) be the smoothed values computed by equally-weighted running means from 5 and 11 observations respectively. The time series of 182 points in length are then computed by subtracting \( \bar{X}_t \) from \( \bar{X}_t \). The frequency responses of various filters are discussed by Holloway (1958). The frequency responses of the present bandpass filter is shown in Fig. 15. The average response of 4–6 day disturbances is about 70.1%. Amplitudes of 4–6 day disturbances are corrected by multiplying by a factor of 1.426.

APPENDIX C  

The Smoothing Effect in Wind Aloft Observations

Upper-air observations are generally taken by tracking a free-rising or a free-falling object. Hori-

---

\( ^8 \) The value used by Yanai and Marakami (1970).

---

Fig. 15. Frequency response function of the bandpass filter.
horizontal wind components are evaluated by measuring the horizontal displacement over given intervals of time. Since the object travels vertically, the measured winds are average values over a vertical distance \( d \). The vertical averaging process will reduce the amplitude of a vertically propagating wave. Taking the form of \( A \cos[\omega t + (2\pi/L)z - \phi] \) for a vertically propagating wave, the smoothing effect can be estimated as

\[
\frac{1}{d} \int_{z-d/2}^{z+d/2} A \cos\left(\frac{2\pi}{L}z' - \phi\right)dz' = (\sin\theta) A \cos\left(\frac{2\pi}{L}z - \phi\right),
\]

where \( \theta = (d/L)\pi \) and \( L \) is the vertical wavelength. It is seen that \( (\sin\theta/\theta) \) is a smoothing factor. The vertical averaging distance is about 1.2 km for standard rawin data in the stratosphere. For \( d \sim 1.2 \) km, the amplitude of a wave will appear greatly diminished when its vertical wavelength is less than 2 km.

REFERENCES


