Reply

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We thank Warren and Schneider for their interest in our work. However, as there seems to be misunderstanding of major points made in Lindzen and Farrell (1977, hereafter referred to as LF), a directed restatement of those results should answer most of the objections raised. Individual arguments will then be addressed.

The main point in LF is that the effects of transport on climate stability (when an ice-albedo feedback exists) depend on the latitudes over which transport is occurring and the effectiveness of the transport. This point was made in order to counter the impression in North (1975) that climate stability was almost independent of the form chosen for transport. All of North's models of transport acted from the equator to the poles. Apart from this, LF and North were in agreement. LF found that heat transport within a latitude belt (θ_1, θ_2; $\theta_2 > \theta_1$) increased the *reduction* in Q needed to bring the ice line to θ_2, but once this Q was reached, sensitivity within (θ_1,θ_2) was also *increased* leading, for sufficiently effective transport, to instability.

The rest of LF explores the implications of the fact that most known mechanisms of heat transport do not act from the equator to the pole. To be sure the application of these notions to an understanding of CLIMAP results was clearly labeled by LF as speculative.

The analysis of the Hadley adjustment is more rigorous and does *not* arise from an attempt to minimize the least square deviation of calculated from observed temperatures. The adjustment is based on the observation (Schneider, 1977; Held and Hou, 1980) that zonally averaged zonal angular momentum at best remains constant from the equator to the pole (in fact, it decreases away from the equator). Now, the equilibrium temperatures in Budyko-Sellers models imply (via the thermal wind relation) angular momenta between the equator and ~30° latitude which *exceed* the angular momentum at the equator. Transports are thus required to correct the angular momentum distribution, and, for internal consistency, such transport must also lead to a flattening of the latitude distribution of temperature (from θ^2 to θ^4 behavior). Note, the temperature flattening and not its precise value is what is

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important (the latter depends on minor and rather inconsequential adjustments in radiative parameters, etc.). Fig. 4 in LF shows that observed temperatures display this flattening. LF refer to this as a "Hadley" adjustment though, as LF note, it need not be due to the Hadley cell alone; nor need this transport be proportional to the intensity of the Hadley cell. A recent discussion of this effect in the context of modeling can be found in Taylor (1980).

The philosophy expressed in LF is that energy balance models should be studied with consideration for the physics underlying the assumed transports. The lack of such considerations has been the major criticism of most energy balance modeling. Warren and Schneider's misstatement of LF's results would have LF guilty of exactly this.

Specific remarks follow.

1) Fit to temperature data

The purpose of LF was to elucidate the transports in general physical terms; there was, therefore, no attempt to best fit a particular data set. This kind of "verification" can be had by judicious choices of albedo and radiative parameters.

2) Relevance of $Q(x_s)$ plots

As noted in Suarez and Held (1976) and more explicitly in Suarez' thesis, only models which display very high sensitivity poleward of 60° latitude in their "annual average" (Budyko) form respond "realistically" to the Millankovitch seasonal mechanism. As noted by LF, such sensitivity would most easily be produced by efficient heat transports at high latitudes.

3) Testing transport parameterizations

LF [as well as North (1975) implicitly, and Stone (1978) explicitly] note that flux is an insensitive and inadequate indicator of the behavior of simple climate models. There are several reasons for this, but one revealing fact is that existing fluxes are almost as great as they can be and models with similar fluxes can give radically different behavior.

4) The "LF-2" model

As it is defined (LF, p. 1492), the "Hadley" heat flux is confined to a region $|\theta| < \theta_h$ over which it eliminates all temperature gradients. In the case of the Budyko model the effect of this flux can be mimicked by setting $S(\theta)$ equal to its average over the region. This observation has regrettably led Warren and Schneider (1979) to confusion, for this strategy fails in the overlapping transport case (LF-2) where a separate flux must be solved for (see Appendix). When the "Hadley" flux is properly implemented there are no variations of temperature inside the region $|\theta| < \theta_h$ such as are shown in Fig. 1c of Warren and Schneider (1980) [such temperature discontinuities would give rise to stability discontinuities inside the Hadley region which are not a feature of LF (Fig. 13)].

It is unfortunate that so unlikely a model has been attributed to the authors (Warren and Schneider, 1979).

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APPENDIX

Solution of the Model with Two Transports and the Hadley Adjustment

The energy balance is expressed as

$$QSA - I + \delta_0(\bar{I}_0 - I) + \delta_a(\bar{I}_a - I) - \frac{dF_h}{dx} = 0, \quad (1)$$

where:

Q = solar constant

$S(x)$ = insolation function $\int_0^1 Sdx = 1$

$A = 1 -$ albedo $= \beta$ for ice free conditions; α for ice

$$\left.\begin{array}{l}\delta_0 = 0 \quad x > x_s\\ \delta_a = 0 \quad x < x_a\end{array}\right\} \text{ constant elsewhere}$$

$$\bar{I}_0 = \frac{1}{x_s}\int_0^{x_s} Idx \quad (2)$$

$$\bar{I}_a = \frac{1}{1 - x_a}\int_{x_a}^1 Idx \quad (3)$$

F_h = Hadley flux chosen so $I = I_h$ constant for $x \leqslant x_h$ and $F_h = 0, x = 0$ and $x \geqslant x_h$.

Other notations are as follows:

x_s = sine of ice line latitude
x_a = sine of latitude where γ_a flux begins
x_h = sine of latitude of Hadley flux extent

$$\phi(x) = \int_x^1 Sdx.$$

Solving for F_h from (1) gives

$$\frac{dF_h}{dx} = QSA + \delta_0\bar{I}_0 + \delta_a\bar{I}_a - (1 + \delta_0 + \delta_a)I_h.$$

Integrating then yields

$$F_h(x) = Q\int_0^x SAdx + \bar{I}_0\int_0^x \delta_0 dx + \bar{I}_a\int_0^x \delta_a dx - I_h\int_0^x (1 + \delta_0 + \delta_a)dx$$

with the boundary condition

$$F_h(x_h) = 0. \tag{4}$$

We will do the case $x_s > x_h$. Eq. (4) becomes

$$Q\beta(1 - \phi(x_h)) + \bar{I}_0\delta_0 x_h + \bar{I}_a\delta_a(x_h - x_a) - I_h[(1 + \delta_0)x_h + \delta_a(x_h - x_a)] = 0,$$

$$I_h = \frac{Q\beta(1 - \phi(x_h)) + \bar{I}_0\delta_0 x_h + \bar{I}_a\delta_a(x_h - x_a)}{[(1 + \delta_0)x_h + \delta_a(x_h - x_a)]}. \tag{5}$$

Outside the Hadley region we have

$$I = \frac{QSA + \delta_0\bar{I}_0 + \delta_a\bar{I}_a}{1 + \delta_0 + \delta_a}. \tag{6}$$

where $\bar{I}_0$, $\bar{I}_a$ can be solved for using the definitions (2) and (3):

$$x_s\bar{I}_0 = I_h x_h + \int_{x_h}^{x_s} I dx,$$

$$(1 - x_a)\bar{I}_a = I_h(x_h - x_a) + \int_{x_h}^{1} I dx.$$

From the expression for I [Eq. 6], performing the indicated integration and some algebra,

$$\left(x_s - \frac{\delta_0 x_h^2}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} - \frac{\delta_0(x_s - x_h)}{1 + \delta_0 + \delta_a}\right)\bar{I}_0 - \left(\frac{\delta_a(x_h - x_a)x_h}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} + \left[\frac{\delta_a(x_s - x_h)}{1 + \delta_0 + \delta_a}\right]\right)\bar{I}_a = \frac{Q\beta[1 - \phi(x_h)]x_h}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} + \frac{Q\beta[\phi(x_h) - \phi(x_s)]}{1 + \delta_0 + \delta_a}, \tag{7}$$

$$-\left(\frac{\delta_0 x_h(x_h - x_a)}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} + \frac{\delta_0(x_s - x_h)}{1 + \delta_0 + \delta_a}\right)\bar{I}_0 + \left((1 - x_a) - \frac{\delta_a(x_h - x_a)^2}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} - \delta_a\left[\frac{(x_s - x_h)}{1 + \delta_0 + \delta_a} + \frac{(1 - x_s)}{1 + \delta_a}\right]\right)\bar{I}_a = \frac{Q\beta(1 - \phi(x_h))(x_h - x_a)}{(1 + \delta_0)x_h + \delta_a(x_h - x_a)} + \frac{Q\beta(\phi(x_h) - \phi(x_s))}{1 + \delta_0 + \delta_a} + \frac{Q\alpha\phi(x_s)}{1 + \delta_a}. \tag{8}$$

Eqs. (5), (7) and (8) can be solved for I_h, $\bar{I}_0$, $\bar{I}_a$, and Eq. (6) yields $I(x,x_s)$ to complete the solution.

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