NOTES AND CORRESPONDENCE

Theory of Atmospheric Tides

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(Manuscript received 25 January 1980, in revised form 25 April 1980)

Abstract

The present extended abstract outlines some important theoretical advances made in the last ten years in the field of atmospheric tides. On most topics included in the lecture, details and references can be found in Lindzen (1979). Only highlights will be sketched here.

1. Situation 10 years ago

The field as it existed ten years ago is described in Chapman and Lindzen (1970). At that time, classical tidal theory (which omitted friction, thermal damping, nonlinearity and the effects of mean winds, but included rotation, spherical geometry, vertical variations in mean temperature, and thermal and gravitational forcing) was fully developed. Most notably, the complete solution to Laplace's Tidal Equation, including Hough modes with positive and negative equivalent depths, had been obtained (Lindzen, 1966; Kato, 1966; Flattery, 1967; Longuet-Higgins, 1967). Also, two major sources of effective thermal forcing, absorption of insolation by H_2O (Siebert, 1961) and by O_3 (Butler and Small, 1963), had been identified. With these forcings and the classical theory, diurnal and semidiurnal surface pressure oscillations were predicted with approximately the amplitudes and phases observed. The theory also predicted diurnal wind oscillations in the mesosphere in agreement with rocket observations. Certain important difficulties were, however, evident:

- The predicted maximum in semidiurnal surface pressure was at 0900 and 2100 local time. Observed maxima occurred closer to 1000 and 2200 l.t.
- The theoretical semidiurnal horizontal winds underwent a 180° phase shift at 28 km which was not observed.
- A more subtle difficulty involved the lunar semidiurnal tide. Sawada (1956) had noted that the predicted surface pressure oscillation was extremely sensitive to details of the mean mesospheric temperature. The sensitivity was not supported by observations. In Chapman and Lindzen (1970) it was noted that infrared damping only modestly reduced this sensitivity. It was also noted that predictions for an isothermal atmosphere (wherein there are no internal reflections) closely matched observations.
- Although viscosity and thermal conductivity were omitted in classical theory, the fact that both effects increased inversely with mean density implied that both would eventually reach dominant importance at thermospheric heights.
- In the absence of damping, tidal amplitudes were predicted to increase inversely as the square root of mean pressure. It thus seemed conceivable that, unless prevented by damping, nonlinearity would become important. However, as noted by Lindzen (1968) the first nonlinear effect was likely to be the convective breakdown of the tide rather than any gradual distortion.
- The importance of mean winds depends on their magnitude relative to the tidal phase speed (*i.e.*, the linear rotation speed of the earth $\sim$400 m/s at the equator). At high latitudes, these speeds are comparable, and should, therefore, be important for higher order semidiurnal modes which have significant amplitudes at high latitudes.
- Although the fact that daily variations constituted the major feature of thermospheric meteorology, no attempt had been made to

consider these variations in the context of tidal theory.

- Somewhat more recently, radio meteor and incoherent scatter data showed that between 90 and 130 km tides were dominated by higher order semidiurnal modes counter to theoretical expectations.

2. Effects of mean winds

Two difficulties mentioned above, the inclusion of mean zonal winds and the inclusion of friction, both lead to equations whose latitude and altitude dependence are non-separable. Lindzen and Kuo (1969) suggested how to solve such equations by means of Gaussian elimination. Their approach has been effectively applied to both of the above problems.

Lindzen and Hong (1974), Miyahara (1975) and Walterscheid and Venkateswaran (1979) have studied the effects of mean winds. More recent studies by T. Aso have been reported at this meeting. Lindzen and Hong (1974) showed that the presence of mean winds gave rise to mode coupling wherein the presence of one semidiurnal mode gave rise to energy in other modes. Such mode coupling lead, among other things, to the reduction of the amplitude of the main semidiurnal mode, to the dominance of higher order modes above the mesopause, and to the displacement of the 180° phase shift in semidiurnal winds towards higher altitudes in the summer (but not in winter). While this last finding was consistent with Reed's (1967) observational analysis which was based on summer data, Wallace and Tadd (1974) found the phase shift to be absent below about 30 km during all seasons.

In addition, Lindzen and Hong found that the inclusion of horizontal variations of mean temperature (consistent with the inclusion of mean winds in thermal wind balance) corrugated the surface in the mesosphere which reflected semidiurnal tidal modes and hence eliminated the possibility of effective constructive and/or destructive interference, thus eliminating the sensitivity of lunar semidiurnal tide to details of mesospheric temperature.

3. Effects of viscosity, thermal conductivity and ion drag

Since 1970 a progression of papers at varying levels of approximation have dealt with this problem. Earlier work dealt with the related (but simpler) problem for internal gravity waves. Lindzen (1970, 1971) attempted to approximate tidal modes by gravity waves and examine the effects of viscosity, thermal conductivity, ion drag and thermospheric heating on these so-called "equivalent gravity modes." The e.g.m. approximation proved reasonably accurate (0 [25%] or better), and was subsequently improved by the inclusion of some of the effects of rotation by Richmond (1975) and Forbes and Hagan (1979).

The e.g.m. results showed that the diurnal tide in the thermosphere could not be forced by heating in the mesosphere and troposphere, but had to be forced within the thermosphere. Moreover, *in situ* thermospheric forcing (mainly due to absorption of solar extreme ultra-violet radiation) appeared adequate to account for thermospheric observations. These results also showed that higher order semidiurnal modes propagating from below the thermosphere would decay markedly above about 115–130 km. The same results showed that the main semidiurnal mode could propagate effectively into thermosphere. Although the amplitude would cease growing with height above about 150 km, almost no decay above this height was predicted and amplitudes of over 100°C for the temperature oscillation were anticipated. Observations soon showed that the actual amplitude was closer to 25°C (depending on phase of solar cycle, etc.).

Subsequent, rather complete calculations by Hong and Lindzen (1976), Forbes and Garrett (1976) and others produced realistic results. The discrepancy in the earlier calculations was due to a sequence of modest errors all operating to diminish predicted amplitudes. The most significant of these errors were:

- Mode coupling in the stratosphere and mesosphere (due to mean winds) reduced the 2,2 mode about 30%.
- Ion drag, because it is more nearly isotropic than unidirectional (which was assumed in e.g.m. calculations), proves more effective.
- Mode broadening due to viscosity and thermal conductivity in a spherical geometry reduced amplitudes at the equator.
- At least at solar maximum, *in situ* forcing is important for the semidiurnal tide and, in general, this forcing is not in phase with the forcing from below.
- More recently, improved evaluations of ozone forcing have reduced its magnitude (Forbes and Garrett, 1979).

While none of the above constituted an error greater than 30%, their product resulted in a 75% reduction of predicted amplitude for the semidiurnal temperature oscillation in the thermosphere.

4. Semi-empirical models for tides in the thermosphere

Accurate predictions of tides in the thermosphere (above about 110 km) have proven difficult because of uncertainties in the semidiurnal tide propagating from below (which depends significantly on the largely unmeasured mean winds in the stratosphere and mesosphere), as well as uncertainties about the precise magnitude of EUV forcing in the thermosphere. There exist practical needs for global descriptions of thermospheric tides, and the question arose as to whether an accurate global description might be obtained from data at two stations. Lindzen (1976) showed that the main semidiurnal tide was weak enough in the neighborhood of 115 km to allow one to accurately describe the semidiurnal tide at these levels in terms of the 2,4 and 2,5 modes which were determinable with incoherent scatter data from Millstone Hill and Arecibo. Garrett and Forbes (1978) have shown that data from two stations can, in fact, be used to determine both diurnal and semidiurnal tides globally. Their approach runs approximately as follows:

1. The diurnal tide in the upper thermosphere is forced entirely *in situ*. In conjunction with a model for the diurnal tide, observations may be used to determine EUV flux.
2. The EUV flux allows the calculation of that part of the semidiurnal tide forced *in situ*.
3. Above 150 km the semidiurnal tide is dominated by the 2,2 and 2,3 modes. Thus, observations of the semidiurnal tide at these levels (two stations needed to determine 2,2 and 2,3) together with the results of item 2 above, allow determination of the 2,2 and 2,3 tides propagating from below.
4. The complete specification of the 2,2 and 2,3 modes allows an improved determination of the 2,4 and 2,5 modes from data in the lower thermosphere.

It should be noted that the modes described above are not the classical modes, but their extensions in a viscous, thermally conducting thermosphere with ion drag. A description of these Hough mode extensions is given in Lindzen, *et al.* (1977).

5. Additional forcing for solar semidiurnal tides

None of the above resulted in any improvement in the phase discrepancy of the solar semidiurnal surface pressure oscillation. Lindzen (1978) noted that the latent heat release associated with the semidiurnal oscillation in tropical rainfall (determined from limited data) was comparable to other semidiurnal forcings and capable of correcting the discrepancy in the phase of the semidiurnal surface pressure oscillation. The presence of this additional tropospheric source also eliminates the pronounced phase shift in horizontal wind at 28 km (viz. item 2 in section 1). However, at the moment, no adequate explanation exists for the oscillation in rainfall.

6. Tides on other planets

In recent years tidal theory has provided useful insights into prominent phenomena on Mars and Venus. For example, the surface tides on Mars, which are normally dominated by the diurnal component, become primarily semidiurnal during a major dust storm. Zurek (1980) has shown that during a dust storm, tidal heating is increased and greatly broadened in height. As on earth, a broad source favors the semidiurnal tide.

Fels and Lindzen (1974) showed how tides on Venus could help force the large zonal winds observed on that planet. At that time, however, the tides, themselves, had not been observed. Now satellite radiance maps clearly show both diurnal and semidiurnal tides at various levels in Venus' atmosphere (Apt, Brown and Goody, 1980).

Fels and Lindzen (1974) also discussed the forcing of mean flows by tides in the earth's atmosphere. Miyahara, at this meeting, presented further results on this matter.

7. Concluding remarks

It should be stressed that tides remain the cleanest example of internal waves in the atmosphere, and are thus a model for our understanding of many phenomena. The importance of tides extends well beyond the relatively narrow speciality.

Acknowledgements

This paper was presented at a meeting cosponsored by the National Science Foundation under Grant No. INT-79-20379. The paper was prepared with support from NSF Grant No. ATM-78-23330.

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大気潮汐理論の問題点

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Abstracts
of
the U.S.-Japan Seminar
on
Atmospheric Tides and Related Phenomena

November 14-16, 1979

Fukuoka, Japan

Sponsored by

National Science Foundation

and

Japan Society for the Promotion of Science

CONTENTS

(*) Reprinted from the Journal of the Meteorological Society of Japan Vol. 58, No. 4, August 28, 1980.

The Atmospheric Tides and Related Phenomena

Preface

The U.S.-Japan Seminar on Atmospheric Tides and Related Phenomena was held on 14th through 16th of November, 1979, in Fukuoka, Japan.

Although the scale of the seminar was rather small, papers presented fully reflected recent progresses in this field to remain to be important references.

The seminar was made possible under financial supports of National Science Foundation of the U.S. and Japan Society of Promotion of Science, for which we are very grateful.

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