SOME REMARKS ON CUMULUS PARAMETERIZATION

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Dr. Sarachik, in the opening paper of this session, has noted that the total precipitation (or equivalently the atmospheric heating) associated with cumulus convection is generally accurately represented by a simple moisture budget:

$$\text{Precipitation} = \text{Evaporation} + \text{Convergence of Moisture} \qquad (1)$$

The relevant convergence appears to be that which occurs in the lower troposphere – below the trade inversion where such a feature exists. Such a budget has been observed in the western Pacific (Ogura and Cho, 1973) and during all phases of GATE (1980). The total cumulus heating is simply the latent heat of condensation, L, times the precipitation.

We shall discuss the above budget later in this note. Clearly, such a budget cannot be true always; otherwise there would be no way for humidity in the lower troposphere to change. For the moment, however, let us ignore this problem. As noted by Sarachik, (1) fails to give us the vertical distribution of heat release. Some additional closure conditions are needed to obtain such a distribution. The question arises as to whether we need a detailed specification of the vertical distribution of cumulus heating. The answer to this question is not entirely clear. Stevens, *et al.* (1977) concluded that the vertical distribution was not too important for fully developed easterly waves because of the important role of cumulus friction. The same insensitivity may not hold for weaker developing waves (Stevens and Lindzen, 1978). Furthermore, the ability of latent heat release in the tropics to force stationary waves in middle latitudes intuitively ought to depend on whether the latent heat is deposited in the lower troposphere where we have prevailing easterlies or at upper levels where prevailing westerlies may exist (Charney and Drazin, 1961). At least in some circumstances, therefore, we *do* expect the vertical distribution of latent heat release to be important.

Among the various closures suggested for obtaining this distribution is that due to Arakawa and Schubert (1974). This particular approach is difficult (if not impossible) to solve for, and its physical basis is not at all clear. An attempt to apply the Arakawa-Schubert scheme to GATE data has been made by Lord (1978). Lord was unable to obtain a solution to the Arakawa-Schubert scheme. However, using simplex methods, Lord obtained answers close (in some sense) to being solutions. A comparison between the predicted distribution of heating (using Lord's approach) and observations is shown in Figure 5 of Sarachik's paper in this volume. The agreement is quite poor. The Arakawa-Schubert scheme predicts a concentration of cloud heating near the surface; observations show fairly uniform heating between the ground and 300 mb. In view of our remarks on tropical forcing of stationary waves we might anticipate serious problems arising from this misrepresentation.

We may next ask whether this failure implies that a more effective parameterization will prove still more complicated than the Arakawa-Schubert scheme. I doubt it. I will attempt to show, in this note, that an almost trivial but plausible scheme provides a rather good simulation of the observed heating. The basic approach is that described in Appendix 1 of Stevens and Lindzen (1979). As shown by Ooyama (1971) and Arakawa and Schubert (1974) (see Sarachik's lecture for further details), cumulus heating can be expressed

$$H = M_c \frac{\partial s}{\partial z} = \rho Q \tag{2}$$

where

Q = Htg. in deg/day

M_c = mass flux in cumulus clouds

$\frac{\partial s}{\partial z}$ = dry static energy gradient in air outside clouds ($s = c_pT + gz$)

The specification of the vertical distribution of heating becomes equivalent to the specification of M_c's distribution. Certain aspects of M_c's distribution are fairly clear: for example, M_c should not extend beyond a height, z_T, where

$$c_pT(0) + Lq(0) = c_pT(z_T) + gz_T \tag{3}$$

where

$$c_pT(0) + Lq(0) = h(0) = \text{moist enthalpy } (s+Lq) \text{ at the ground}$$

and

$$c_pT(z_T) + gz_T = s(z_T) = \text{dry static energy at } z_T.$$

Equation (3) defines z_T. Equation (1) gives us that

$$\int_0^{z_T} M_c \frac{\partial \theta}{\partial z} dz = L(E + \text{conv. of water vapor}) \tag{4}$$

The simplest choice of M_c consistent with (3) and (4) would be to make M_c a constant between $z = 0$ and $z = z_T$. Such a choice would lead, via (2), to a better fit to the observed heating during GATE than that obtained by Lord. However, it would do poorly for the Marshall Islands. Following an admittedly *ad hoc* procedure, one can do much better. In Figure 1 we show s, h and h* (saturated moist enthalpy) *vs.* p for GATE and the western Pacific (during the Marshall Islands nuclear tests of the late 1950s). (Note that s is very nearly linear in p – rather than z. This implies that $\partial s/\partial z \propto \rho$ and hence Q will follow M_c rather than M_c/ρ.) Clearly, air above the minimum in h cannot participate significantly in cumulus convection. We shall assume (counter to the most straightforward arguments) that all air below the minimum can participate in cumulus convection,[1] and that the

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[1]Conventional reasoning (Holten, 1972) would hold that only air with values of h such at h* at some greater height is less than h can become convectively buoyant. However, Augstein, *et al.*, (1974) show that at any instant the minimum in h arises from a precipitous drop in h from larger values below, and that all the air below can be convectively unstable (*viz.* Figure 2). The layer in which the precipitous drop occurs is identified with the trade inversion. The broad minimum in Figure 1 seems likely to be the result of averaging over a trade inversion which is moving up and down.

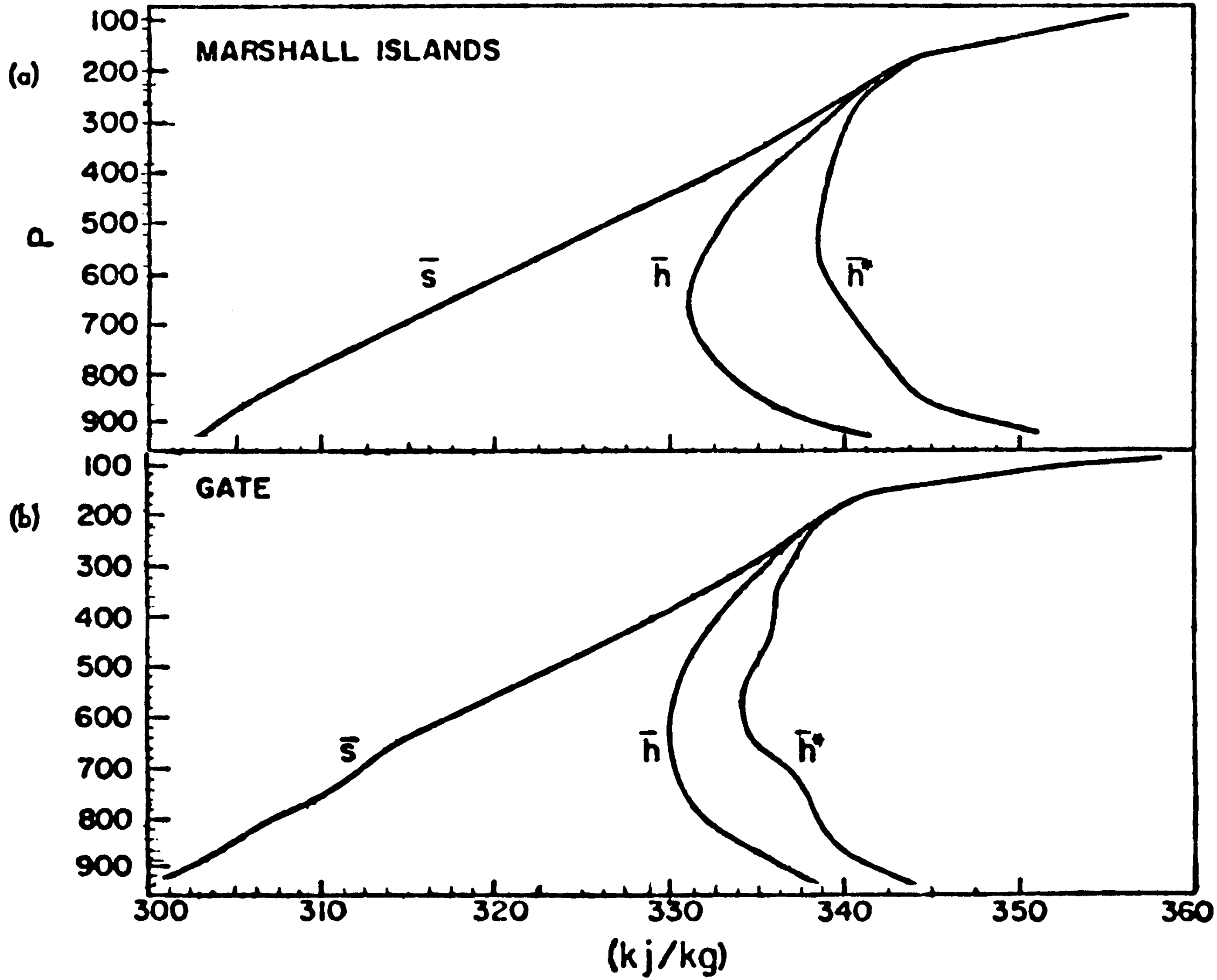

Figure 1. Dry static energy, $\bar{s}$, moist static energy, $\bar{h}$, and saturation moist static energy, h^*, in units of kj kg^{-1} *vs.* height (mb). From Fig. 4.7 in Lord, 1978.

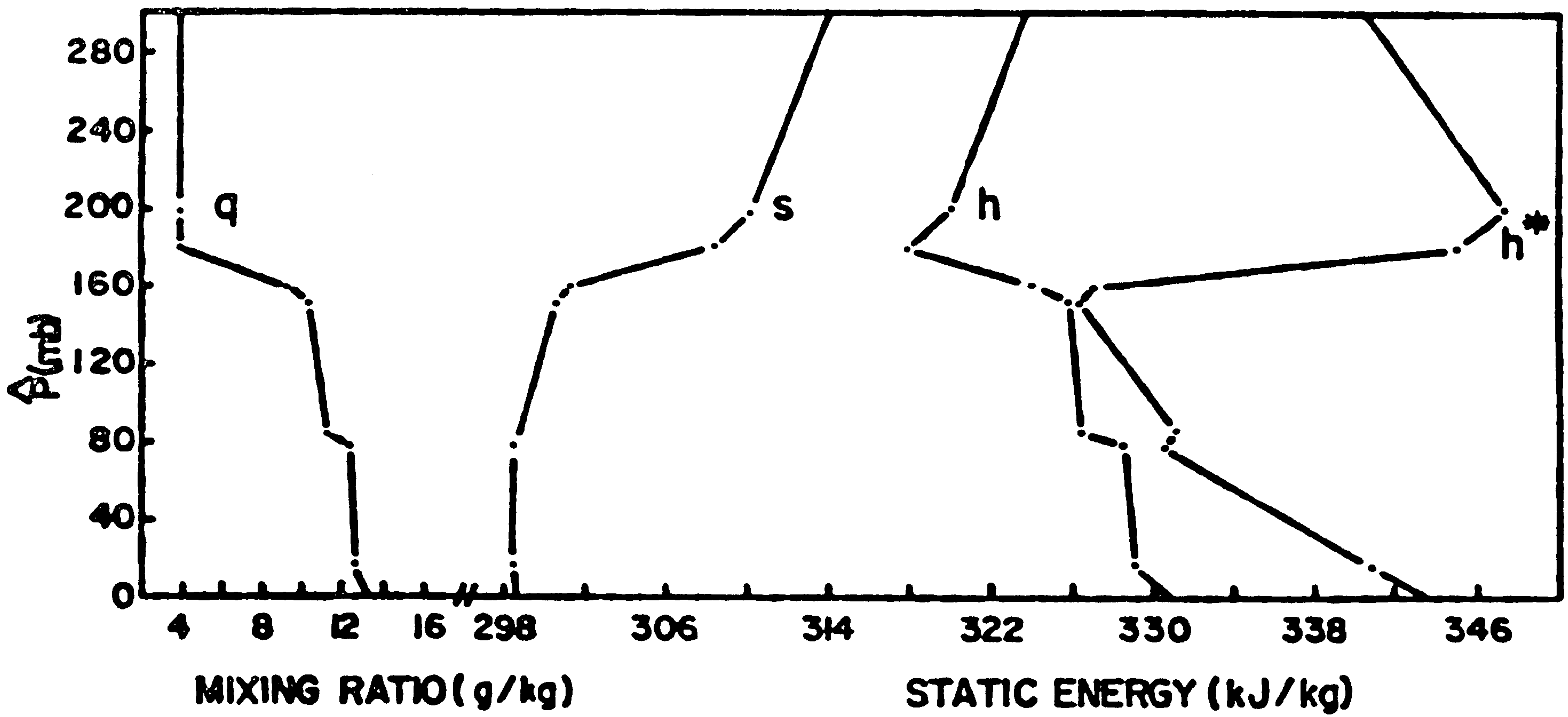

Figure 2. Vertical profiles of mixing ratio, dry static energy, moist static energy, and saturation static energy for the ship Planet, February 7-12, 1969 (Augstein, *et al.*, 1974).

moisture convergence referred to in Eq. (1) is that which takes place below the minimum in h. We next assume that portion of M_c which arises from evaporation, $M_{co} = E/q_o$ (where E = surface evaporation and q_o = specific humidity of surface air) detrains at z_T (*viz.* Eq. (3)), while that portion of the air converged between some z and δz, $\delta(\rho w)$ gives rise to a $\delta M_c = \delta(\rho w)$ which detrains at that z_d where $s(z_d) = h(z)$. This procedure completely specifies the vertical distribution of M_c. The above procedure is unambiguous when low level convergence is occurring. Its application is schematically illustrated in Fig. 3. Additional considerations must be applied when we have divergence. Indeed, when divergence exceeds evaporation, drying must occur. For the purposes of this brief note divergence will be ignored.

We shall use data from the Marshall Islands (Reed and Recker, 1971) and from the third phase of GATE (Lord, 1978) to calculate the vertical distribution of cumulus heating. For both GATE and the Marshall Islands it was the case that evaporation was small (~20% of convergence in trough regions), and, hence, we will ignore M_{co}. Figure 1 gives the vertical distribution of s and h for both cases. The distributions do not change markedly during the passage of an easterly wave. Hence this figure can be used to determine at what height air converged below h's minimum will eventually detrain. Figure 4 shows ω (p–velocity ~ $-\rho w$) *vs.* p for the convergent phases of easterly waves in the Marshall Islands (taken from Fig. 8 of Reed and Recker, 1971) and during GATE (taken from Fig. 5.6 of Lord, 1978). Note that during GATE convergence (increasing $\rho\omega$ with height) was confined far more closely to the surface than during the Marshall Islands tests. We now calculate M_c using the approach described above and in Fig. 3. The results are shown in Figs. 5 and 6. Also shown is the cumulus heating given by Eq. (2) – using $\partial s/\partial z$ from Fig. 1. Note that the vertical distribution of heating is very different in the two cases – largely but not totally reflecting the different distributions of M_c. The differences in M_c arise from differences in the large-scale low-level convergence.

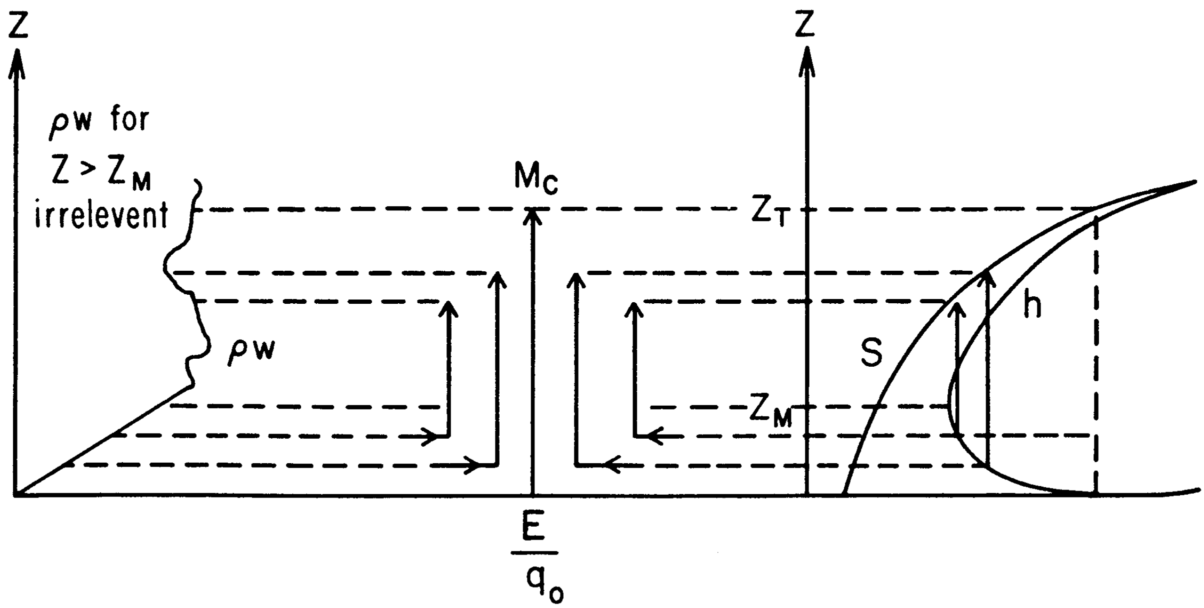

Figure 3. Schematic illustration of how M_c is related to distribution of s, h and $\rho\omega$ ($\sim -\omega$).

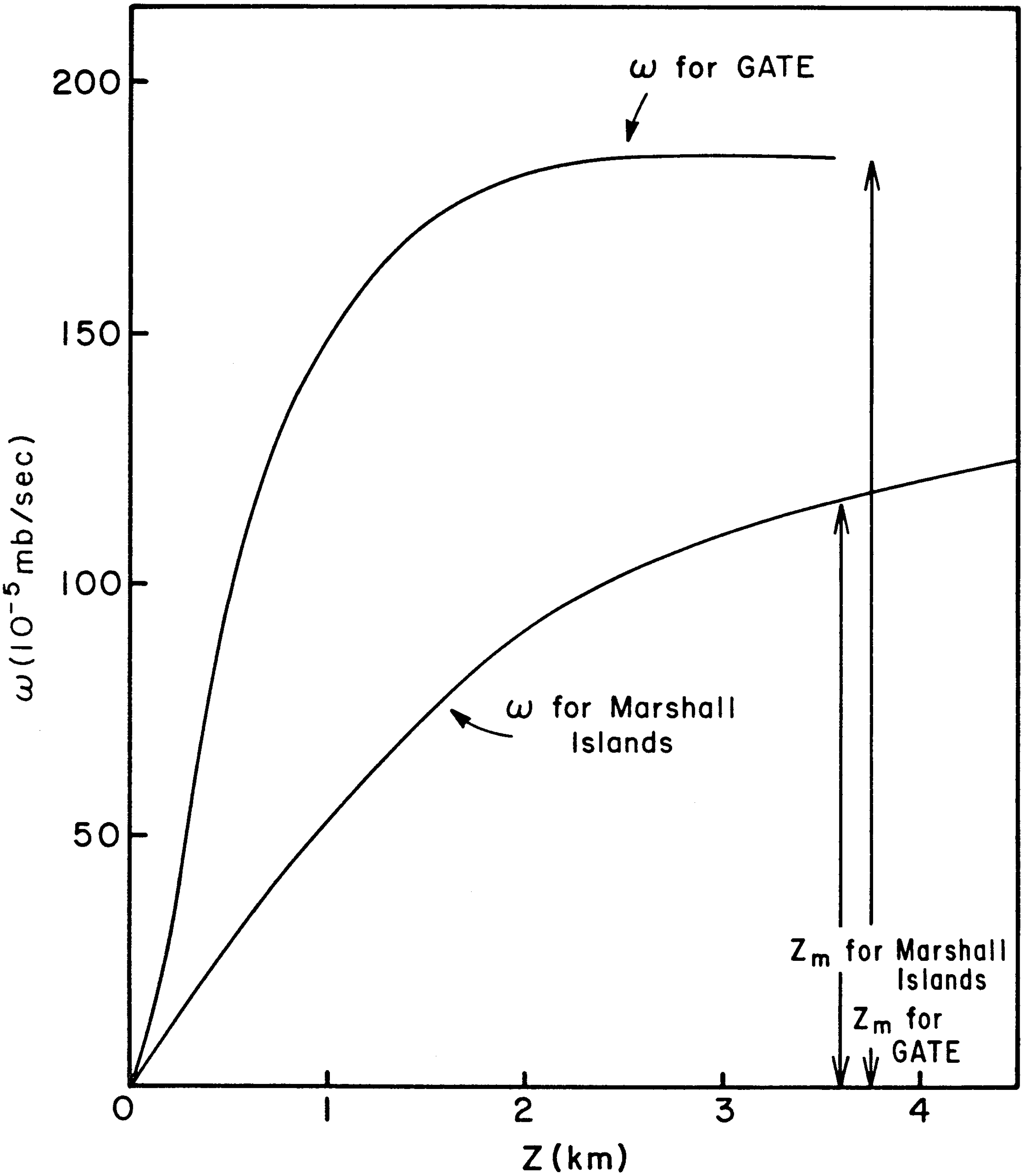

Figure 4. Vertical distribution of $-\omega$ (10^{-5} mb/sec) during convergent phase of easterly wave.

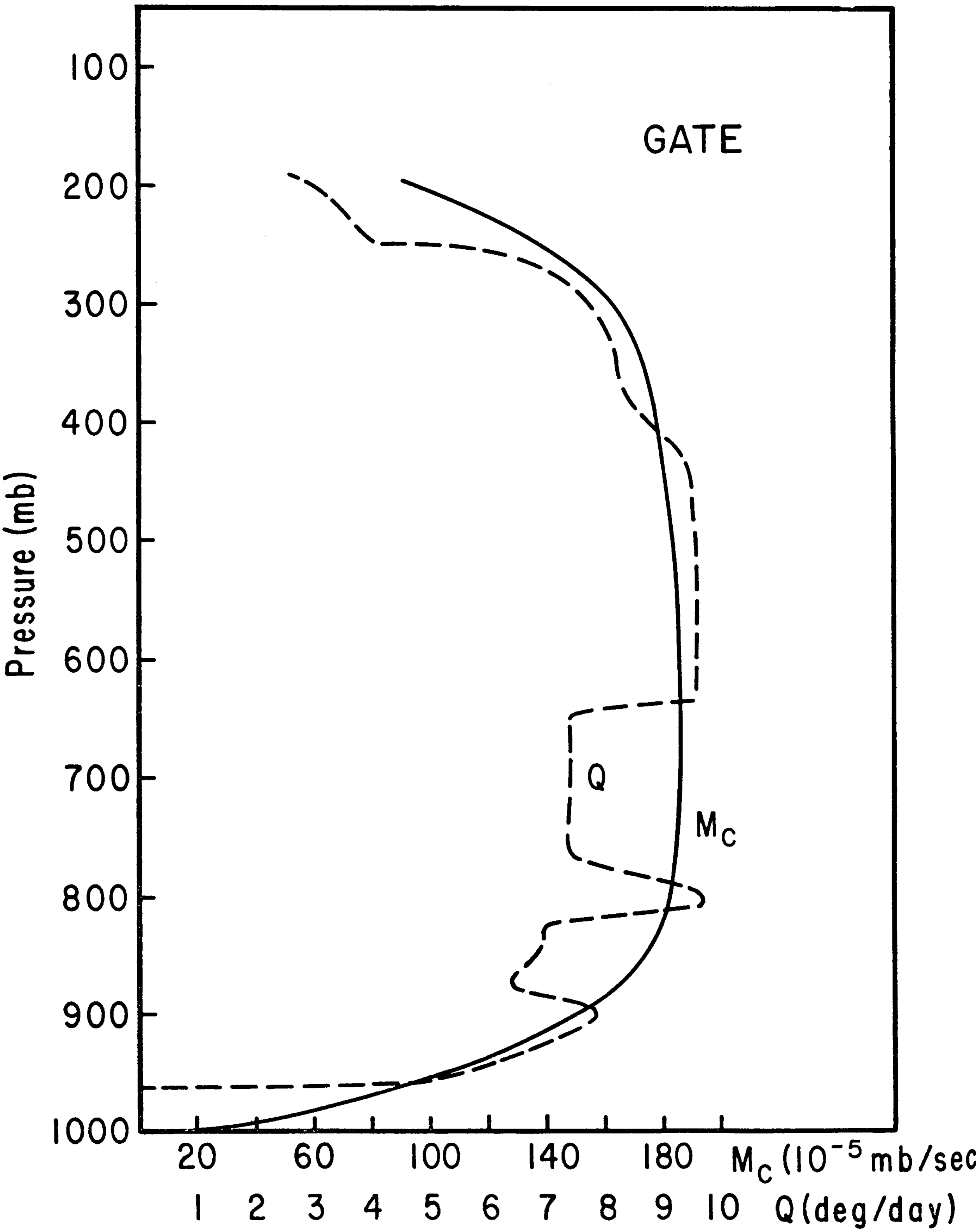

Figure 5. Predicted M_c and Q for GATE.

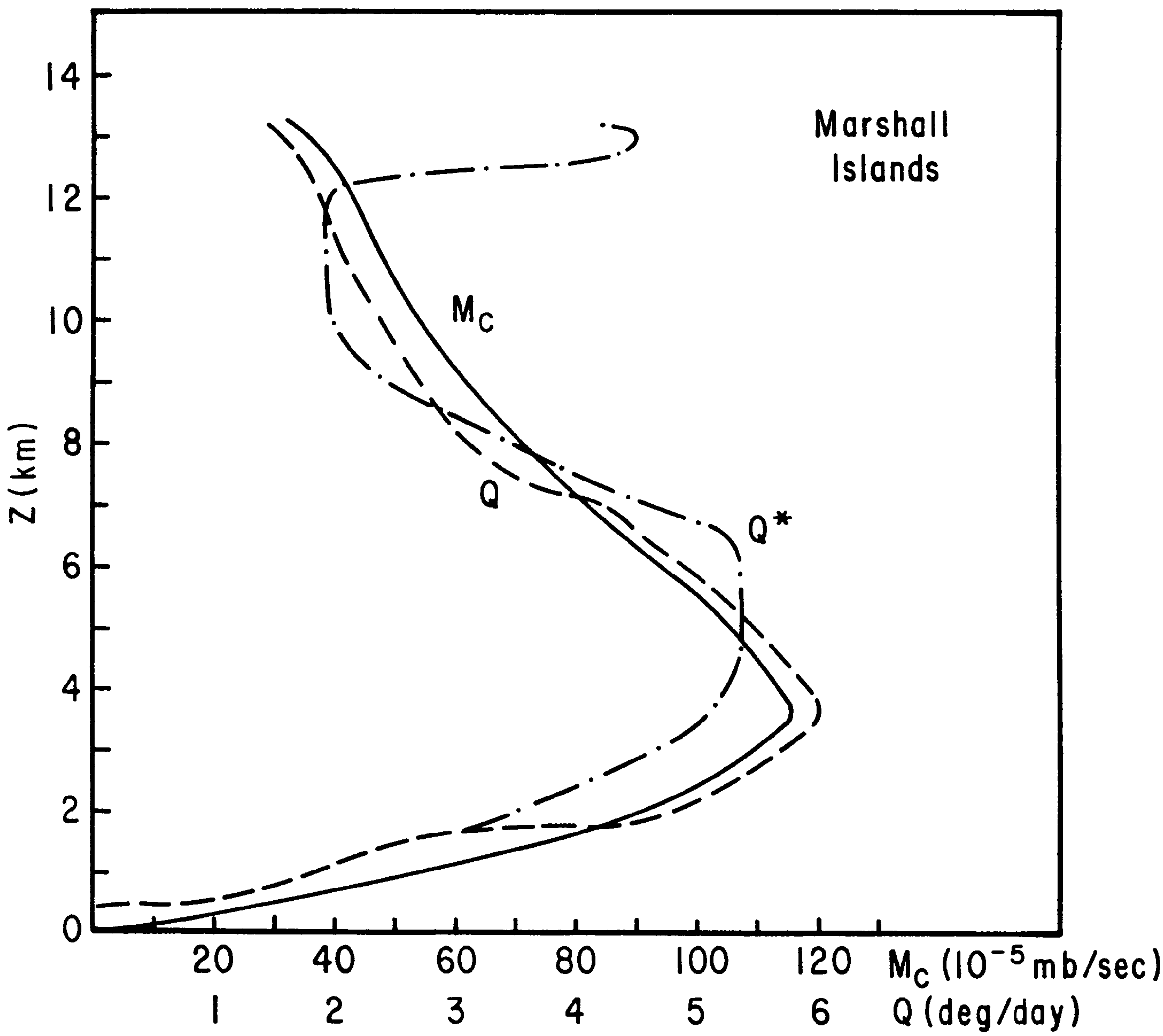

Figure 6. Predicted M_C, Q and Q* for Marshall Islands.

For the GATE case large-scale convergence is confined to a region well below the minimum in h. The converged air, therefore, has a large value of h and does not detrain until it reaches the upper troposphere. In between there is a large region of constant M_C. For the Marshall Islands large-scale convergence occurs throughout the region below the level of minimum h, and hence detrainment begins shortly above this level. Interestingly, in both cases the predicted profiles are very similar to the observational estimates of cumulus heating (*viz.* Fig. 14 in Reed and Ricker, 1971 and Fig. 5 in Sarachik). This measure of agreement in two very different cases supports the notion that the vertical distribution Q depends strongly on the distribution of low-level convergence. In contrast, the Arakawa-Schubert scheme, wherein the distribution of Q depends strongly on the distribution of ω at upper levels (Stark, 1976), yields extremely poor agreement with observed distributions. Thus, in addition to being so obscure, the Arakawa-Schubert scheme seems to be in conflict with observations.

To be sure, the results in Figs. 5 and 6 are not in perfect agreement with observations – obviously observations are not perfect either. First there is a small underestimate of total heating in both cases (~0 (10-20%)) which almost certainly results from ignoring evaporation. Finally, in Fig. 5, for the Marshall Islands, the peak in Q is 1-2 km lower than observed, while in Fig. 6, for GATE, the low-level wiggles are somewhat more extreme than those observed. These modest discrepancies are associated with small variations in $\partial s/\partial z$ with height (*viz.* Fig. 1, and Eq. (2)). While such variations of $\partial s/\partial z$ are probably not of fundamental consequence (and probably not very measurable), they do effect the detailed structure Q. In Fig. 7 we show the observed zonally averaged distribution of s (from Oort and Rasmusson, 1971) for the equatorial region. The use of this profile rather than that in Fig. 1 (for the Marshall Islands) does not greatly alter the distribution of M_c, but does lead to a new heating distribution – labelled Q* in Fig. 5. Q* peaks at an altitude in agreement with observation but has a second (unobserved) peak in the upper troposphere. There is no reason to worry about this upper level peak; the heating would rapidly alter s so as to eliminate the peak.

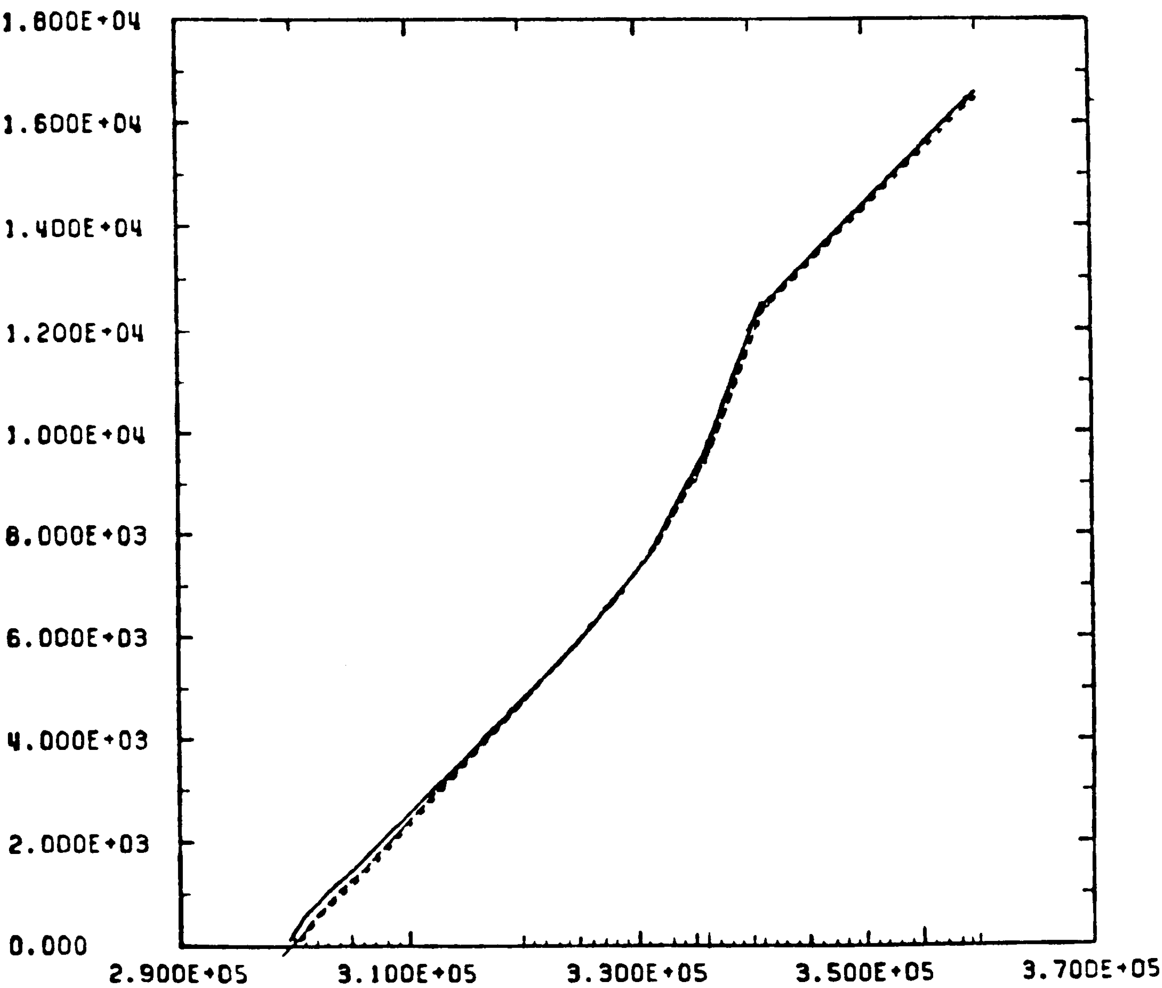

Figure 7. s *vs.* z from Oort and Rasmusson (1971).

In conclusion, our results suggest that the physically accurate parameterization of cumulus heating may be relatively simple – rather than a hopelessly difficult task. There does remain one difficulty mentioned at the beginning of this note. Namely, the detailed validity of the moisture budget given by Eq. (1). Intuitively, it seems that the development of deep instabilities from previously stable configurations might prove difficult in the presence of such a budget constraint. In addition, we know that shallow cumulus clouds do *not* rain efficiently. Thus it might suffice to replace (1) with a relation

$$\text{Precipitation} = f(z_T)\left\{\text{Evaporation} + \text{Convergence of Moisture}\right\} \tag{5}$$

where (3) defines z_T, and $f(z_T)$ is less than one, approaching one as z_T approaches the upper troposphere. At the moment, the determination of $f(z_T)$ is *ad hoc*, but, it should not be difficult to tune.

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