Errata and Supplements

Dynamics in Atmospheric Physics

(2^{nd} printing, 1993)

Page 1 On the second line of the second paragraph, replace 'atempt' with 'attempt.'

Page 6: On line 7, replace '$\mathcal{A}$ is due ...' with '$\mathcal{A}$, in these models, is due ...'.

Page 6: Add to the end of the last paragraph the following: 'In reality, this is unlikely to be a good approximation.'

Page 7: Replace Equation 2.4 with

$$\text{div flux} = -F[T]$$

Page 11: The ordinate in Figure 2.2 should be labelled T(°C) rather than τ(°C).

Page 15: Append to the end of Chapter 2 the following exercise:

Exercise

2.2 Trace the behavior in Figure 2.4 to the various assumptions — especially the use of annual averages. Was it appropriate to replace annual means with equinoctial values? How would matters have changed, had we correctly taken means?

Page 26, 7 lines from bottom: Add the following to the end of the paragraph: 'What would happen if α_4 were to equal $2\times10^{-7}\text{sec}^{-1}$?'

Page 64: Add to the end of item 3 the following: 'It also lasts longer there. What might be going on?'.

Page 82: Append to the end of Chapter 5 the following exercise:

Exercise

5.1 With reference to Figures 5.1-5.14, note significant differences between the Northern and Southern Hemispheres. Comment on these differences where possible. Pay particular attention to Figure 5.12. Comment on the different patterns of seasonality.

Page 86, last two lines: The sentence "The stress tensor, ..." should be "The stress tensor, σ_{ij}, represents the flux of i-momentum in the *minus* j-direction (recall that $\vec{n}$ is the outward normal whereas we are considering the force exerted on S by the fluid outside R)."

Page 88: The definition of δ_{ij} was omitted. δ_{ij} is the Kronecker Delta, where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$.

Page 92, 3 lines from bottom: 'homog., incomp. fluid' should be 'homogeneous, incompressible fluid'.

Page 94: The quantity $\frac{\vec{\omega}(\vec{\omega}\cdot\vec{R})}{\omega^2}$ in Figure 6.1 should be $\frac{\vec{\omega}(\vec{\omega}\cdot\vec{r})}{\omega^2}$

Page 99, middle of the page: The sentence beginning 'In order to exploit Ro$\ll$ 1 in order to' should begin 'In order to exploit Ro$\ll$ 1 to'.

Page 99: Append to the end of Chapter 6 the following exercise:

Exercise

6.1 Evaluate dissipation for both molecular and eddy coefficients of viscosity. For what length scales does each reach 1 degree C per day?

Page 100: On line 4, before the word 'Nevertheless', insert the sentence, 'The two are related; for example, the storm paths along which travelling cyclones travel are significantly determined by the planetary scale waves forced by inhomogeneities in the earth's surface.'

Page 110: The reference to Holton, 1979, near the top of the page, refers to the second edition. The page reference in the third edition (Holton, 1992) is page 67.

Page 112: Equation 7.27 should be

$$\phi^* = \tan^{-1}\{[(1+2R)^{1/2} - 1]^{1/2}\}.$$

Page 132, line 4: Add the following parenthetical remark following 'northward excursion of ϕ_0': (Recall that the intensity of the Hadley circulation is proportional to $(\overline{\Theta}-\overline{\Theta}_E)$; *viz* Equation 7.42.).

Page 135: Append the following to Question 7.5:

Question 7.5 is fairly open ended, but it can be approached relatively simply. Needless to say, I don't expect a truly analytic solution, but one can get fairly far with relatively simple calculations. Let's focus on the simpler, symmetric case. Let be the latitude separating rising streamlines from descending stream lines. Let the upper poleward branch of the Hadley circulation consist in a well mixed bundle of streamlines, whose angular momentum is characteristic of the angular momentum of the streamlines that have risen up to the latitude in consideration: i.e., for $\phi < \phi_m$,

$$M_b(\phi) \approx \frac{\int_0^\phi M(\phi)\cos\phi d\phi}{\int_0^\phi \cos\phi d\phi}$$

where $M_b(\phi)$ is the angular momentum of the upper bundle of streamlines, and $M(\phi)$ is the angular momentum of a streamline originating at the surface at latitude ϕ. For simplicity, we assume that there are equal updrafts per unit area throughout the upwelling region. For $\phi > \phi_m$,

$$M_b(\phi) \approx \frac{\int\limits_0^{\phi_m} M(\phi)\cos\phi d\phi}{\int\limits_0^{\phi_m} \cos\phi d\phi}.$$

The above allow us to calculate $U(H)$ and $\bar{\Theta}$ as in the original argument, and to evaluate ϕ_H and $\bar{\Theta}(0)$. For purposes of estimation, we might choose $\phi_m \approx 0.5\phi_H$. One might have to do a bit of iteration in order to get everything to work out. At this stage, you might want to see how well you are replicating Figure 7.6, and discuss remaining differences.

The asymmetric case is treated similarly, but is more complicated because the changes wrought by considering the bundle of streamlines are much greater so that iteration becomes more cumbersome. However, there is not too much trouble in estimating how much the easterlies will be reduced at the equator, and how the surface winds will be changed.

As you play with the problem in the above fashion, you will develop a better idea of how to do the problem correctly,' but the above should suffice for the homework.

Page 136: On the sixth line from the bottom, there should be a comma between the words 'clear internal'; i.e., it should be 'clear, internal'.

Page 138, last line: The right hand side of the equation should read $k(\omega_0) + \left.\frac{dk}{d\omega}\right)_{\omega_0} \omega$.

Page 139: On the last line, the sentence 'Wave motions will be excited in the fluid above the plate.' should be replaced with 'Wave motions will be excited in the fluid above the plate with frequency, σ, where $\sigma = kc$.'.

Page 141: Change the line above Equation 8.1 from 'which is, in fact, our dispersion relation.' to 'which is, in fact, our dispersion relation. Note that ℓ is the vertical wavenumber, where $\ell = 2\pi/\text{vwl}$.'.

Page 142: Immediately following Equation 8.6, 'Thus c_{p_x} and c_{g_z}' should be corrected to 'Thus c_{p_x} and c_{g_x}'.

Page 153: On the sixth line, add to the sentence ending with '$z \to \infty$' the following: 'if we assume a rigid boundary at $z = 0$'.

Page 166: At the end of the third line of footnote 9, 'on hour' should be 'one hour'.

Page 191: Add to the end of item (a) the following sentences: 'Note, however, that the choice $n = 0$ is incompatible with the channel geometry. $n = 0$ actually implies a meridionally unbounded fluid.'

Page 197, 5 lines from bottom: 'effective ℓs (meridional wavenumbers) and smaller hs than would global' should be 'effective ℓ's (meridional wavenumbers) and smaller h's than would global'.

Page 199, 4 lines from bottom: 'T_0s' should be 'T_0's'.

Page 200, 2 lines from bottom: The sentence beginning 'So what is' should start a new paragraph.

Page 203: The reference in Exercise 9.3 to Table 10.1 should be to Table 9.1 instead.

Page 207: The unnumbered equation following Equation 10.16 should be

$$w^* = -\frac{1}{p_0}\left(\underbrace{\frac{\partial p'}{\partial t} + U_0\frac{\partial p'}{\partial x}}_{ik(U_0-c)p'=ik\rho_0(U_0-c)\Phi'} + \underbrace{w\frac{dp_0}{dz}}_{-\frac{p_0}{H}w}\right)$$

Page 212, 6 lines from bottom: '$w'\frac{\partial u'}{\partial z}$' should be '$w^*\frac{\partial u'}{\partial z^*}$'.

Page 214, third line after Equation 10.31: Add to the parenthetical remark the following sentence: 'Note, as well, that the second term on the right hand side of Equation 10.31 is simply the inverse of the group velocity divided by the damping time, a^{-1}; i.e., the distance the wave has propagated in a damping time.'

Page 218: Append to the end of Chapter 10 the following exercise:

Exercise

10.4 Show that for internal gravity wave propagation in the presence of small damping, that the attenuation of the wave in the direction of propagation depends exponentially on the ratio of the wave travel time as determined by the group velocity in the direction of propagation to the characteristic damping time.

Page 248: Equation 12.46 should be

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + e^{z^*}\frac{\partial}{\partial z^*}(e^{-z^*}w^*) = 0$$

Page 261: The following paragraph should be appended to the end of Section 13.2.6:

Before ending this section, a comment is in order on the Helmholtz velocity profile. The discontinuity in U at $z = 0$ is easily handled by the matching conditions (Equation 13.14). However, this ease tends to obscure the fact that the velocity discontinuity in the basic state leads to a *pair* of delta function contributions to d^2U_0/dz^{*2} at $z = 0$, and associated contributions to the underlying full equation for perturbations (i.e., Equation 10.8). In the next section, we will be concerned with issues like changes in sign for d^2U_0/dz^{*2}. It will prove essential to keep in mind the delta function contributions that have been effectively disguised in the present treatment.

Page 263: Equation 13.39 should be

$$\begin{aligned}\left(\frac{\partial}{\partial t} \right. &+ \left. \bar{u}\frac{\partial}{\partial x}\right)\left(\nabla_H^2\Phi' + e^{z/H}\frac{\partial}{\partial z}\left(\epsilon e^{-z/H}\frac{\partial \Phi'}{\partial z}\right)\right) \\ &+ \frac{\partial \Phi'}{\partial x}\left(\beta - \frac{\partial^2 \bar{u}}{\partial y^2} - e^{z/H}\frac{\partial}{\partial z}\left(\epsilon e^{-z/H}\frac{\partial \bar{u}}{\partial z}\right)\right) = 0.\end{aligned}$$

Page 266: The last term in Equation 13.44 should be

$$\int_0^\infty \int_{y_1}^{y_2} e^{-z/H}|\Phi'|^2 \frac{\bar{q}_y}{\bar{u} - c} dy dz = 0.$$

Page 266: Equation 13.45 should be

$$c_i \left\{ -\int_{y_1}^{y_2} \epsilon P \frac{\partial \bar{u}}{\partial z} dy \bigg|_{z=0} + \int_0^\infty \int_{y_1}^{y_2} P \frac{\partial \bar{q}}{\partial y} dy dz \right\} = 0.$$

Page 268: Equation 13.50 should be

$$v = \frac{\partial \psi}{\partial x}.$$

Page 269: Equation 13.59 should be

$$u = -\frac{\partial \psi}{\partial y} = \frac{ASt}{k(1+S^2t^2)} \sin[k(x - Syt)]$$

Page 269: Equation 13.60 should be

$$v = \frac{\partial \psi}{\partial x} = \frac{A}{k(1+S^2t^2)} \sin[k(x - Syt)]$$

Sections A.3 and A.4 Replace Sections A.3 and A.4 with the following

A.3 Numerics

The grid is specified as follows:

$$z_k = k\Delta, k = 1, \ldots, K+1, \qquad A.13$$

where the mesh size Δ is given by

$$\Delta = \frac{z^*_{top}}{K+1}. \qquad A.14$$

We approximate the second z^*-derivative by the standard formula

$$\tilde{w}_{z^*z^*} \approx \frac{w_{k+1} - 2w_k + w_{k-1}}{\Delta^2}, \qquad A.15$$

where a grid notation has been introduced: $w_k = \tilde{w}(z_k)$. The finite difference version of Equation A.3 is then

$$w_{k+1} + (\Delta^2 Q_k^2 - 2)w_k + w_{k-1} = \Delta^2 F_k \qquad A.16$$

for $k = 1, \ldots, K$. At $z = 0$ we may specify $w_0 = w_{bot}$. Thus at level 1, A.16 becomes

$$w_2 + (\Delta^2 Q_1^2 - 2)w_1 + w_{bot} = \Delta^2 F_1,$$

or

$$w_2 + (\Delta^2 Q_1^2 - 2)w_1 = \Delta^2 F_1 - w_{bot}, \qquad A.17$$

For our upper boundary condition, we have two choices: a rigid lid or the radiation condition. In the case of the latter, we assume the top is in a region of slowly varying Q^2 and above any source of forcing.

In the case of a rigid lid, $w_{K+1} = 0$, and A.16 becomes

$$w_{K-1} + (\Delta^2 Q_k^2 - 2)w_K = \Delta^2 F_K. \qquad A.18$$

For the radiation condition, A.9 becomes

$$\frac{w_{K+1} - w_{K-1}}{2\Delta} \approx -iQ_K w_K,$$

which when combined with A.16, becomes

$$2w_{K-1} + (\Delta^2 Q_k^2 - 2\Delta i Q_K - 2)w_K = 0. \qquad A.19$$

The above system is easily solved using the up-down sweep method (which is simply gaussian elimination). To begin, we introduce two new vectors, α_k and β_k, related as follows:

$$w_k = \frac{\beta_k - w_{k+1}}{\alpha_k} \qquad A.20$$

The up-sweep portion of the algorithm consists in determining α_k and β_k. To do this, we substitute A.20 (for $k = K - 1$) into A.16 (where we have defined $a_k \equiv \Delta^2 Q_k^2 - 2$ and $b_k \equiv \Delta^2 F_k$).

$$w_{k+1} + a_k w_k + \frac{\beta_{k-1} - w_k}{\alpha_{k-1}} = b_k,$$

or

$$w_{k+1} + (a_k - \frac{1}{\alpha_{k-1}})w_k = b_k - \frac{\beta_{k-1}}{\alpha_{k-1}}. \qquad A.21$$

Rewriting A.20 as follows

$$w_{k+1} + \alpha_k w_k = \beta_k, \qquad A.22$$

and comparing A.21 with A.22, we immediately get

$$\alpha_k = a_k - \frac{1}{\alpha_{k-1}} \qquad A.23a$$

$$\beta_k = b_k - \frac{\beta_{k-1}}{\alpha_{k-1}}. \qquad A.23b$$

Our lower boundary condition automatically determines α_1 and β_1.

$$\alpha_1 = a_1$$

$$\beta_1 = b_1 - w_{bot},$$

and A.23 then gives us the remaining α_k's and β_k's. A.20 will now give us w_k if we have w_K. We get this from the upper boundary condition. With a lid, it is readily shown that

$$w_K = \frac{\beta_K}{\alpha_K}$$

For the radiation condition, it can be shown that

$$w_K = \frac{-2\beta_K}{a_K + 2\Delta i Q_K - 2\alpha_K}.$$

The above algorithm solves for the complex values of w_k at each level. In practice, it is more convenient to look at the amplitudes and phases where

$$amplitude(w_k) = \left((\mathrm{Re}(w_k))^2 + (\mathrm{Im}(w_k))^2\right)^{1/2}$$

and

$$phase(w_k) = \arctan\left[\frac{Im(w_k)}{Re(w_k)}\right].$$

Page 303: The title of the book by J.T. Houghton should be *The Physics of Atmospheres*.

Addition to Chapter 1

Finally, I hope, through this book, to communicate the obvious fact that dynamic meteorology is not so much a body of canonical results, but rather an active research field in a state of flux.

Supplement to Chapter 14

14.6 Geometric stabilization

Both Equation 14.33 and Figure 14.1 suggest an alternative approach to stabilization. Figure 14.1 clearly shows the existence of a short-wave cutoff for baroclinic instability in a 2-level model. Instability disappears if

$$\frac{k^2\lambda^2}{2} > 1.$$

Now, if our fluid is confined within a channel of width L, then we will have a meridional wavenumber, ℓ, in addition to the zonal wavenumber, k. Thus, k in the above equation, must be replaced by the total wavenumber, $K = (k^2 + \ell^2)^{1/2}$, and the above condition becomes

$$\frac{K^2\lambda^2}{2} > 1.$$

Moreover, $\ell \geq \pi/L$, and, hence,

$$\frac{K^2\lambda^2}{2} > \frac{(\pi/L)^2\lambda^2}{2} > 1.$$

Thus one might *geometrically* stabilize the fluid by confining it in a sufficiently narrow channel. Similarly, for a fixed L, on could stabilize the fluid by raising the upper boundary and thereby increasing λ.

The situation is not quite so simple as the above suggests. If $\beta \neq 0$, then the continuous problem (as opposed to the 2- level problem) does not have a short-wave cutoff. The continuous problem is known as the Charney Problem (Charney, 1947). However, a special case of the continuous problem wherein $\beta = 0$ or, more generally, $\overline{q}_y = 0$, and where the fluid has an upper boundary at a finite height (known as the Eady Problem (Eady, 1947)) does have a short-wave cutoff. Lorenz (1962), noting the relevance of the Eady Problem to baroclinic instability in a rotating annulus, showed that reducing the width of the annulus could stabilize the waves. In the Eady Problem, instability arises from the delta-function contributions to $\overline{q}_y$ at the top and bottom boundaries.

As it turns out, baroclinic waves in the atmosphere are meridionally confined, not only by the finite extent of the earth, but by the jet like structure of the mean zonal wind (Ioannou and Lindzen, 1986, 1990). Lindzen (1992) has recently suggested that the atmosphere could be stabilized with respect to baroclinic instability, while maintaining surface temperature gradients, by eliminating $\overline{q}_y$ in the bulk of the troposphere while concentrating $\overline{q}_y$ at some upper surface whose height is sufficiently great. This height turns out to be of the order of the tropopause. Observations (Hoskins, et al, 1985) do indeed suggest that at midlatitudes $\overline{q}_y$ is much smaller in the bulk of the troposphere than it is at either the surface or at the tropopause. The implications of this sort of *geometric* stabilization are currently being explored.

Additional references

Holton, J.R. (1992). *An introduction to dynamic meteorology*, 3rd edition, Academic Press, San Diego, 507 pp.

Hoskins, B.J., M.E. McIntyre, and A.W. Robinson (1985). On the use and significance of isentropic potential vorticity maps. *Q.J. Roy. Met. Soc.*, **111**, 877-946.

Ioannou, P., and R.S. Lindzen (1986). Baroclinic instability in the presence of barotropic jets. *J. Atmos. Sci.*, **43**, 2999-3014.

Ioannou, P., and R.S. Lindzen (1990). W.K.B.J. approximation of the stability of a frontal mean state. *J. Atmos. Sci.*, **47**, 2825-2831.

Lindzen, R.S. (1992). Baroclinic neutrality and the tropopause. *J. Atmos. Sci.*

Lorenz, E.N. (1962). Simplified dynamic equations applied to the rotating-basin equations. *J. Atmos. Sci.*, **19**, 39-51.