The Interaction of Waves and Convection in the Tropics

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(Manuscript received 9 June 2002, in final form 9 July 2003)

ABSTRACT

Interest in tropical waves and their interaction with convection has been rekindled in recent years by the discovery, using satellite infrared data to track high clouds, that such waves closely display the dispersive properties of linear, inviscid wave theory for an atmosphere with a resting basic state and equivalent depths between 12 and 60 m. While several current approaches focus on internal modes in the atmosphere, this is inconsistent with the absence of internal modes in the atmosphere, which is characterized by a single isolated eigenmode and a continuous spectrum. It will be shown, using an extremely simple approach to convection, that the observed properties of waves are consistent with a continuous spectrum. The approach assumes that the total convection is determined by mean evaporation, but that the convection is patterned by zero-averaged perturbations to triggering energy following the recent approach of Mapes. This is, perhaps, the simplest hypothesis that can be applied. The observed convection associated with the migrating semidiurnal tide is used to calibrate the time scale for the convective response to patterning, which is the only adjustable parameter in this formulation. It is shown that this time scale leads to not only the observed phase of the semidiurnal heating but also the observed phase lead of low-level convergence in tropical waves vis-à-vis the convective heating. Finally, it is shown that this phase is sensitive to the equivalent depth, which it is suggested is the basis for the selection of equivalent depth. Reasonable simulations of observed waves are readily obtained.

1. Introduction

An intriguing feature of tropical waves is the fact that a wide spectrum of such waves seems to be characterized by equivalent depths in the range of about 12–60 m. This is illustrated in Fig. 1 taken from Wheeler and Kiladis (1999), and was earlier noted by Takayabu (1994). A notable exception is the Madden–Julian oscillation (MJO; in that its dispersive properties do not correspond to a particular equivalent depth), but there are other exceptions as well, including tides. We will return to the exceptions later in this paper; for the moment we will concentrate on the 12–60 m waves. The notion of “equivalent depth” arises in tidal theory and is discussed in a more general context in Lindzen (1967) and in Lindzen and Matsuno (1968)—all of whom focus on equatorial waves of the sort investigated by Wheeler and Kiladis (1999). A pedagogical treatment may be found in Lindzen (1990). In section 2, we review the theory of linearized waves on a static basic state—from which the concepts and terminology for tropical waves arises. Equivalent depth is a measure of vertical wavelength (or of exponential scale, depending on the sign and magnitude of the equivalent depth). An equivalent depth of 40 m corresponds approximately to a vertical wavelength of 8 km. The possible importance of such equivalent depths was noted by Stevens and Lindzen (1978, hereafter SL) who pointed out that a quarter wavelength, which was about the depth of the tropical convective boundary layer, might maximize convergence in such a layer. More recently, Wheeler and Kiladis (1999) showed with satellite data that equivalent depths in this neighborhood actually dominated the spectrum. Early attempts to explain the prevalence of particular equivalent depths are reviewed in section 3. It remains tempting to associate these equivalent depths with internal normal modes, but as noted in section 2, the atmosphere does not, in general, possess such normal modes. Rather, from the beginning it was recognized that the existence of such preferred equivalent depths was likely to arise from the interaction of the waves with cumulus convection and the “effective” atmospheric heating associated with the latent heat release in these clouds. However, in one fashion or another, the attempts described in section 3 proved unsatisfactory. These attempts mostly sought explanations based on instabilities arising from the wave–convection interaction.

In section 4 of the present paper, another approach is taken based on a simplification of the “triggering” approach to convection presented by Mapes (2000). Here, convection, in the gross, is in statistical equilibrium with evaporation, but low-level convergence can act so as to
pattern this convection, with convection, itself, acting to guarantee that the low convergence is consistent with the redistribution of convection. It should be stressed at the outset that this is simply an hypothesis; however, it turns out to be an hypothesis whose implications appear to be consistent with various observations.

In this approach, we usefully distinguish patterning provided by disturbances that are primarily forced by mechanisms other than convection from those that are forced by the patterned convection itself. Among the former is the solar semidiurnal tide, which we will use to determine the characteristic time scale for convective patterning. For the latter, the low-level convergence provided by the excited disturbance must be consistent with the heating needed to force the disturbance. Assuming that there is a characteristic time for the patterning to occur, the convergence must lead the heating in phase by a specified amount given from the tidal study.

This is confirmed by a recent data analysis by Straub and Kiladis (2003). In section 5, classical atmospheric wave theory (for an unbounded atmosphere without internal normal modes) is used to examine the conditions for self-consistent patterning of convection by waves forced by the patterned convection. Consistency leads to the selection of the observed equivalent depths [though for shorter period waves, the small heating associated with congestus clouds described in Mapes (2000) improves agreement]. Moreover, the vertical structure associated with these equivalent depths agrees with the observed structure in that it follows the shape of the heating in the troposphere while taking the form of upward-propagating internal waves in the lower stratosphere. Finally, in section 6, we discuss the implications of the present results, as well as some remaining difficulties in the theory.

2. Classical atmospheric wave theory

The applicability of classical atmospheric wave theory to tropical waves is by no means self-evident. However, the fact that observed tropical waves display the dispersive properties obtained from classical atmospheric wave theory (see Fig. 1) suggests that it is a reasonable place to start. This theory dates back to Laplace (1825), and its description in its present form can already be found in Lamb (1932). The version of the theory where the spherical earth is approximated by an equatorial beta plane is given in Lindzen and Matsuno (1968). All this material is covered in current textbooks (Andrews et al. 1987; Lindzen, 1990), and there is little point in repeating this material here. However, for convenience we will sketch the theory, emphasizing points of special relevance to the present study.

Beginning with the linearized equations of motion for a shallow perfect gas on a rotating sphere, we can reduce the equations to a single equation for a particular variable; for example, vertical velocity $w$. Given that the coefficients of the differential equation are independent of time $t$ and longitude $\phi$, and that the equation is separable in its colatitude $\theta$ and altitude $z$ dependence, we can obtain solutions of the form

$$w(\theta, \phi, z, t) = e^{i(tz)} e^{i(\phi+\phi_0)} \sum y_n(x) \Theta_n(\theta),$$

where $x = \int_0^z (dz/H)$, and $H = RT/g$ is the local scale.
height, where $R$ is the gas constant for air, $T$ is the basic unperturbed temperature (taken to depend only on $z$ because we are taking the basic state to be static), and $g$ is the acceleration of gravity. The height in scale heights is $x$. The equations for $y_n(x)$ and $\Theta_n(\theta)$ are given by the following, where $L$ and $\mathfrak{M}$ are operators where the subscript indicates the appropriate variable and the superscripts show the parametric dependences (on the frequency, zonal wavenumber, rotation rate $\Omega$, earth’s radius $a$, static stability $\Gamma$, etc.). The separation constant (known as the equivalent depth) is $h_n$, and $F^{\omega,s}$ is the forcing associated with frequency $\omega$ and wavenumber $s$:

$$L_y^{h_n=\omega,T}\{\Theta_n\} = 0 \quad \text{(Laplace’s tidal equation);}$$

$$\mathfrak{M}y_n^{h,T}\{y_n\} = F^{\omega,s}(x) \quad \text{(vertical structure equation),}$$

where $F^{\omega,s}(x)$ is the projection of $F^{\omega,s}(x, \theta)$ on $\Theta_n(\theta)$.

In the present paper, we will primarily use the vertical structure equation, which is characteristically of the form

$$\frac{d^2y_n}{dx^2} + \lambda^2 y_n = F_n,$$  \hspace{1cm} (4)

where

$$\lambda^2 = \frac{1}{h_n T_0} \left( \frac{dT_0}{dz} + \frac{g}{c_p} \right) - \frac{1}{4H^2}. \quad (4a)$$

Essentially, the equivalent depth (which is the separation constant) is a measure of the vertical wavenumber (or, obviously, the vertical exponential scale when $\lambda^2$ is negative), and depending on the relation of $h_n$ to zonal wavenumber, frequency, etc., the wave will essentially be an internal gravity or Rossby wave or some combination of the two. The relation between equivalent depth and vertical wavelength for $T_0$ characteristic of the troposphere is shown in Fig. 2.

It will be important to distinguish between the use of classical wave theory for forced and free waves.

**Free versus forced waves**

For *forced waves*, we are given $\omega$ and $s$. Laplace’s tidal equation is solved for $h_n$ and $\Theta_n$, where $h_n$ is an eigenvalue and $\Theta_n$ is an eigenfunction. The forcing is
expanded in these eigenfunctions (known as Hough functions), and the vertical structure equation is solved for the response to each component of the forcing, the components being the projections of the forcing on the Hough functions. If there exists a complete set of vertical eigenfunctions, then one could just as well have expanded the forcing in terms of these, but, as we shall note, this is not the case for the present problem.

For free waves, the vertical structure equation is solved in the absence of forcing. The eigenvalues are the equivalent depths of the fluid system. For shallow water, the only eigenvalue is the depth of the fluid. For a stably stratified liquid with a lid, the equivalent depths correspond to an infinite set of vertical modes. For the unbounded atmosphere, there is generally only a single eigenvalue, corresponding to a Lamb mode with an equivalent depth of about 10 km. There is also a continuous spectrum. For each equivalent depth and zonal wavenumber, Laplace’s tidal equation is solved for the eigenfrequencies \( \omega \) and the associated Hough functions. For each Hough function, one obtains a relation between frequency and wavenumber.

The name “equivalent depth” was chosen by analogy with the shallow water case where the equivalent depth was the actual depth of the fluid, and there was no need for a vertical structure equation.

Despite the fact that the atmosphere generally is found to have only a single equivalent depth (~10 km), many tropical waves are observed to behave as though they had a relatively unique equivalent depth of around 12–60 m as seen in Fig. 1 from Kiladis and Wheeler (1999). This new equivalent depth is generally attributed to the interaction of waves with tropical convection. Note as well that the MJO is not associated with a particular equivalent depth but rather with a specific period. While the MJO is not the focus of the present paper, it should be mentioned that there have been many suggestions as to how this period arises including the observation that this is the expected period for planetary-scale baroclinic instability, which might, in turn, pattern tropical convection (Straus and Lindzen 2000). There is also a long record of attempts to account for the MJO with variations of wave–conditional instability of the second kind (CISK) mechanisms (see section 3; Lindzen 1974b; Chang and Lim 1988; Wu 2003).

It should also be noted that recent attempts to interpret the Wheeler–Kiladis results in terms of internal vertical normal modes (Mapes 2000; Majda and Schefter 2001; Emanuel et al. 1994) are inconsistent with the spectral properties of the vertical structure equation for realistic atmospheres without lids. Traditional arguments that the tropopause acts as a lid are inconsistent with the low reflectivity of the tropopause, as well as the fact that the tropopause is not absolutely horizontal. Similarly, vertical normal modes such as those calculated by Fulton and Schubert (1985) are also based on the assumption of a perfectly reflecting upper boundary, and as noted by Lindzen et al. (1968), this leads to spurious resonances. To be sure, the set of vertical modes thus obtained constitute a complete set, which can be used to expand solutions, but no physical meaning attaches to such modes since the essential lid does not exist for the real atmosphere, and the failure of such modes to satisfy an appropriate upper boundary condition leads to Gibbs-type phenomena.

3. Early approaches to the interaction of waves and cumulus convection

The initial approach to such waves was the so-called wave-CISK theories (Lindzen 1974b; Yamasaki 1969; Hayashi 1970). These followed the approach of Charney and Eliassen (1964) in assuming that, if a large-scale dynamic system could lift air to the lifting condensation level, then a cooperative interaction between convection and the large-scale disturbance could lead to the amplification of the disturbance. In the Charney–Eliassen version of CISK, the lifting was due to Ekman pumping, while in wave-CISK, the lifting comes from the wave field itself. Maximum lifting at the lifting condensation level implied an equivalent depth of 10 m (with a quarter wavelength corresponding to about 500 m). This led to the suggestion that a spectrum like that displayed in Kiladis and Wheeler (1999) should exist—though with a smaller equivalent depth (Lindzen 1974a).

In these early approaches, the cumulus mass flux was taken to be proportional to convergence at 500 m. However, the constant of proportionality was generally unknown. To remedy this, Cho and Ogura (1974) sought to determine with observations the relation between cumulus mass flux and vertical velocity at the lifting condensation level. They found that the cumulus mass flux was approximately 4 times the ambient vertical mass flux at the lifting condensation level.

Although more recent discussions tend to ignore this, there was a substantial reassessment of wave-CISK over the following 5 yr. However, even in the 1960s, A. Eliassen (1975, personal communication) noted a basic problem with the very concept of CISK that was associated with his name: namely, that the lowest 2 km of the tropical atmosphere formed a turbulent trade wind boundary layer in which air was constantly being lifted above the lifting condensation level—even in the absence of any larger-scale system. In general, the break-out of deep convection is limited by the presence of a trade inversion (or more generally, the convective inhibition energy; Mapes 2000).

As noted by SL and Lindzen (1988), the ratio found by Cho and Ogura (1974) became unity if one considered ambient vertical mass flux at 2 km instead of 500 m. This led to an approach to cumulus parameterization wherein local cumulus mass flux was taken to be determined by evaporation and large-scale convergence within the trade wind boundary layer (Lindzen 1988; Geleyn et al. 1982). The resulting parameterization, modified for use with the European Centre for Medium-
Range Weather Forecasts (ECMWF) model, has come to be known as the Tiedke parameterization, though the parameterizations used by the ECMWF have evolved since. The geometry involved is schematically illustrated in Fig. 3. It was noted by SL that tropical waves were more nearly characterized by an equivalent depth of 30 m, which corresponded approximately to a vertical wavelength of 8 km, with a quarter wavelength (where one would expect a maximum in convergence) corresponding to the depth of the convective boundary layer (note that different papers associate the same vertical wavelengths with somewhat different equivalent depths because of the use of different basic-state $T$s). Unfortunately, SL found that their interaction was unable to produce instability except for gravity waves corresponding to squall systems.

In Stevens et al. (1977), emphasis shifted from wave-CISK to a view of equilibrated waves whose convergence field below 2 km served to simply reorganize convection that would occur anyway. As noted by Reed and Recker (1971) and many since, the amplitude in precipitation of tropical waves tends to equal the mean precipitation (where mean refers to a mean over the wave considered), suggesting a reorganization of existing precipitation rather than the production of additional precipitation. This is illustrated in Fig. 3. Consistent with this observation, Stevens et al. (1977) assumed that the amplitude of the wave forcing was approximately equal to the mean latent heating. Equivalently, if the average cumulus mass flux is $M_c$, then equilibration occurs when the zero-average wave contribution to the cumulus mass flux $M'_c$ has an amplitude equal to $M_c$. Thus, convection patterned on one scale can be repatterned on smaller scales, much as schematically illustrated in Fig. 4. This situation will be discussed further in the next section.

A question has long remained as to whether there actually is a causal relation between large-scale convergence and cumulus mass flux or whether it is simply a necessary balance (Arakawa and Schubert 1974; Emanuel 2000) in an equilibrated system. The work of SL already noted that wave-CISK really no longer worked with the mass budget parameterization. The purpose of the present paper is to examine the behavior of tropical waves from a perspective more nearly in line with equilibrium views.

4. Wave patterning of convection

Our present approach is a very substantially simplified version of the convective triggering approach described by Mapes (2000). In general,

$$M_c = \frac{E}{q} + \nabla \cdot \rho V,$$

where $E$ is evaporation and $q$ is specific humidity in the mixed layer, and the divergence is evaluated below the top of the convective boundary layer.

In the absence of large-scale convergence, we expect convection to be randomly occurring with a spatially uniform probability distribution. In the mass budget approach, $M_c$ responds to directly determined convergence within the convective boundary layer. However, in the patterning approach, convection automatically provides self-consistent low-level convergence, but perturbations to convergence determine the pattern of convection (Mapes 2000, it should be noted, emphasized other perturbations). Of course, if the perturbation provides more convergence, the convection will not have to provide as much. This situation is somewhat analogous to the situation in Benard convection where small irregularities in the bottom plate can determine the plan form of the convection (Koschmieder 1993).

Note that in both the mass budget and the patterning approaches, waves do not change the total amount of convective activity. In the patterning approach, the low-level fields of the wave perturbation biases the random breakdowns of convective inhibition energy (CIE) produced by boundary layer turbulence so as to pattern the convection that would occur anyway. Thus, the mean amount of convection is essentially determined by the mean evaporation. While there is evidence that squall
systems play an important role in the convection itself, other systems ranging from gravity waves to easterly and Kelvin waves to the Hadley and Walker circulations serve primarily to pattern the convection that would otherwise exist. Moreover, all the sources of patterning can simultaneously coexist as schematically illustrated in Fig. 4.

Recall that the amplitude in precipitation of tropical waves tends to be about equal to the mean precipitation averaged over the wave, suggesting a reorganization of existing precipitation rather than the production of additional precipitation. Note that precipitation rate and $M_0$ are closely related since the moisture rising in the cumulus tower condenses due to adiabatic cooling. It is tempting to assume that the latent heat thus released serves to heat the atmosphere. However, as shown by Arakawa and Schubert (1974) and Ooyama (1971), cumulus convection does not directly heat the ambient atmosphere. Rather, that portion of the mean vertical velocity that is carried in cumulus towers also does not contribute to adiabatic cooling of the ambient atmosphere. Thus, we must subtract this part from the adiabatic cooling; that is, the adiabatic cooling term becomes

$$\left(\rho w - M_0\right) \frac{\partial \theta}{\partial z}. \quad (6)$$

The term, $M_0 (\partial \theta / \partial z)$, constitutes an effective cumulus heating term. The patterning of the convection gives rise to a contribution to the effective cumulus heating in the form of the pattern. The contribution of the patterning to the mean is, however, zero. This effective heating forms an essential link in the interaction of large-scale dynamics with convection since it serves as a forcing for motions. However, as previously mentioned, it proves useful to distinguish two different modes of interaction. In the first, we are dealing with motion systems that have their origin in processes separate from effective cumulus heating. These motion systems, however, provide low-level convergence with the resulting effective cumulus heating modifying the motion system.

The second mode involves self-excitation where, for example, motion systems forced by effective cumulus heating provide low-level convergence that, in turn, triggers the convective pattern that forces the wave. Self-excitation requires that the phase of the patterning be such as to produce the wave required for the patterning. We will discuss this further in the next section where it will be seen that this leads to the selection of a particular equivalent depth (or depths).

An important and well-studied example of the first mode of interaction is the solar semidiurnal migrating tide (Chapman and Lindzen 1970). Forcing of this tide is primarily due to insolation absorption by ozone (Butler and Small 1963) and water vapor (Siebert 1961). Such forcing leads to approximately the observed amplitude of the surface pressure oscillation, but phase is about 1 h off (maxima at 0900, 2100 UTC instead of 1000, 2200 UTC). Lindzen (1978) and Hamilton (1981) showed that the observed semidiurnal component of rainfall provided additional forcing that would correct the discrepancy. While it is easy to imagine local factors causing daily variations in rainfall, there is reason to expect that the semidiurnal component is primarily due to patterning by the preexisting global migrating tide. In both Lindzen (1978) and Hamilton (1981), when daily variations in precipitation were Fourier decomposed at individual stations, it was found that the diurnal (24 h) component varied in local phase according to the nature of the station (land, atoll, island, etc.) as one might expect for local factors. On the other hand, the semidiurnal (12 h) component was found to have approximately the same phase in local time everywhere as would be characteristic of forcing primarily by the preexisting migrating tide. However, the calculated convergence due to the global migrating tide was one order of magnitude less than needed to account for the observed precipitation. It was already noted by Lindzen (1978) that this implies that patterning rather than direct forcing of the convective pattern is involved. However, in contrast to tropical waves, the tidal component of rainfall is only a fifth of mean rainfall. Assuming that patterning is involved, this would imply that the time needed for the convective response to the patterning perturbation is long compared to the tidal time scale (12 h/2π). Indeed, one can use the amplitude of the semidiurnal tide in rainfall to estimate the characteristic response time. The simple calculations used to make this estimate also have two additional implications that can be checked in order to test the patterning hypothesis. First, the ratio of wave time scale (period/2π) to convective response time also determines the phase lag between the effective heating and the low-level convergence responsible for the patterning. Thus, one can immediately check if this phase lag is such as to correct the discrepancy in the observed semidiurnal tide. Second, the finite convective response time also implies that there must be a specific phase lead for low-level convergence relative to effective heating for tropical waves of the sort described by Wheeler and Kiladis (1999), if these waves do, in fact, involve the hypothesized patterning mechanism. The recent analysis of Straub and Kiladis (2003) allows us to check this at least for equatorial Kelvin waves. As we will see, there is quantitative agreement in both cases supporting the present form of the patterning hypothesis. Note, that consistent with our attempt to consider the simplest possibility, we have taken the convective response time to be independent of the horizontal scale and magnitude of the perturbation. Presumably, shortcomings in this assumption should manifest themselves in obvious discrepancies in our results.

To crudely analyze this situation, we will take the vertical velocity within the convective boundary layer
Fig. 5. Perturbation cumulus mass flux as a fraction of mean cumulus mass flux vs time scaled by wave period. Also shown is the Fourier projection of the solution on frequency $\omega$: (a) $a/\omega = 0.16$; (b) $a/\omega = 5$.

to be given by $w \sin(\omega t)$. Let $M'_c$ be the cumulus mass flux responding to wave patterning induced by perturbations in low-level convergence, while $\overline{M}_c$ is the mean mass flux. Let $a^{-1}$ be the characteristic response time of $M_c$ to the patterning provided by the perturbation in $w$. Patterning is taken to concentrate convection in regions where $w$ is positive and suppress convection where $w$ is negative. The following equation roughly describes how we expect $M'_c$ to behave:

$$\left( \frac{1}{a} \frac{d}{dt} + 1 \right) M'_c = \overline{M}_c \text{ sgn}(\sin(\omega t)), \quad (7)$$

where $\text{sgn}(x) = 1$ for $x > 0$, and $\text{sgn}(x) = -1$ for $x < 0$.

For convenience, we will let $\omega t = x$, so that the above equation becomes

$$\left( \frac{\omega}{a} \frac{d}{dx} + 1 \right) M'_c = \overline{M}_c \text{ sgn}(\sin x). \quad (7a)$$

Although $M'_c$ will, of course, be distorted from a sine wave, its impact on the wave will be associated with its projection on the sinusoidal $\omega$ component.

Figures 5a and 5b show the behavior of $M'_c/\overline{M}_c$ for $a/\omega = 5$ and 0.16. In general, as $a/\omega$ becomes large, $M'_c/\overline{M}_c$ approaches one, and the phase lag goes to zero. This is already evident in Fig. 5a. On the other hand, as $a/\omega$ becomes small, $M'_c/\overline{M}_c$ decreases, and the phase lag for convection approaches 90°. For the solar semidiurnal tide, Lindzen (1978) finds that $M'_c/\overline{M}_c = 0.2$. This, as can be shown, corresponds to $a/\omega = 0.16$, or $a^{-1} \approx 11.94$ h. The phase lag is about 81.8°. The effective heating associated with this phase lag is, indeed, what is needed to correct the phase of the semidiurnal tide forced by ozone and water vapor heating alone. This offers some confidence that the value of $a$ determined by means of the semidiurnal tide is reasonable. It should be added that this value is also compatible with $M'_c/\overline{M}_c$ being on the order of unity for tropical waves with periods on the order of 5 days or longer. Note that for $|a/\omega| \approx 1$, the phase lag becomes essentially $|\omega/a| \times 90°$. This is illustrated in Fig. 6.

The waves studied by Straub and Kiladis (2003) have periods in the neighborhood of 3–5 days. From Fig. 6, we see that there must be a low-level phase lead in convergence [relative to outgoing longwave radiation (OLR)] of about 40° if patterning is to be appropriate. This is, in fact, what they find, though the time resolution of their data is only just adequate to determine such a phase lead albeit with some uncertainty.

5. Explicit calculation of equivalent depth for self-consistent patterning of convection

Our final task in this paper is to show that the patterning mechanism acts to select the observed range of
equivalent depths as well as to replicate the observed structure of the tropical waves—within the context of classical atmospheric wave theory. We will solve Eq. (4) using a forcing distribution corresponding to the effective cumulus heating for a continuous range of $h$ in order to see if a particular choice of $h$ leads to a consistent phase for the low-level convergence to pattern the convection needed to produce the patterned itself. If the phase is inconsistent, it is essentially analogous to pushing a swing at a frequency different from the swing’s natural frequency, and the result will be to cancel the oscillation. As can be deduced from Fig. 6, the appropriate phase depends on the period of the oscillation. For long periods, the low-level patterned convergence should be approximately in phase with the effective convective heating; however, for shorter periods (about 5 days), there should be a discernible phase lead for the convergence.

We will basically follow the analysis of SL. We will consider the linearized equation for vertical structure of $\bar{w}$ (the vertical velocity in log $p$ coordinates, $w^*$, weighted by $\exp(-x/2)$), where $x = \ln(p/p_s)$, $p$ is pressure, and $p_s$ is surface pressure:

$$\frac{d^2\bar{w}}{dx^2} + \lambda^2 \bar{w} = \frac{R}{gh} \exp\left(-\frac{x}{2}\right)Q(x),$$

(8)

where

$$\lambda^2 = \frac{S}{h} - \frac{1}{4}$$

and

$$S = \frac{R}{g} \left(\frac{dT}{dx} + \kappa T\right).$$

The equivalent depth is $h$, $T$ is the basic-state temperature, $g$ is the acceleration of gravity, $\kappa = R/c_p$, $R$ is the gas constant for air, $c_p$ is the heat capacity of air at constant pressure, and $Q(x)$ is the vertical distribution of effective convective heating. Stevens and Lindzen (1978) took $S$ to be constant (620 m), which plausibly replicates the gross thermal structure of the tropical troposphere. We will also use a more detailed specification of the thermal structure given by

$$S(x) = S_1 + (S_2 - S_1) \left[1 + \tanh\left(\frac{x - x_1}{\delta_1}\right)\right]$$

$$+ (S_3 - S_2) \left[1 + \tanh\left(\frac{x - x_2}{\delta_2}\right)\right]$$

$$+ (S_4 - S_3) \left[1 + \tanh\left(\frac{x - x_3}{\delta_3}\right)\right],$$

(9)

which allows us to represent the reduced stability in the mixed layer and the trade wind boundary layer, as well as the increased stability in the stratosphere. Figure 7 shows the distribution of basic temperature with height for the following choice of parameters: $S_1 = 5$ m, $S_2 = 200$ m, $S_3 = 500$ m, $S_4 = 2400$ m, $x_1 = 0.0625$, $x_2 = 0.25$, $x_3 = 2.2$, $\delta_1 = 0.01$, $\delta_2 = 0.03$, and $\delta_3 = 0.25$. For the heating distribution we take

$$Q = e^{zh} \sin\left(\frac{\pi x - x_z}{x_T - x_z}\right),$$

(10)

which closely follows the form observed by Reed and Recker (1971) and Yanai et al. (1973) for $h = -0.33$, $z_o = 0.15$, and $z_T = 2.01$. For the small values of $h$ (equivalent depth) that we will consider, there is negligible difference between $w$ in $z$ or log $p$ coordinates (Lindzen 1990). Hence we can take $w = 0$ for our lower boundary condition. For the upper boundary condition we take the radiation condition applied at $x = 10$. Our vertical resolution is 0.005; that is, we have a total of 2000 levels. Note that for the present analysis, the magnitude of $Q$ is irrelevant; only its shape matters. Thus, we ignore the multiplicative constant that should be in expression (10).

Our procedure will be to solve Eq. (2) for values of $h$ ranging from 1 to 100 m (using Gaussian elimination as described in Lindzen (1990); if the user wishes to use the algorithm in Lindzen (1990), he should obtain an errata sheet from the author). In order for the wave to be consistent with the effective convective heating it triggers, the phase of $w$ in the boundary layer will have to be somewhat greater than zero (i.e., it must lead heating) in order to allow for the response time inferred in section 2. For periods greater than a few days, this is generally small. Figure 8 shows the variation of phase with $h$. We see that approximately appropriate values are found for $h$ around 15 m and between about 35 and 45 m. While the behavior shown in Fig. 8 does not change much for reasonable changes in basic state or heating profile, such changes can lead to appropriate values occurring throughout the region $12 < h < 60$ m. For example, in Fig. 9, we show results using the basic state and heating from SL. This is a very different...
basic state from what we refer to as our standard basic state. Here $S = 620$ m and there is no attempt to delineate either the boundary layer or the stratosphere. The heating differs from that adopted in this paper in that $z_c = 0.07$ (instead of 0.15). There are obvious changes to the solution, but none alter our earlier conclusions either qualitatively or even quantitatively. It is important to recognize that one does not, in general, need precise phase consistency for low-level convergence. While what one takes as “close enough” is, to some extent, subjective, the underlying principle is clear. For example, if the low-level convergence is off by $30^\circ$, the patterned heating will still reinforce the wave for about three cycles, after which it will begin cancelling the wave. Thus, for a packet with three cycles, $30^\circ$ may be close enough, though not for longer wave trains.

While the preceding analysis yields results consistent with Wheeler and Kiladis (1999) for relatively long periods, for relatively short periods (about 5 days), consistency requires a phase lead on the order of $30^\circ$, and this would lead to equivalent depths somewhat larger than suggested by the data. The results presented by Mapes (2000) show that we may have misrepresented the effective cumulus heating. Mapes stresses that the break out of cumulus towers is accompanied and somewhat preceded by the development of congestus clouds whose precipitation is smaller than that of the taller towers (see Fig. 10 of Mapes). The inclusion of effective heating of this sort turns out to be able to bring the above results into good agreement with observations. As long as the congestus heating is shallower and weaker than, and somewhat in advance of, that due to the towers, the details are relatively unimportant.

To show this, we will add the following to our expression for $Q$:

$$Q_{\text{congestus}} = Ke^{i\phi} \sin\left(\frac{x - x_c}{c_w - x_c}\right),$$

where $x_c = 0.75$.

For $K$, we will take 0.2. For $\phi$, we will try 0 and $\pi/6$. The latter choice of phase lead is appropriate for relatively short periods, while zero is more nearly appropriate for longer periods. Note that Mapes (2000) finds a lead time, and a given lead time corresponds to a smaller phase lead as the period gets longer. The results for both the SL basic state, and what we have referred to as the standard basic state, are shown in Figs. 10 and 11. We see that for both basic states, consistent and nearly consistent phases are found in the range of $h$ between 15 and 60 m. It should be understood that the present approach to the interaction of waves with convection consists in finding consistent phases since inconsistent phases will lead to the self-destruction of waves with the corresponding equivalent depths. Such self-destruction will, however, be slow for nearly consistent phases. It is evident that the presence of a rel-
Fig. 11. Same as Fig. 9, with congestus heating. Dashed horizontal lines indicate range of phases that are sufficiently close to being consistent so as to be associated with minimal suppression.

Fig. 12. Vertical structure of $w$ for $h = 18$ m: (a) amplitude; (b) phase.

6. Discussion and summary

In this paper, we have examined the hypothesis that low-level convergence can sequentially pattern convection that would occur anyway due to evaporation. (By sequential, we simply mean that the patterning by a perturbation of a given scale will pattern the convection already patterned by larger scales.) It was further assumed that patterning is associated with a time scale that is largely independent of wave scale or amplitude, this being the simplest situation to consider. In support of the proposed mechanism, we found that it leads to the correct phase of the semidiurnal component of the observed daily variations of precipitation and to the observed equivalent depths and vertical structures of tropical waves as analyzed by Wheeler and Kiladis (1999) and Straub and Kiladis (2003). (It should be added that the time scale found for patterning, about 12 h, is longer than the time it typically takes for the decay and re-

A relatively small amount of congestus heating, even in the absence of a phase lead for such heating, significantly changes the phase lead of the low-level convergence. The reason is mainly that the shallowness of the congestus heating allows more wave leakage into the boundary layer since leakage depends on the ratio of the vertical wavelength of the wave (in the absence of forcing) to the thickness of the forcing (Lindzen 1966).

Finally, Figs. 12 and 13 show the vertical structure of the vertical velocity field for waves with $h = 18$ m and $h = 52$ m. We see that within the troposphere, the waves mostly follow the distribution of the effective heating, but above this region they behave like vertically propagating disturbances. This is exactly what is found by Straub and Kiladis (2003, see their Fig. 3). The mathematical reason for this is simply that for thick forcing and large $\lambda^2$ the solution to (8) consists approximately of a balance between the second term on the left-hand side of the equation and the forcing (i.e., the particular solution), while above the forcing the balance must be between the two terms on the left-hand side (the homogeneous solution). Note as well, that there is more wave leakage for $h = 52$ m than for $h = 18$ m; here again, this is because the ratio of the vertical wavelength to the thickness of the forcing is greater. It should finally be noted that the vertical structure of the temperature field will differ from that of the vertical velocity. This is because the wave temperature field is proportional to the difference between the second term on the left-hand side of Eq. (8) and the forcing, and hence, gives more emphasis to the homogeneous solution. Thus, the temperature field displayed in Straub and Kiladis (2003) is, not surprisingly, more structured than the vertical velocity field. This property of the temperature field is also evident in Stevens et al. (1977).
generation of an individual cumulus tower. It is more nearly the characteristic time for mesoscale cloud complexes. This suggests that patterning is organizing these complexes rather than individual clouds, and is doing so in the shortest time consistent with the formation of these complexes regardless of the amplitude or scale of the perturbation.) There were several important steps to our argument.

1) Using the solar semidiurnal tide as an example of patterning by low-level convergence, we obtained an estimate for the time scale for such patterning to occur. This, of course, will be longer than the time scales for squalls, which are likely characteristic of the convection itself. In addition, the patterning hypothesis led to the prediction of the observed phase for the semidiurnal component of convective precipitation. The assumption that the time scale arrived at from considering the zonal wavenumber 2 tide would also be characteristic of other scales tested in the second step.

2) In this step we check whether the phase lead required by step 1 for self-excited equatorial waves corresponds to observations. As it turns out, the phase lead observed by Straub and Kiladis (2003) is essentially what is called for by the patterning time derived from the semidiurnal tide. This provides important support for both the notions that convection is patterned and that waves are forced by the resulting “effective cumulus heating.”

3) Finally, patterning requires only a consistent (or nearly consistent) phase in the trade wind boundary layer, and the paper simply shows that phase varies substantially with equivalent depth, hovering around acceptable phases for the observed equivalent depths. This serves to preferentially select these equivalent depths over other equivalent depths that do not present consistent phases.

Note that the fact that the congestus clouds play a role in the cumulonimbus response to dynamic patterning (at least for relatively short periods), and that the inclusion of congestus heating is important in order to achieve consistency in phase between patterning and cumulus heating at the observed equivalent depth when short periods are considered suggests that the interaction of waves and convection may be more subtle than anticipated.

Despite our success in accounting for many observed features of tropical waves, as noted in Stevens et al. (1977), solutions of simple inviscid linear theory for tropical waves suffer from one significant drawback: for observed values of rainfall (i.e., effective cumulus heating), amplitudes of temperature and horizontal velocity oscillations are too large. Models commonly replicate tropical easterly waves, but with reduced wave components of rainfall. Stevens et al. (1977) showed that these problems could readily be eliminated by the inclusion of a simple model for cumulus momentum transport (Schneider and Lindzen 1980). Since then, there has been much interest and controversy over the form or even the existence of so-called cumulus friction. Sar- deshmukh and Hoskins (1987), for example, argued that there was no evidence for any such phenomenon. Tung and Yanai (2002a,b), however, have recently presented evidence to the contrary. Until these issues are resolved, the present approach must be considered somewhat tentative. Nevertheless, as noted earlier, the dispersion relation observed by Wheeler and Kiladis (1999) suggests that such physics remains relevant, and provides a plausible starting point for the study of such waves.

Acknowledgments. The preparation of this paper was supported by Grant DE-FG02-93ER61673 from the Department of Energy. Conversations with K. Emanuel, M. Yanai, and G. Kiladis are gratefully acknowledged, as are the helpful comments of the reviewers of a previous version of this paper.
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