Consistent Vertical and Horizontal Resolution

RICHARD S. LINDZEN

Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts

MICHAEL FOX-RABINOVITZ

Laboratory for Atmospheres, Goddard Space Flight Center, Greenbelt, Maryland

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ABSTRACT

Simple physical relations (namely, the Rossby ratio between vertical and horizontal scales in quasi-geostrophic flow and the dispersion relation for internal gravity waves) are used to estimate the vertical resolution consistent with a given horizontal resolution. Using these relations we find that virtually all large scale models and observing systems have inadequate vertical resolution. In models, the excess horizontal resolution can lead to increased model "noise" rather than improved accuracy. In observing systems, the finer horizontal scales can be severely misrepresented.

1. Introduction

We have long known that a given spatial resolution demands a minimum time resolution in order to avoid numerical instability (Courant et al. 1928). The mechanistic basis for this instability is reviewed in appendix A. Briefly, the instability arises when the spatial resolution defines modes whose time scale is too short to be resolved with the existing time steps.[1] *It should be emphasized that the striking implication of the CFL condition is that it calls for a time step that is usually much smaller than the time scale associated with the dominant spatial scale of the phenomena being described.* The point is that as long as we resolve the smaller scales for which the CFL condition may be violated, these scales will eventually be excited by nonlinearity or even roundoff error.

More recently, it has been noticed that a similar consistency requirement exists between vertical and horizontal resolution. It was noted by Hong and Lindzen (1976) that excessive horizontal resolution could resolve modes whose vertical wavelength might be too small to be resolved with the existing vertical resolution, and that this situation could lead to spatial instability (Lindzen 1970). When the radiation condition is used, this can actually lead to spurious amplitude growth with height; when there is a lid, the manifestation is more complicated. When there are so few levels that spurious growth can't manifest itself, one simply obtains incorrect solutions (Lindzen et al. 1968). *Here too, the vertical resolution may be adequate for the horizontal scales one is physically concerned with. Smaller scales, however, are inevitably generated in the course of integration.* Despite this awareness, little evidence in the literature shows that any serious attempt is being made to achieve any consistency between horizontal and vertical resolution in general circulation models, in models for numerical weather prediction, or even in so-called mechanistic models (for stationary waves, sudden warmings, etc.). This is despite the fact that the problem was early recognized by Charney (1949).

The purpose of the present note is to develop criteria for such consistency and to examine current models in the light of these criteria. The criteria developed also apply to the resolution of observing systems, and we shall examine this as well.

2. Consistency relation

In general, the atmosphere supports a variety of physical wave mechanisms, and it is difficult, therefore, to uniquely establish a consistent relation between horizontal and vertical resolution. We shall separately consider the quasi-geostrophic regime and the scaling appropriate to gravity waves.

a. *Quasi-geostrophic flows*

If one restricts oneself to waves which are quasi-geostrophic, then a relation between horizontal and vertical

Corresponding author address: Dr. Richard S. Lindzen, Center for Meteorology & Physical Oceanography, M.I.T., 77 Massachusetts Avenue, Cambridge, MA 02138.

[1] To be sure, one can avoid explicit numerical instability by using semi-implicit time differencing; however, failure to satisfy the CFL condition is inevitably accompanied by gross inaccuracy even in the absence of numerical instability.

scales based on considerations of the Rossby radius of deformation exists. The basic equation for quasi-geostrophic flow is the geostrophic conservation of pseudopotential vorticity on horizontal surfaces (Charney 1973). The pseudopotential vorticity, q, on a beta-plane is given by

$$q = \frac{1}{f_0}\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right) + f_0 + \beta y + e^{(z^*/H_0)}\frac{\partial}{\partial z^*}\left[\frac{f_0}{N^2}e^{(-z^*/H_0)}\frac{\partial\Phi}{\partial z^*}\right] \quad (1)$$

where

- Φ geopotential height
- H_0 characteristic scale height
- f_0 characteristic Coriolis parameter
- β characteristic value of df/dy
- z^* $= H_0 \ln(p_s/p)$
- p pressure, and
- p_s surface pressure.

The inversion of (1) yields the velocity and temperature fields. Within (1), horizontal scales, ΔL, and vertical scales, Δz, are related by the famous expression

$$\Delta L = \frac{N}{f_0}\Delta z. \quad (2)$$

Commonly, we take $\Delta z = H_0$, and refer to ΔL as the Rossby radius. All vertical scales may appear, however, and each scale is associated with its own radius. Similarly, each horizontal scale is associated with its own "Rossby depth." This symmetry is different from the situation in hyperbolic systems where one cannot have "too much" time resolution. For quasi-geostrophic systems one needs a balance between horizontal and vertical resolution (see appendix A). In practice, however, most existing numerical models and observing systems have inadequate vertical resolution compared to horizontal resolution; we will, therefore, emphasize the need for vertical resolution.

Now $f = 2\Omega \sin\phi$, where Ω is the earth's rotation rate, and ϕ is the latitude. Thus at the poles, $f = 2\Omega$, but Eq. (2) is singular at the equator. On the basis of equatorial β-plane results, Matsuno (1966) and Lindzen (1967) have shown that in the neighborhood of the equator the counterpart of ΔL is the geometric mean of the Rossby radius at the poles and the radius of the earth, a; i.e.,

$$(\Delta L)^2 = \frac{N}{2\Omega}\Delta z a. \quad (3)$$

In general, Eqs. (2) and (3) suggest that for a given ΔL (which we will identify with a horizontal grid interval) there will be different values for Δz in the tropics and at higher latitudes. Equivalently, if we fix Δz, then there will be different values of ΔL in the tropics and at higher latitudes. If one uses a grid point model in spherical coordinates with uniform angular grid intervals (i.e., $\Delta\phi = \Delta\lambda$, where λ is the longitude), then for a given $\Delta\phi$, the consistent Δz will decrease as one approaches the equator (as already noted); it will also decrease as one approaches the poles since a given $\Delta\lambda$ will be associated with smaller ΔL. This last problem is avoided in spectral models by the use of triangular truncations. In grid point models, the problem is dealt with by using heavy spatial filtering poleward of 60° latitude—apparently regardless of model resolution. Such filtering represents one approach toward eliminating excess horizontal resolution. We will discuss this further later in this paper.

TABLE 1. Vertical resolutions corresponding to various choices of $\Delta\phi$ and ΔL.

$\Delta\phi$ (deg)	Δz (km) Eqtrl	Δz (km) Local 60°	Δz (km) Local 45°	Δz (km) Local 22.5°	Δz (km) Stat. wv. extratrop.
1	$1.4\ 10^{-2}$	0.34	0.39	0.28	0.78
2	$5.6\ 10^{-2}$	0.68	0.78	0.55	1.55
4	$2.24\ 10^{-1}$	1.35	1.55	1.10	3.10
8	0.896	2.7	3.10	2.19	6.20

Expressing horizontal resolution in terms of $\Delta\phi$, (3) becomes

$$\Delta z = \frac{2\Omega a}{N}(\Delta\phi)^2 \quad (4)$$

while (2), locally evaluated, becomes

$$\Delta z = \frac{\Omega a \sin 2\phi}{N}\Delta\phi \quad (5)$$

where

$$\Delta L = a\cos\phi\Delta\lambda = a\cos\phi\Delta\phi.$$

Taking

$$\Omega = \frac{2\pi}{86\,400\ \mathrm{s}}$$

$$a = 6\,400\ \mathrm{km}, \quad \text{and}$$

$$N = \frac{3\pi}{300\ \mathrm{s}},$$

Eq. (4) becomes[2]

$$\Delta z \approx 1.4 \times 10^{-2}(\Delta\phi_{\mathrm{deg}})^2. \quad (6)$$

Note that Δz, in the tropics, depends on the square of $\Delta\phi$. The dependence is more clearly revealed in Table 1. Table 1 also shows numerical values for Δz at 22.5°, 45°, and 60° based on Eq. (5).

[2] In Eq. (3), the quantity, $\Delta\phi$ is in radians; the conversion factor, $\pi/180$, has been included in Eq. (6).

The numbers in Table 1 are not meant to be precise, but they are roughly relevant, and the $\Delta^2\phi$ dependence in Eq. (6) implies that only models with relatively coarse horizontal resolution are likely to have adequate vertical resolution in the tropics. The situation is better in midlatitudes. As we will see in section 3, *local* vertical resolution requirements in midlatitudes are sometimes marginally met in actual models, although the situation may, in fact, be deceptive. Even when waves at 45° are locally adequately resolved in the vertical, in fact that they are inadequately resolved in the tropics should, within a few days, influence results at 45°.

Finally, the present considerations are not restricted to fully nonlinear global models. They are also relevant to linear models for stationary waves, limited area models, mesoscale models, etc. For these cases, however, special consideration must again be given to the choice of ΔL. For example, linear (and other) stationary wave models typically restrict themselves to zonal wavenumbers 1–3 (Jacqmin and Lindzen 1985). The value of ΔL is thus primarily determined by meridional resolution. In this case we would have

$$\Delta L = a\Delta\phi,$$

since $\Delta\lambda$ is no longer germane. The relation for Δz will still be different in the tropics and at higher latitudes. Equation (4) remains appropriate in the tropics, but at the poles we now have[3]

$$\Delta z = \frac{2\Omega a}{N}\left(\frac{\pi}{180}\right)\Delta\phi. \tag{7}$$

Evaluating Eq. (7), we obtain the following relation between Δz (in kilometers) and $\Delta\phi$ (in degrees):

$$\Delta z \approx 0.776\Delta\phi. \tag{8}$$

These results are also illustrated in Table 1; evidently, stationary wave models with adequate resolution in the extratropics will still be problematic in the neighborhood of the equator.

For limited area models, our local results for midlatitudes, as given by Eq. (5), are appropriate. Clearly, vertical resolution requirements for such models increase as ϕ decreases.

b. Gravity waves

Although most models focus on the synoptic and planetary scales, the fact is that these models generally use the primitive equations and, therefore, also contain internal gravity waves. The approximate relation between vertical and horizontal scales for such waves is derived from the dispersion relation (see Lindzen 1981) which yields

$$\Delta z \approx \frac{c - U}{N} \tag{9}$$

where c is the phase speed of wave, and U is the speed of the basic flow. Superficially, Eq. (9) suggests that Δz is independent of ΔL. On the other hand, U varies throughout the atmosphere, and, for most internal gravity waves there will be critical surfaces where $U - c = 0$. Equation (9) would appear to call for unlimited vertical resolution near such surfaces. In practice, the situation is more complicated. To be sure, the explicit resolution of behavior near a critical surface is the most demanding problem from the point of view of resolution, and the easiest to develop an explicit criterion for. When damping is present, c has an imaginary part, c_i, and the quantity $c - U$ will not have a smaller magnitude than c_i. Thus Eq. (9) can be replaced by

$$\Delta z_{\min} \approx \frac{c_i}{N}. \tag{10}$$

Usually damping is represented by a damping rate, σ_i, corresponding to a damping time, $\tau = (\sigma_i)^{-1}$. The damping rate is related to c_i by the relation

$$c_i = \frac{\sigma_i}{k} = \sigma_i \Delta L$$

where k is the horizontal wavenumber. We may use the above relation to rewrite Eq. (10) as

$$\Delta z_{\min} \approx \frac{\sigma_i}{N}\Delta L. \tag{11}$$

Let us take

$$\Delta L = a\Delta\phi$$

which is appropriate at the equator, and ignore the effects of converging meridians. If we rather arbitrarily set $\sigma_i = 1/10$ days, express $\Delta\phi$ in degrees, and express $\Delta z_{\min}$ in kilometers, we get the following expression for $\Delta z_{\min}$:

$$\Delta z_{\min} \approx 0.619\ 10^{-2}\Delta\phi. \tag{12}$$

Even without tabulation, it is evident that the vertical resolution requirements for gravity waves near critical surfaces (and almost all meteorologically important gravity waves are likely to have critical surfaces) are beyond the expectations of any model for either the general circulation or for weather forecasting. This would still be true even if we took $\sigma_i = 1/1$ day.

In this connection, it is worth examining the relation between vertical eddy viscosity and σ_i. Crudely speaking, we have

$$\sigma_i \approx \frac{\nu}{\Delta^2 z}.$$

[3] Clearly the condition for Δz will become more restrictive as one moves equatorward.

Taking $\nu = 10^6$ cm^2 s^{-1} and $\Delta z = 1$ km $= 10^5$ cm, we get $\sigma_i = 10^{-4}$ s^{-1} $\approx (3$ h$)^{-1}$. For this choice of ν, $\Delta z = 1$ km is compatible with $\Delta\phi \approx 2°$ at the equator; however, at the poles the situation would be even worse. That said, it must be noted that such a choice for ν would be regarded as excessively large except within the boundary layer.

The above vertical resolution requirement applies to the neighborhood of critical surfaces. Away from such surfaces, the vertical resolution requirements are likely to be far less demanding. The trouble is that we do not, in general, know what c a priori. Also, it is possible for errors at critical surfaces to propagate throughout the domain. This is especially true when the waves originate from instabilities at critical surfaces, but modest damping may so attenuate a gravity wave propagating toward a critical surface, that errors at the critical surface won't matter. This depends on the damping and the time it takes the wave to travel across some relevant domain at its group velocity—which in turn depends on horizontal scale. This situation is very case-dependent and doesn't readily lend itself to a general estimate.

c. *General remarks: observing systems*

The above criteria for consistent vertical resolution may seem unreasonably demanding. Observed systems are commonly large in scale with relatively slow vertical variation. Such systems, however, require only coarse horizontal resolution, which in turn is consistent with coarse vertical resolution. When models have finer horizontal resolution, motions at the finer scales will inevitably develop and if these motions are not adequately resolved in the vertical then these scales will contribute only to the "noise" field and degrade the overall accuracy of the solution. The criteria for consistent resolution for both quasi-geostrophic systems and for internal gravity waves are based on well-established physical relations which are clearly manifested in analytic solutions for baroclinic instabilities (Lindzen et al. 1980), internal Rossby waves (Charney and Drazin 1961), and internal gravity waves.

This being the case, the conditions for consistent resolution must also apply to observing systems. Clearly, if observing systems with fine scale horizontal resolution are not accompanied by sufficient vertical resolution then phenomena with fine horizontal scales will be incorrectly observed, and the observations will provide incorrect initial conditions for models.[4] In the following section, we will examine how existing models and observing systems fare according to the above criteria.

[4] It should also be noted, in passing, that these physical arguments always refer to Δz. This suggests that levels separated by uniform Δp's will tend to be problematic since the Δz's corresponding to the upper levels will inevitably be large.

3. Relation to existing models and observing systems

The earliest numerical baroclinic models using quasi-geostrophic or other filtered equations (Kibel 1957; Thompson 1961) had rather coarse horizontal resolution ($\Delta L \sim$ 300–500 km) and only 2–3 levels in the vertical. These characteristics were related to the limited computational resources then available, but it was commonly believed that for the forecasting of synoptic scale processes with filtered models such resolution would be adequate. The local results for 45° in Table 1 suggest that this conclusion was not too far off concerning consistency between vertical and horizontal resolution—though these models certainly had other major shortcomings. Other early arguments suggested that horizontal resolution should be at least of the order of the mean distance between observating stations, which for Europe and the United States approximately 200–300 km. The fact that synoptic systems had finer scales than the interstation separation suggested that even filtered models would need still finer horizontal resolution for numerical accuracy.

The first primitive equation (PE) models (Shuman 1962; Shuman and Hovermale 1968; Mintz 1965; Arakawa 1970; Smagorinsky 1963) still had rather poor spatial resolution ($\Delta L \sim$ 300–400 km with 2–6 levels), although the need for a substantial increase in resolution was generally appreciated. The improvement of PE NWP and GC models in recent decades has been tightly connected with increasing numerical resolution (along with better parameterization of physical processes and improved initial data analyses). That said, however, it must also be recognized that the emphasis has been on increasing horizontal resolution. In an influential paper, Robert (1974) advocated a modest increase in vertical resolution (8–10 levels—not evenly spaced in pressure), but placed considerably more stress on the need for 150 km horizontal resolution in forecast models. Such emphasis was defensible up to a point. Since then, however model horizontal resolution has been increasing substantially; values of ΔL from 50–100 km are not unheard of in large scale models, while values of ΔL from 30–60 km are commonly found in operational mesoscale and hurricane models. At the same time, many models added additional levels, though these levels were frequently used to improve coverage of the stratosphere and the planetary boundary layer (PBL) rather than to increase vertical resolution. Indeed, average vertical resolution has been increasing relatively slowly even during the post-FGGE period. The estimates in section 2 suggest that it may be time for renewed emphasis on vertical resolution.

We show in Fig. 1 the vertical and horizontal resolutions used in actual NWP and GC models during the pre-FGGE period. For purposes of comparison we also reproduce the consistent resolutions displayed in Table 1. Figure 2 shows the same information for the post-FGGE period. Figure 1 includes filtered as well

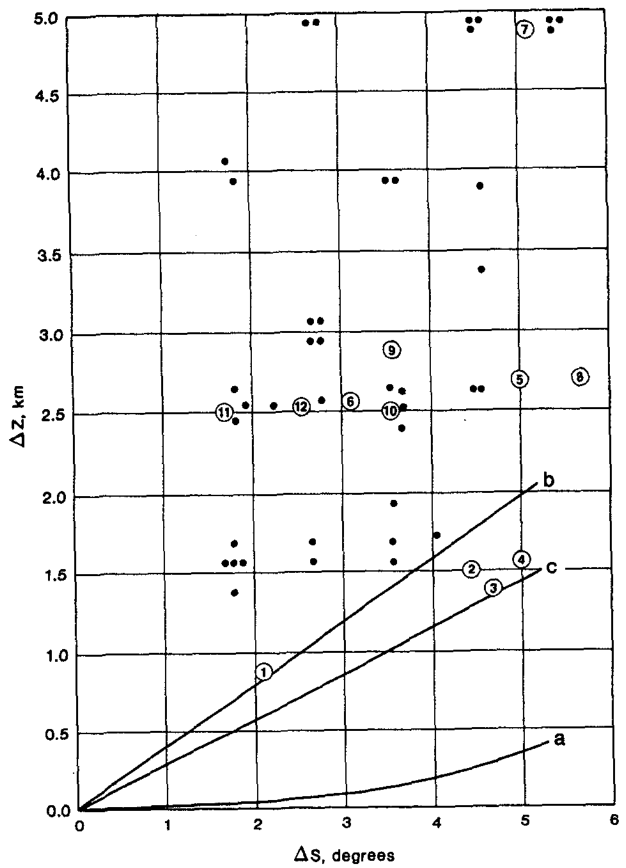

FIG. 1. Vertical resolution (in km) and horizontal resolution (in deg) used in numerical weather prediction and general circulation models of the pre-FGGE period. Points are taken from Döös (1970, 1974). Numbered points refer to models listed in appendix B. Curve a corresponds to the minimum vertical resolution needed in the neighborhood of the equator. Curve b corresponds to the minimum local resolution at 45°. Curve c corresponds to the minimum local resolution at 22.5°.

as PE models. Typical resolutions in the early 1970s were $\Delta L \sim 300$–400 km and $\Delta z \sim 2.5$ km. By the middle and late 1970s there were slight improvements to $\Delta L \sim 200$–400 km and $\Delta z \sim 1.5$–2 km. It should be noted that the average Δz's recorded in Figs. 1 and 2 are generally for the troposphere below 10–12 km. Vertical resolution is usually poorer in the stratosphere. The main point made in Fig. 1 is that almost all points are above all our relevant consistency curves. Only some *experimental* models with relatively poor horizontal resolution ($\Delta L \sim 500$ km) and with relatively fine vertical resolution ($\Delta z \sim 1.25$ km) have roughly consistent resolution away from the equator. The GFDL model has locally consistent resolution at 45°. None has consistent quasi-geostrophic resolution near the equator, and none comes close to consistent resolution for internal gravity waves with critical surfaces. Turning to Fig. 2, we see a continuing tendency to increase horizontal resolution relative to vertical resolution. Almost all post-FGGE results lie above any of the theoretical consistency curves, although a few models are locally consistent at 45°. According to our estimates (see Table 1), the ratio $r = \Delta z/\Delta^2\phi$ should be O(0.015) near the equator. For the models in Fig. 2, $r \approx 0.5$–1.0. Recent reports (WMO 1987) have demonstrated improved large scale model performance with increased vertical resolution. This is especially true for the SKYHI model at the Geophysical Fluid Dynamics Laboratory at Princeton (Mahlman and Umscheid 1987) with $\Delta z = 1$ km. These authors, in fact, claim to resolve internal gravity waves, which is entirely possible away from critical surfaces. Unfortunately, as noted in section 2, inadequate resolution at critical surfaces acts in many cases to produce incorrect results at all levels.

Insufficient vertical resolution is particularly evident in operational mesoscale and hurricane models where $\Delta L \sim 30$–60 km, and $\Delta z \sim 0.75$–1.25 km. It is interesting in this regard to note that the Limited-area Fine Mesh (LFM) 6-level model of the National Meteorological Center (NMC) seems to show no forecasting advantage over NMC's large scale spectral 18-level model.

Finally, we note that the high-resolution linear stationary wave model of Jacqmin and Lindzen (1985) ($\Delta\phi = 1.5°$, and $\Delta z = 1$ km) has consistent resolution outside the tropics according to Table 1, but has inadequate vertical resolution near the equator. Consistent with this, their solutions were ill behaved near the equator until heavy damping was used in this region. Similar results have been reported for other stationary wave models (Nigam et al. 1986).

Turning to observing systems, it must be immediately recognized that there are far more systems than

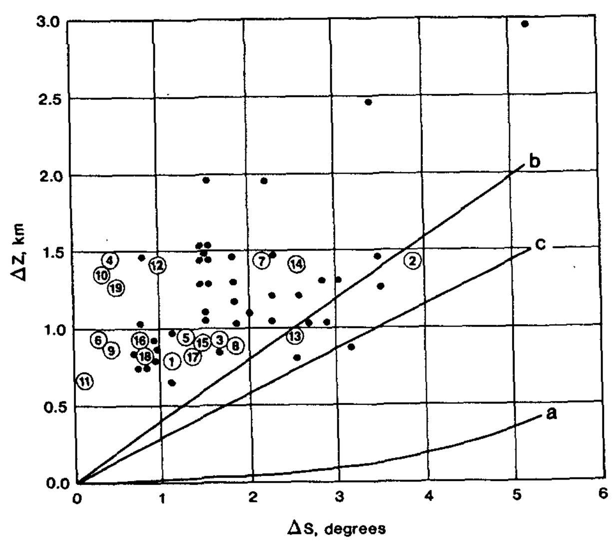

FIG. 2. The same as Fig. 1 but for post-FGGE models described in WMO (1987). Numbered points refer to models listed in appendix C.

we can reasonably discuss. We will therefore restrict ourselves to the main observing systems for large scale weather prediction and climate analysis: namely, the conventional rawindsonde and satellite systems. Rawindsondes provide data primarily at mandatory levels for which the characteristic vertical resolution is approximately 1.5 km. Additional data from other significant levels is provided and sometimes used. More commonly, not even all the mandatory levels are used. For purposes of this paper, however, we will take $\Delta z = 1.5$ km for rawindsonde data. The conventional rawindsonde network is notoriously unevenly distributed over the globe. Characteristic horizontal resolution over the United States, Europe, and South and Eastern Asia is typically $\Delta L \approx 2°–3°$. Over less densely covered areas like Africa, South America, Siberia and Northern Canada, $\Delta L \approx 5°–7°$, and over most of the globe, $\Delta L > 7°$. The horizontal resolution associated with satellite radiance data is, in general, much better. For current systems, $\Delta L \approx 1°$ (Susskind et al. 1984; Reuter et al. 1988), although this data strongly depends on cloud coverage, and is generally less accurate than rawindsonde data. In the future, we may anticipate even higher horizontal resolution from such systems. The proposed Earth Observing System anticipates $\Delta L \approx 0.5°–0.8°$ (NASA 1984). The vertical resolution associated with satellite radiance data is not altogether clear. Straightforward inversion techniques yield significantly worse vertical resolution than is obtained with rawindsondes; however, it is argued that some retrieval techniques do provide useful data at the conventional mandatory levels. Somewhat optimistically, we will accept $\Delta z \approx 1.5$ km for satellite data. The horizontal and vertical resolution provided by rawindsonde and satellite systems is summarized in Fig. 3, where we again include the curves for consistent resolution from Table 1. As can be seen, the vertical resolution associated with mandatory levels is consistent with a horizontal resolution of 4° near 45°, and of 9° near the equator. Over much of the globe vertical resolution is consistent (for quasi-geostrophic components of the circulation) with the horizontal density of rawindsonde data. Unfortunately, it is also clear that this horizontal resolution is frequently inadequate to even identify important meteorological systems. Over the data-dense regions, even rawindsonde data requires more vertical resolution. Even with generous estimates for vertical resolution, it is evident that much of the horizontal resolution provided by satellite radiance data is not likely to be quantitatively useful. It only leads to the horizontal resolution of systems for which the data provide incorrect vertical structures (i.e., real vertical structures are aliased to the resolved vertical scales); this, in turn, can lead to meaningless diagnoses for these scales since eddy heat fluxes and other diagnostics depend critically on the detailed phase structure of waves. It should be emphasized that although the larger horizontal scales may be associated with slow vertical variation, the same is not, in general, true for the finer scaled features. Neither major observing system appears capable of generally resolving internal gravity waves. It is also unclear whether the possibility that some internal gravity wave may be adequately resolved is useful in practice.

4. Conclusions

In this paper we have developed simple physical criteria for the vertical resolution consistent with horizontal resolution in both models and observation systems. The criteria are based directly on well-established notions of Rossby radius and gravity wave dispersion. We then proceed to examine existing numerical models for forecasting, general circulation, mesoscale circulation and stationary waves in the light of our criteria.

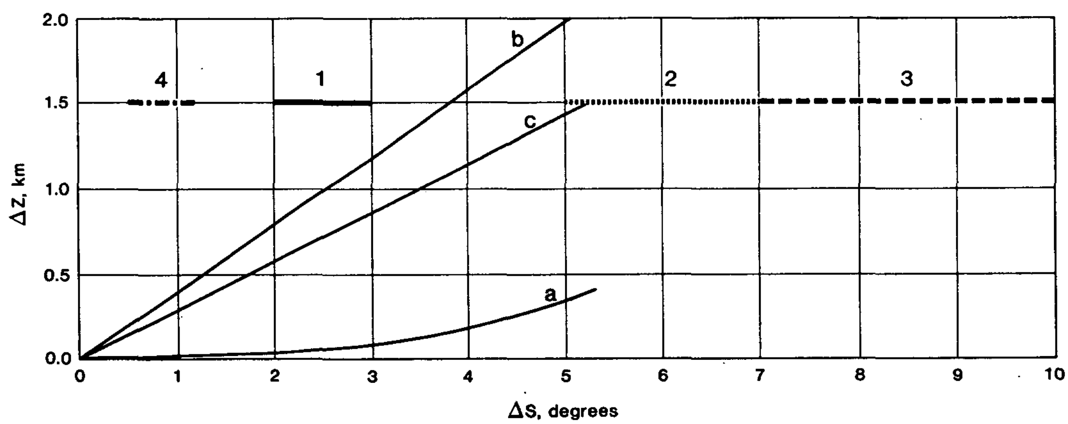

FIG. 3. Typical horizontal and vertical resolution for rawindsonde (R), and satellite radiance (S) observing systems. The solid line, 1, corresponds to R for the United States, Europe, and Southern and Eastern Asia; the dotted line, 2, to R for Africa, South America, Siberia, and Northern Canada; the dashed line, 3, to R for oceanic regions (this line should really continue well beyond the edge of the figure); the dot-dashed line, 4, to S for current and projected systems. The curves for theoretically consistent resolution are identical to those in Fig. 1.

In almost all cases model vertical resolution is smaller than demanded by consistency. This is especially so near the equator and for gravity waves. The same shortcomings are found to exist in the major observing systems (notably, the global network of rawindsondes and satellite radiance observing systems).

In view of the above, one might plausibly ask why existing models work. This question is not easy to answer with any degree of certainty, but three possibilities are readily identifiable.

1) As far as we can tell, all existing models suffer from noisiness when started with real data and/or without smoothing and substantial damping. In the light of our criteria, it would appear that the initialization of data amounts to a smoothing operation which serves to minimize the extent to which inconsistently resolved scales are initially excited. Thus, the development of these inconsistently resolved scales via nonlinearity, roundoff errors, etc., must take a finite amount of time. Inconsistent resolution then joins a number of other factors in limiting predictability.

2) Similarly, the use of smoothing and damping (especially diffusive damping) within models serves to decrease the actual horizontal resolution—presumably to a degree consistent with the vertical resolution. This is certainly the case with polar filtering. At best it would appear that we are engaged in the wasteful activity of introducing excessive horizontal resolution (relative to vertical resolution) and then, in effect, throwing it away.[5] By "substantial damping" we mean damping sufficient to cause the decay of a particular scale in a time short compared to the time over which one is predicting. It is interesting to note in this connection that fourth order horizontal diffusion in the ECMWF model provides substantial damping for scales less than 500 km (A. Simmons personal communication 1989). Reference to Fig. 2 shows that this leads almost exactly to consistent resolution.

3) It appears sometimes to be the case that nonlinear processes actually smooth fields naturally in the neighborhood of the equator so as to diminish vertical resolution requirements. Nigam et al. (1986) have observed that GCM's seem to require less vertical resolution to depict stationary waves in the tropics than do linear models. There is, thus far, no compelling evidence that this is a general effect in all regions.

In view of our arguments, it would seem more reasonable for future models to seek a better balance between vertical and horizontal resolution. In view of our results for the neighborhood of the equator, it would also appear that this would require nonuniform horizontal resolution with much poorer resolution near the equator (at least for a fixed vertical resolution). We should emphasize that higher horizontal resolution is desirable; our only point is that just as higher horizontal resolution must inevitably be accompanied by higher time resolution, it must also be accompanied by higher (and in many cases much higher) vertical resolution. In this connection, the danger of models whose vertical structure is described by levels separated by approximately uniform Δp is evident, since at upper levels these models will have large Δz.

Similarly, in order to properly describe the atmosphere (and provide suitable initial conditions for models), data must also have a balance between vertical and horizontal resolution along the lines illustrated in Table 1. For radiosonde data, consideration should be given to the explicit use of data from all possible levels, and not just some of the mandatory levels. The situation seems more difficult with satellite radiance data. It may be well to recognize in this case that high horizontal resolution may not, in practice, be meaningful. New satellite techniques involving Doppler lidar wind sensing (NASA 1984) seem potentially more consistent in this regard.

Finally, it should be noted that neither models nor global observing systems seem likely to properly resolve internal gravity waves. To be sure, some gravity waves may be locally well resolved; however, as already noted, inadequate resolution at critical surfaces will commonly lead to errors everywhere, including regions where gravity waves are adequately resolved locally. In view of this, it is worth considering where the assumed advantage of primitive equation models actually lies. It is conceivable that balanced, filtered models may yet offer some advantages.

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APPENDIX A

Interpretation of C-F-L Condition

At the heart of the C-F-L condition is the obvious fact that one cannot march forward without adequate resolution, and that the resolution needed in time is determined by the spatial resolution. The literature contains several approaches to this central result. All begin with the canonical hyperbolic equation

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2}. \tag{A1}$$

The approach most common in the meteorological literature (see Thompson 1961) simply uses centered dif-

[5] It is conceivable that the extra resolution might facilitate the introduction of initial data. This might account for the improved performance sometimes noted when horizontal resolution is increased. Recent findings, however, tend to show no substantial model improvement when horizontal resolution is increased beyond a few degrees without also increasing vertical resolution (WMO 1987).

ferences in t and x to approximate (A1) and shows that the numerical solution will have exponential growth in time if $c\Delta t/\Delta x > 1$. An alternative approach, due to Courant et al. (1928) notes that when $c\Delta t/\Delta x > 1$, the numerical domain of influence for a point (x, t) is smaller than the domain determined by the characteristics of (A1), and hence the numerical solution cannot converge to the actual solution as $\Delta x \to 0$.

A third, more restrictive, approach is most illustrative in the present case. Consider a situation where $p(0) = p(L) = 0$ (i.e., vibrations of a string). Equation (A1) can be solved by separation of variables:

$$p = \sum_{1}^{\infty} f_n \sin\left(\frac{n\pi x}{L}\right). \qquad \text{(A2)}$$

Using finite differences in x is equivalent to truncating (A2) at a finite N where

$$N \approx \frac{2L}{\pi \Delta x}. \qquad \text{(A3)}$$

The equation for f (dropping the subscript, n) is

$$f_{tt} + c^2\left(\frac{n\pi}{L}\right)^2 f = 0. \qquad \text{(A4)}$$

If one approximates (A4) with centered time differences, one again develops numerical instability for those N's where [using (A3)] $c\Delta t/\Delta x > 1$. The important point here, however, is that one would also obtain spuriously growing solutions if $c^2 < 0$, and the system were elliptic. One might then associate t with a spatial dimension rather than time. The appropriate condition would involve the absolute value of c. This is the present situation, where Δt, above, is identified with vertical resolution, and Δx is identified with horizontal resolution. The symmetry of bounded elliptic systems would also demand that Δt not be too small relative to Δx—in contrast to hyperbolic systems where $\Delta t/\Delta x$ can be made arbitrarily small. In actual models and observing systems it turns out that horizontal resolution determines the needed vertical resolution.

The nature of such numerical instabilities is most clearly illustrated by the mathematically trivial situation where our horizontal resolution is sufficient to resolve a solution whose vertical behavior is expected to be exponential decay with a characteristic scale which, for convenience, is taken to be unity. In a simple separable problem, the vertical structure might be described by an equation like the following:

$$\frac{df}{dz} = -f \qquad \text{(A5)}$$

with $f(0) = 1$. Equation (A5) has the solution

$$f = e^{-z}.$$

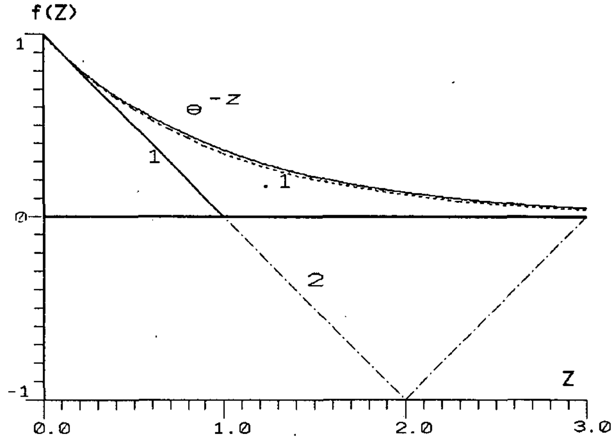

FIG. 4. Finite difference approximations to e^{-z}.

Differencing Eq. (A5) in the most obvious way, we get

$$\frac{f_{n+1} - f_n}{\Delta z} = -f_n \qquad \text{(A6)}$$

with $f_0 = 1$. In Fig. 4 we show the analytic solution as well as solutions to Eq. (A6) for various choices of Δz. For $\Delta z = 0.1$, the solution to Eq. (A6) is a reasonable approximation to the exact solution. For $\Delta z = 1$, the numerical solution is at best only schematically correct. For $\Delta z = 2$ we have a "$2\Delta z$" oscillation rather than exponential decay, while for $\Delta z > 2$ (not shown in Fig. 4), we have an unstable oscillation. One simple point which needs emphasis is that although the condition for spatial instability is $\Delta z > 2$, solutions with $2 > \Delta z > 1$ are almost totally inaccurate. In this connection it is interesting to note a recent result of Lorenz (1989) wherein be discovered that for difference solutions satisfying conditions for numerical stability, one obtained chaotic behavior—not related to solutions of the underlying continuous equations—unless still finer resolution was used. Finally, it should be noted that if one has substantially inadequate resolution, then increasing the resolution does not actually do much to increase the accuracy of the numerical solution. Solution accuracy only begins to improve once one gets close to adequate resolution, and then the improvement is rapid. This is clearly seen in the calculations of Lindzen (1970).

APPENDIX B

Selected Pre-FGGE Models

The list of models comes from Döös (1970, 1974).

1. GFDL global 18-level model using modified Kurihara grid, United States of America
2. GISS global 9-level model, United States of America

3. NCAR global 12-level model, United States of America
4. NMC global 8-level model, United States of America
5. Global 5-level model, Union of the Soviet Socialist Republics
6. Global 5-level general circulation model, United Kingdom
7. Global 3-level model, Japan
8. Global 5-level spectral model, Australia
9. Filtered 4-level model, Canada*
10. PE 6-level model, Federal Republic of Germany
11. PE 5-10 level model, France*
12. PE 5-level model, Sweden*

Note: The results for the models maked by an asterisk are taken from Döös (1970). The other results are from Döös (1974). The numbers correspond to those shown in Fig. 1.

APPENDIX C

Selected Post-FGGE Models

The list of models comes from WMO (1987).

1. ECMWF 19-level spectral model (with T203)
2. Spectral 9-level model, Australia
3. Spectral 15-level model, Canada
4. Fine-mesh limited area 9-level model, People's Republic of China
5. Spectral 15-level model, France
6. Fine-mesh 15-level model, France
7. Moist 9-level model, Federal Republic of Germany
8. Spectral 16-level model, Japan
9. Very fine-mesh 16-level model, Japan
10. 8-level typhoon model, Japan
11. 20-level mesoscale model, Norway
12. Regional 9-level model, Sweden
13. Spectral 15-level model, Union of the Soviet Socialist Republics
14. Hemispheric 10-level model, Union of the Soviet Socialist Republics
15. Global 15-level model, United Kingdom
16. Regional fine-mesh 15-level model, United Kingdom
17. NMC Spectral 18-level model, United States of America
18. NMC Regional nested grid 16-level model, United States of America
19. 10-level Movable fine-mesh hurricane model, United States of America

Note: The numbers of the above items corresponds to those shown in Fig. 2.

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