Effect of Daily Variations of Cumulonimbus Activity on the Atmospheric Semidiurnal Tide

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ABSTRACT

It is suggested that an additional source of semidiurnal forcing due to daily variations in tropical rainfall could correct the discrepancy between the calculated phase (based on forcing due to insolation absorption by ozone and water vapor) and that observed for the surface pressure oscillation. It is also shown that the 180° phase shift in horizontal wind oscillations at 28 km which current calculations predict, but which is not observed, would be eliminated by the proposed additional forcing. The magnitude and phase of the required rainfall oscillation is calculated and found to be consistent with existing observations. Finally, it is shown that the convergence field due to the tide could not directly account for the rainfall oscillation.

1. Introduction

In a review of atmospheric tides (Chapman and Lindzen, 1970), it was noted that forcing due to insolation absorption by ozone and water vapor appeared adequate to account for observed magnitudes of migrating semidiurnal and diurnal oscillations in the atmosphere. Nevertheless, there were some slightly worrisome discrepancies between theory and observations for the semidiurnal tide:

1) Theory predicted surface pressure maxima at 0900 and 2100 LT while observations showed the maxima to occur approximately 44 min later.

2) Theory predicted a 180° phase shift in semidiurnal horizontal wind oscillations at an altitude of about 28 km (with a concomitant null in amplitude). While data (Reed, 1972) suggested such a phase shift below 50 km, the evidence suggested no such phase shift as low as 30 km.

The first discrepancy was considered at some length by Lindzen and Blake (1971). They showed that dissipative effects and surface heating could not account for the discrepancy. They also argued that a consideration of the effect of mean winds would not provide the answer. This last point was confirmed in detail by Lindzen and Hong (1974). Two remaining possibilities were suggested: either an additional source of forcing had been neglected or the phase of forcing by ozone heating had been miscalculated.

Assuming that the pressure oscillation produced by this additional source would be less than 0.6 mb, Lindzen and Blake noted that an additional heating with maxima at 0300 and 1500 LT (i.e., 3 h later than maximum heating due to insolation absorption by ozone and water vapor) would be of nearly optimal effectiveness in properly altering the phase of the surface pressure oscillation. However, even with this choice of phase for the additional heating, that heating would have to be able to produce, by itself, a semidiurnal surface pressure oscillation of about 0.4 mb amplitude (almost half the amplitude of the observed oscillation).[1] This seemed intuitively excessive to Lindzen and Blake who, therefore, favored the second possibility. Unfortunately, Blake (1972, personal communication), after carefully including the diurnal variation of ozone itself, found that the phase of the surface pressure oscillation thus produced was negligibly different from that predicted in more primitive calculations (the main effect being a small reduction in ozone forcing magnitude). Given the above, one is forced to give greater consideration to the first possibility. It is the main purpose of this paper to determine whether daily variations in tropical rainfall (and associated variations in latent heat release) could constitute the needed additional forcing. In Section 2, I show that the needed additional forcing would be produced by an oscillation in precipitation whose amplitude in the neighborhood of the equator would be about 0.12 cm day^{-1} [as compared to a mean precipitation of about 0.6 cm day^{-1} (Sellers, 1966)]. Analyses of the observed daily variation of rainfall rates are available only for a few tropical stations: it is shown in Section 3 that for these stations the amplitudes and phases of the semidiurnal variations in precipitation are approximately what is needed. Indeed, this is entirely consistent with a commonly mentioned feature

[1] As will be shown in Section 2, a contribution to the tide of such a magnitude will produce the required change of phase in the total oscillation with maximum heating any time between 0230 and 0430 LT.

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of tropical rainfall; namely, maxima in both the afternoon and between midnight and dawn (Finkelstein, 1964). This afternoon maximum, however, is often claimed not to exist over open sea (Jacobson, 1976). We shall see, however, that this is not particularly germane to the semidiurnal Fourier component. It is difficult, at this stage, to go much further than this: we have neither a global analysis of observed precipitation nor a theory for the daily variation of precipitation. With reference to the latter, it is shown in Section 4 of this paper that the semidiurnal tidal winds themselves (as forced by ozone and water vapor heating as well as an assumed latent heat source) would *not* be capable of converging sufficient moisture to produce the required oscillation in precipitation. In other words, a CISK mechanism does not appear to be involved [a current discussion of CISK may be found in Stevens and Lindzen (1978)]. The finding by Brier and Simpson (1969) that $S_2(p)$ at the ground correlates significantly with variations in cloud activity likely arises from the role of clouds in forcing $S_2(p)$.

The second discrepancy was considered by Lindzen and Hong (1974) who found that the effect of mean zonal winds (varying with both altitude and latitude) was to produce mode coupling between the main 2,2 mode and the 2,3 mode during solstices. This, in turn, caused the level of 180° phase shift to move above 30 km during summer at extratropical and higher latitudes. Such behavior was consistent with Reed's (1972) observational analysis which was based on summer data at 30°N. However, Wallace and Tadd (1974) on the basis of extensive radiosonde data showed that a phase shift near 28 km was generally absent—independent of season and latitude. Thus, it would appear that the explanation of Lindzen and Hong is probably incomplete. Although the two discrepancies are sometimes thought of separately, Green (1965; Chapman and Lindzen, 1970, pp. 132–135) has shown that the 180° phase shift at 28 km is directly attributable to the presence of the bulk of the thermal forcing being above 28 km (as would be the case if most of the forcing were due to ozone). If the latent heat source, suggested here, is real, then the bulk of semidiurnal forcing resides in the troposphere and no 180° phase shift at 28 km is expected. The results of some simple classical tidal calculations, illustrating this point, are presented in Section 4 as well.

2. Calculation of needed latent heat source

Existing tidal calculations based on forcing due to insolation absorption by H_2O and O_3 tend to predict a semidiurnal surface pressure amplitude of about 1.16 mb at the equator with maxima at 0900 and 2100 LT. Observations suggest a maximum of comparable amplitude but with maxima at 0940 and 2140 LT. From Fig. 1 we see that an oscillation with an amplitude of about 0.4 mb and a maximum any place between

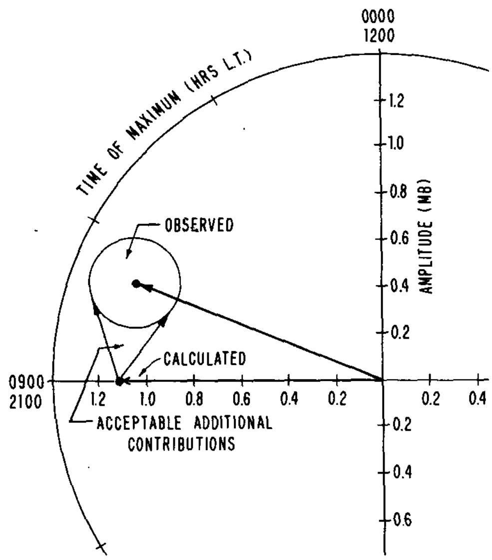

Fig. 1. Harmonic dial for semidiurnal surface pressure oscillation. Shown are the observed oscillation at the equator and the calculated response to forcing due to insolation absorption by ozone and water vapor. The vector connecting the above represents the needed additional contribution. Because of uncertainties and variations in the observed oscillation, the additional source may have a range of phases.

about 1130 and 1330 LT (and between 2330 and 0130), if added to the currently predicted oscillation, will reasonably simulate the observed oscillation. The bulk of the semidiurnal surface pressure oscillation is due to the main 2,2 Hough mode, and for this mode, the phase of the surface pressure oscillation precedes the heating by 3 h. Thus, the additional heating should have a maximum between 1430 and 1630 LT (and between 0230 and 0430 LT), and if the additional heating is due to latent heat release then we anticipate that the semidiurnal component of surface rainfall variations will have maxima several minutes after the required heating maxima. The question addressed in this section is what magnitude of oscillation in rainfall is required to produce an 0.4 mb surface pressure oscillation.

It will suffice, for the above purpose, to restrict ourselves to the 2,2 Hough mode. Following classical tidal theory (Chapman and Lindzen, 1970) we shall use as our dependent variable

$$G=-\frac{1}{\gamma p_0}\frac{Dp}{Dt}, \tag{1}$$

where p is the pressure, γ the ratio of heat capacities $(=c_p/c_v=1.4)$ and p_0 the unperturbed pressure. For the 2,2 mode, we may write

$$G=y_{2,2}(x)\Theta_{2,2}(\theta), \tag{2}$$

where $\Theta_{2,2}$ is a Hough function, θ the colatitude and

$$x=\int_0^z \frac{dz}{H}, \tag{3}$$

the height in scale heights, z the altitude,

$$H=RT_0/g, \qquad (4)$$

the local scale height and $T_0(z)$ the unperturbed temperature. $y_{2,2}$ satisfies

$$\frac{d^2y_{2,2}}{dx^2}+\left[\frac{\kappa H+dH/dx}{h_{2,2}}-\frac{1}{4}\right]y_{2,2}=\frac{\kappa J_{2,2}}{\gamma g h_{2,2}}e^{-x/2}, \qquad (5)$$

where $\kappa=(\gamma-1)/\gamma=2/7$, $h_{2,2}$ is the equivalent depth of 2,2 mode (≈ 7.852 km) and $J_{2,2}(x)$ is the vertical structure of that part of the semidiurnal forcing proportional to $\Theta_{2,2}(\theta)$.

Requiring $w=0$ at $x=0$ (where w is the vertical velocity) implies

$$\frac{dy_{2,2}}{dx}+\left(\frac{H}{h_{2,2}}-\frac{1}{2}\right)y_{2,2}=0 \quad \text{at} \quad x=0. \qquad (6)$$

As $x\rightarrow\infty$ we require either boundedness for y or a radiation condition depending on whether the bracketed quantity in Eq. (5) is negative or positive as $x\rightarrow\infty$. Once one obtains $y_{2,2}(x)$ it is an easy matter to relate all other fields to $y_{2,2}(x)$ and $\Theta_{2,2}(\theta)$ (see Chapman and Lindzen, 1970). In particular, if we write the 2,2 contribution to the surface pressure oscillation as

$$S_{2,2}(p)=\delta p_{2,2}(0)\Theta_{2,2}(\theta), \qquad (7)$$

then

$$\delta p_{2,2}(0)=i(\gamma/\sigma)y_{2,2}(0)p_0(0), \qquad (8)$$

where $\sigma=4\pi/24$ h.

There remains the specification of $J_{2,2}$ in Eq. (5). In this section we restrict ourselves to that part of $J_{2,2}$ which might be due to semidiurnal variations in tropical

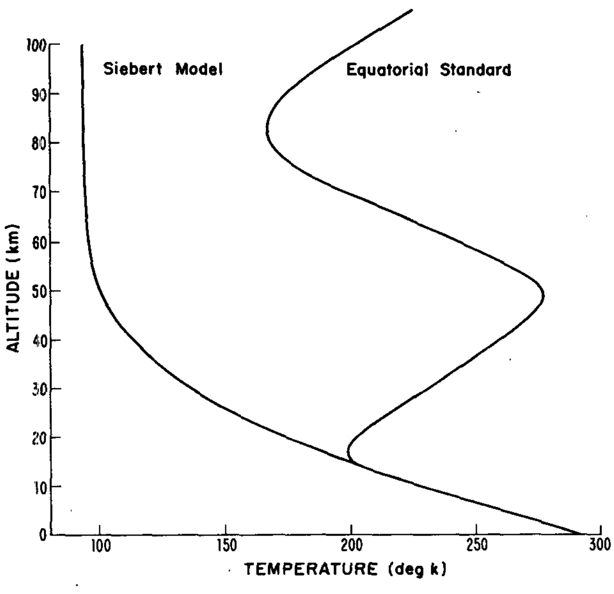

FIG. 2. Unperturbed temperature vs altitude. Shown are an equatorial standard profile and a Siebert model. See text for details.

rainfall. Following Arakawa and Schubert (1974) we write

$$J_{2,2}=\frac{c_p}{\rho_0(x)}M_{c;2,2}(x)\left(\frac{dT_0}{dz}+\frac{g}{c_p}\right), \qquad (9)$$

where ρ_0 is the unperturbed density, and $M_{c;2,2}(x)$ the vertical structure of that part of the semidiurnal variation in deep cumulus mass flux proportional to $\Theta_{2,2}(\theta)$. Using (9) we may rewrite the right-hand side of (5) as

$$\frac{\kappa J_{2,2}}{\gamma g h_{2,2}}e^{-x/2}=\frac{g(dH/dx+\kappa H)}{\gamma h_{2,2}}e^{x/2}\frac{M_{c;2,2}}{p_0(0)}. \qquad (10)$$

Moreover, as may be inferred from Green (1965), the 2,2 response does not depend strongly on any but the grossest features of the forcing distribution. Thus, we shall model M_c with a non-entraining cloud which detrains at that level where the moist enthalpy of surface air equals the moist enthalpy of the ambient air (conditional instability is assumed). It suffices to assume no ambient moisture at this level, in which case this requirement is

$$c_pT_0(0)+Lq=c_pT_0(z_T)+gz_T, \qquad (11)$$

where L is the latent heat of condensation, q the specific humidity of surface moist layer and z_T the detrainment height. The fact that the cumulus heating is entirely due to the latent heat of precipitation can be shown to imply (Stevens and Lindzen, 1978)

$$M_{c;2,2}=\frac{P_{2,2}}{q}, \qquad (12)$$

where $P_{2,2}$ is the proportionality constant for the semidiurnal component of the precipitation proportional to $\Theta_{2,2}(\theta)$ (the phase lag between precipitation and latent heating has been ignored). The units for $P_{2,2}$ are g cm^{-2} day^{-1}.

The last thing required for the solution of Eq. (5) is the specification of $T_0(z)$. We shall solve (5) numerically [following a procedure described in Chapman and Lindzen (1970)] for a slightly modified equatorial standard atmosphere shown in Fig. 2. However, it is of some interest to solve Eq. (5) analytically for an atmospheric model first used by Siebert (1961). In this model, we take

$$\mathcal{S}\equiv\kappa H+\frac{dH}{dx}=\text{constant}. \qquad (13)$$

The distribution $T_0(z)$ for $\mathcal{S}=0.77045$ km, which is also shown in Fig. 2, is seen to reasonably approximate the equatorial standard atmosphere within the troposphere, but markedly misrepresents matters above the tropopause. When (13) holds, it can be shown that

(11) implies

$$x_T=\frac{\kappa Lq}{g\mathcal{S}}. \tag{14}$$

In addition, the quantity in brackets in Eq. (5) is constant and negative, so that (for the 2,2 mode)

$$\mu^2=\frac{1}{4}-\frac{\mathcal{S}}{h_{2,2}}\approx 0.15188. \tag{15}$$

Solving for $y_{2,2}$, one obtains (in view of the assumed constancy of $M_{c;2,2}$ below x_T)

$$y_{2,2}=\begin{cases} ae^{\mu x}+be^{-\mu x}+\dfrac{\mathbf{R}h_{2,2}}{\mathcal{S}}e^{x/2}, & x<x_T \\ ce^{-\mu x}, & x>x_T \end{cases} \tag{16}$$

where

$$\mathbf{R}\equiv\frac{g\mathcal{S}}{\gamma h_{2,2}p_0(0)}M_{c;2,2},$$

and a, b and c are constants to be determined by the lower boundary condition [Eq. (6)] and by the continuity of $y_{2,2}$ and $dy_{2,2}/dx$ at $x=x_T$. Only a and b are needed for our purposes; these turn out to be

$$a=-\frac{\mathbf{R}h_{2,2}}{\mathcal{S}}\frac{(\frac{1}{2}+\mu)\exp(x_T/2)}{2\mu\exp(\mu x_T)}, \tag{17a}$$

$$b=\frac{\mathbf{R}h_{2,2}}{\mathcal{S}}\left\{\frac{-\dfrac{H(0)}{h_{2,2}}+\dfrac{\left(\mu+\dfrac{H(0)}{h_{2,2}}-\dfrac{1}{2}\right)(\frac{1}{2}+\mu)\exp(x_T/2)}{2\mu\exp(\mu x_T)}}{\left(-\mu+\dfrac{H(0)}{h_{2,2}}-\dfrac{1}{2}\right)}\right\}. \tag{17b}$$

One may calculate c using Eq. (17). Finally, using Eq. (8) we have

$$\frac{\delta p_{2,2}(0)}{p_0(0)}=i\frac{\gamma}{\sigma}\frac{\mathbf{R}h_{2,2}}{\mathcal{S}}\left\{-\frac{(\frac{1}{2}+\mu)}{2\mu}\exp[x_T(\tfrac{1}{2}-\mu)]+\left[\frac{-\dfrac{H(0)}{h_{2,2}}+\left(\mu+\dfrac{H(0)}{h_{2,2}}-\dfrac{1}{2}\right)\dfrac{(\frac{1}{2}+\mu)}{2\mu}\exp[x_T(\frac{1}{2}-\mu)]}{\left(-\mu+\dfrac{H(0)}{h_{2,2}}-\dfrac{1}{2}\right)}\right]+1\right\}. \tag{18}$$

The following are typical tropical values of relevant parameters: $T(0)=17°C$ [which implies $H(0)=8.455$ km] and $q=12.3$ g kg^{-1}. For these choices $x_T=1.1633$ (corresponding to $z_T=8.8364$ km) and

$$\frac{\delta p_{2,2}(0)}{P_0(0)}\approx i\frac{P_{2,2}}{200.7 \text{ g cm}^{-2}\text{ day}^{-1}}. \tag{19}$$

$P_{2,2}$ is frequently expressed in units of cm day^{-1}; its numerical values in either g cm^{-2} day^{-1} or in cm day^{-1} are identical since the density of water is 1 gm cm^{-3}.

Recalling that we want

$$\left|\frac{\delta p_{2,2}(0)\Theta_{2,2}(\theta)}{P_0(0)}\right|\approx 0.4\times 10^{-3},$$

Eq. (19) implies a need for

$$|P_{2,2}|\approx\frac{0.0803}{\Theta_{2,2}(0)}\text{ cm day}^{-1}=0.06905\text{ cm day}^{-1}, \tag{20}$$

with maximum precipitation occurring between 0230 and 0430 LT (and, of course, between 1430 and 1630 LT). Times, a few minutes later than these, may be expected given the expected phase lag between surface precipitation and latent heat release.

Of course, the Siebert profile is sufficiently unrealistic to call into question the quantitative accuracy of (20). For the above choice of q, and the equatorial standard atmosphere, one obtains $z_T=9.3182$ km.

Numerical solution of Eq. (5) yields

$$\frac{\delta p_{2,2}(0)}{P_0(0)}\approx i\frac{P_{2,2}}{321.18\text{ cm day}^{-1}}, \tag{21}$$

implying the need for

$$|P_{2,2}\Theta_{2,2}(0)|\approx 0.1285\text{ cm day}^{-1}. \tag{22}$$

This need for a larger amplitude of semidiurnal precipitation oscillation arises from the fact that the bracketed quantity in Eq. (5) actually becomes positive above the tropopause—permitting the leakage of significant amounts of energy.

The somewhat unrealistic choice of a non-entraining cloud model also affects our results. Typical observed distributions of M_c peak below our choice for z_T and decay smoothly to zero. With a Siebert model for T_0, such M_c distributions do slightly reduce the effectiveness of a given $P_{2,2}$ in producing surface pressure oscillations; however, for the equatorial standard atmosphere, effectiveness is actually increased since the bulk of the forcing is further removed from the stratosphere where propagation is possible. The consideration of such details at this stage, however, seems inappropriate. We merely wish to determine whether the required precipitations are roughly

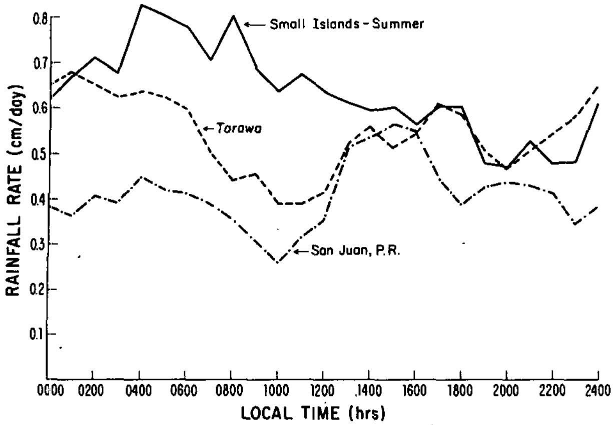

FIG. 3. Rainfall rate vs time. Shown are results from a collection of small Pacific islands in summer (after Jacobson, 1976), from San Juan, P. R. (after Ray, 1928) and from Tarawa (after Finkelstein, 1964).

compatible with observations, and our calculations are sufficient to determine that the required semidiurnal precipitation is on the order of 0.1 cm day^{-1} at the equator and must have its maxima in the ranges 0230–0430 and 1430–1630 LT (or a few minutes later). It should also be noted that these are the values called for at the equator; $\Theta_{2,2}(\theta)$ behaves roughly as $\sin^3\theta$ so that smaller values are expected at higher latitudes. Finally, even before looking at the data in detail, we should note that the called for amplitudes in precipitation rate are much smaller than the mean precipitation rate at the equator [$\sim$0.6 cm day^{-1} (Sellers, 1966)]. This is encouraging; if it were not the case, we would be forced to reject the mechanism without further consideration.

3. Observed daily variations in tropical rainfall

Observation of rainfall rates remains difficult and of questionable accuracy, especially over the sea. Not surprisingly, studies of the daily variations of tropical rainfall are rare and usually rather primitive. Notable among the few papers dealing with this subject are Ray (1928), Finkelstein (1964), Brier and Simpson (1969), Jacobson (1976) and Inchauspe (1970). The problem of determining daily variations in rainfall is complicated by the fact that rainfall does not vary smoothly through the day at individual stations. The general procedure has been to collect rainfall events according to hour of occurrence over many years in order to produce a pattern of daily variation. In most of the above papers, the results are discussed qualitatively with emphasis on the occurrence (or non-occurrence) of rainfall maxima during the late afternoon and early morning. As a rule, Fourier analyses of the daily variations are not presented.

In Fig. 3 some examples of daily variations in rainfall are displayed. The results from San Juan are typical of large islands. Clear maxima appear near 0400 and 1500 LT—suggestive of a strong semidiurnal variation. The lumped results of Jacobson (1976) for a set of small tropical Pacific islands during summer are also shown; here only an early morning maximum appears. The results from Tarawa (a particular small Pacific atoll) show a rather amorphous picture. What should be stressed is that none of these curves shows a pure

TABLE 1. Fourier decomposition of the daily variation in rainfall rate at various tropical stations. The quantities A_1 and A_2 refer to the diurnal and semidiurnal amplitudes (cm day), respectively.

Station and reference	Latitude	Mean rainfall rate (cm day^{-1})	A_1 (cm day^{-1})	Time of maximum (LT)	A_2 (cm day^{-1})	Times of maxima (LT)
Small Pacific Islands-summer (Jacobson, 1976)	7–18°N	0.63	0.126	06.79	0.052	04.22 16.22
Djarkata (Brier and Simpson, 1969)	7°S	0.504	0.343	18.19	0.182	04.6 16.6
Canton Island (Finkelstein, 1964)	3°S	0.216	0.048	00.32	0.040	03.71 15.71
Tarawa (Finkelstein, 1964)	2°N	0.543	0.081	00.32	0.074	03.24 15.34
Apia (wet) (Finkelstein, 1964)	14°S	1.044*	0.102	22.15	0.082	05.27 17.27
Apia (dry) (Finkelstein, 1964)	14°S	0.348*	0.028	07.87	0.006	09.79 21.79
Lauthala Bay (wet) (Finkelstein, 1964)	18°S	1.044*	0.35	17.95	0.241	03.73 15.73
Lauthala Bay (dry) (Finkelstein, 1964)	18°S	0.348*	0.084	21.73	0.015	04.87 16.87
San Juan, P.R. (Ray, 1928)	18°N	0.449	0.051	17.29	0.068	03.67 15.67

* Finkelstein (1964) did not report absolute rainfalls for these stations; he merely noted that average annual rainfall exceeded 100 inches. We have arbitrarily taken the mean rainfall to be 150 inches year^{-1} during the wet half of the year, and 50 inches year^{-1} during the dry half.

diurnal sine wave; hence, we may expect all the curves to have significant semidiurnal Fourier components. The appearance of two maxima is in no way crucial to the presence of a strong semidiurnal Fourier component. What our Fourier analysis of these and other data show is that the superficial differences between various patterns of daily variation in rainfall manifest themselves in large differences in the amplitude and phase of the diurnal component of the rainfall; the amplitudes and, to a larger extent, the phase of the semidiurnal components are notably coherent from station to station.

The results of Fourier analyses for a representative array of cases are shown in Table 1. Error bars, while unavailable, are likely to be large. Nevertheless, it is evident that with the exception of Apia in the dry season, all stations display significant semidiurnal oscillations with maxima between 0300 and 0530, compatible with our requirements. Although the amplitude of the semidiurnal oscillation at most stations is somewhat less than we require, there are two stations where the amplitude is larger than we need and, in general, the semidiurnal amplitude at all stations is of the same order of magnitude as our requirement for an additional semidiurnal source. In view of the uncertainties in the data, this is perhaps the most we could ask for at this stage. Certainly, however, the Fourier analysis of larger collections of data might improve matters, though it appears that meaningful analyses will call for longer records than are generally available.

As has already been noted, the diurnal component is rather incoherent; it is also generally not much larger than the semidiurnal component (as compared to the ratio of diurnal to semidiurnal forcing for the conventional sources). Therefore, there is little reason to expect that tropical latent heat release will contribute significantly to the migrating diurnal tide.

The question remains as to why tropical rainfall behaves in the manner found. To this question we do not, at the moment, have any satisfying answer. It has been suggested by Brier and Simpson (1969) that the rainfall oscillation is forced by the tide. Their suggestion, however, was based on 1) a correlation between the amplitudes of pressure and rainfall oscillations, and 2) the assumption that the surface pressure oscillation was entirely forced by insolation absorption with O_3 playing the major role. The fact that the rainfall oscillation, itself, could force a large contribution to the surface pressure oscillation diminishes the strength of their argument.

In Section 4 we will show that the rainfall oscillation cannot be forced by the tide in any simple way; i.e., the convergence of moisture by the tide is an order of magnitude too small to account for the needed rainfall.

4. More detailed tidal calculations

In this section we describe in greater detail the calculations for an equatorial standard atmosphere first mentioned in Section 2. In the absence of a posited latent heat source, our calculations are identical to those described in Lindzen (1968) and Chapman and Lindzen (1970) for forcing by insolation absorption by ozone and water vapor. Newtonian cooling is included, but without photochemical acceleration (see p. 162 of Chapman and Lindzen). When a latent heat source is included, it is of the form described in Section 2. The magnitude of the forcing for the 2,2 mode is given by Eq. (22) so as to produce the needed contribution to the surface pressure. The phase of the heating is taken to be such that maximum heating occurs at 0330 and 1530 LT.

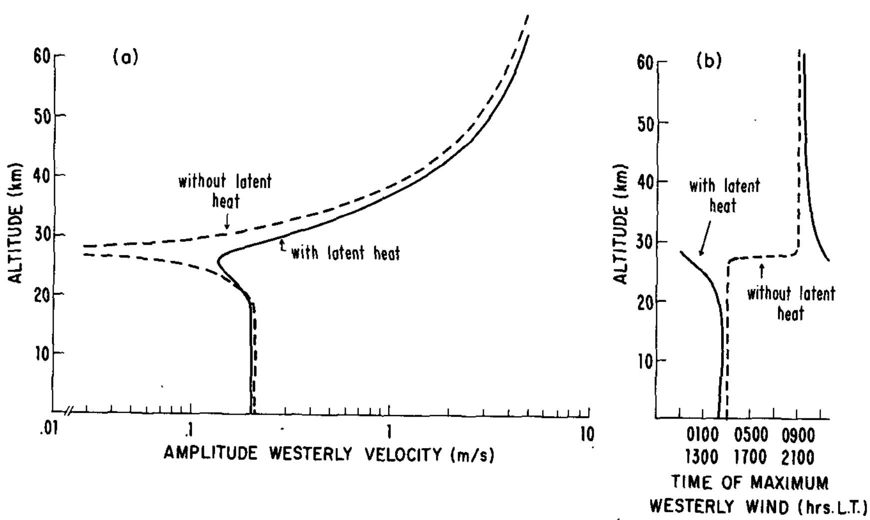

Fig. 4. Calculated semidiurnal oscillation in westerly wind over the equator with and without the inclusion of latent heat forcing (ozone and water vapor forcing present in both cases): (a) amplitudes (m s^{-1}), (b) phase (hour of maximum).

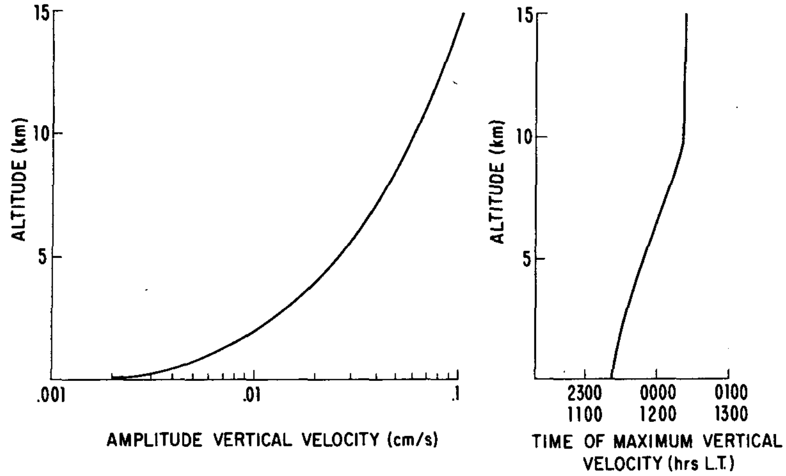

FIG. 5. Calculated amplitude and phase of the semidiurnal oscillation in vertical velocity over the equator as functions of altitude (latent heat forcing included).

There remains the question of whether the latent heatsource might not significantly excite higher order Hough modes. To investigate this question we have assumed that the semidiurnal rainfall oscillation has the same latitudinal distribution as mean rainfall. We have further assumed that the forcing is associated purely with tropical cumulonimbus convection. As the symmetric component of the latitudinal distribution of such rainfall we have used the distribution chosen by Schneider and Lindzen (1977), i.e.,

$$\text{precipitation} \propto \exp\{-[(|y|-0.1)/0.3]^2\}, \quad (23)$$

where $y=\cos\theta$. Expanding (23) in terms of normalized Hough functions yields

$$\exp\{-[(|y|-0.1)/0.3]^2\} \approx 0.7202\Theta_{2,2}(\theta) - 0.1933\Theta_{2,4}(\theta) - 0.0178\Theta_{2,6}(\theta)\cdots. \quad (24)$$

The contributions to higher order modes is even less than in the case of the conventional sources, and hence, is of little importance below the mesosphere. At mesospheric heights and above the neglect of the effects of mean winds (Lindzen and Hong, 1974) and of dissipation (Hong and Lindzen, 1976; Lindzen *et al.*, 1977) are likely to be more serious problems. It has been suggested by Wallace (1975) that even midlatitude rain has a significant semidiurnal variation, and this could, indeed, result in the forcing of higher order modes. However, for the purposes of this paper we have ignored this possibility. The latitudinal distribution of semidiurnal rainfall which is used in this paper is obtained from Eqs. (22) and (24):

$$P(\text{semidiurnal}) \approx 0.1105 \text{ cm day}^{-1}\,\Theta_{2,2}(\theta) - 0.0297 \text{ cm day}^{-1}\,\Theta_{2,4}(\theta) - 0.0027 \text{ cm day}^{-1}\,\Theta_{2,6}(\theta)\cdots. \quad (25)$$

The inclusion of a latent heat source, as described above, leads to a surface pressure oscillation at the equator of amplitude 1.14 mb [assuming $p_0(0)=1013.2$ mb] with maxima at 0940 and 2140 LT. In the absence of the posited latent heat source we obtain an amplitude of 1.18 mb with maxima at 0900 and 2100 LT.

We next turn to the semidiurnal oscillation in westerly wind at the equator. [Results for northerly wind and for other latitudes are easily obtained from the 2,2 mode structures described in Chapman and Lindzen (1970); all are proportional to the equatorial results.] In Fig. 4 we show the predicted results with and without the latent heat source. Fig. 4a shows amplitude vs. altitude, while Fig. 4b shows phase vs. altitude. We see that the inclusion of the latent heat forcing eliminates the node at 28 km as well as the 180° phase shift, as was suggested in Section 1. The results with latent heat forcing are much more in line with the data analyses of Harris *et al.* (1962) and, more recently, of Wallace and Tadd (1974). This adds to the plausibility of such an additional source. Moreover, at greater heights (>40 km), it is interesting to note that results with and without the latent heat source differ almost negligibly.

Finally, we examine the convergence field produced by the semidiurnal tide. In Fig. 5 we show the altitude distribution of the amplitude and phase of the semidiurnal vertical velocity field (including latent heat forcing). This vertical velocity field will, of course, lead to the divergence and convergence of moisture. Our question is whether this convergence and divergence is sufficient to account for the semidiurnal rainfall. The convergence of moisture is easily estimated since moisture is confined to approximately the lowest 2.5 km of the atmosphere and is moderately mixed in this region (below the trade inversion). If q is a characteristic specific humidity in this region, then moisture convergence is given approximately by

$$\rho_0(2.5 \text{ km})w(2.5 \text{ km})q.$$

If the resulting quantity is comparable to the needed precipitation (g cm^{-2} s^{-1}) then the tide may very well be generating part of its own heating. Now from (22) we have

$$|P| \approx 0.1285 \text{ g cm}^{-2} \text{ day}^{-1},$$
$$\approx 1.487\times10^{-6} \text{ g cm}^{-2} \text{ s}^{-1}.$$

From Fig. 5 we have $|w(2.5 \text{ km})| = 1.23\times10^{-2}$ cm s^{-1}, $\rho_0(2.5 \text{ km}) = 0.745\times10^{-3}$ g cm^{-3} and $q\approx 12.3\times10^{-3}$. Thus

$$|\rho_0(2.5 \text{ km})w(2.5 \text{ km})q| = 1.127\times10^{-7} \text{ g cm}^{-2} \text{ s}^{-1}$$

is more than an order of magnitude less than the required $|P|$, and the above suggested possibility can be rejected. Moreover, the phase of w is also inconsistent with what is needed and observed in precipitation. The possibility remains that the tide might act to trigger squall line instabilities of the sort described in Lindzen (1974) and in Stevens and Lindzen (1978), thus indirectly producing a semidiurnal variation in rainfall; however, no detailed picture of such a mechanism has been developed.

5. Conclusions

Noting the discrepancy between the phase of the observed solar semidiurnal surface pressure oscillation and that predicted by all calculations of the tidal response to insolation absorption by ozone and water vapor, we suggest that this discrepancy may be due to the presence of an additional source due to the release of the latent heat of a semidiurnal oscillation in tropical rainfall. We show that this would be the case for a semidiurnal oscillation in rainfall rate with amplitude $\sim$0.12 cm day^{-1} at the equator and with maxima occurring between 0230 and 0430, and between 1430 and 1630 LT (or a few minutes later to take account of the lag between surface precipitation and condensation). A limited analysis of available observations shows that a semidiurnal oscillation of roughly the correct magnitude and phase does indeed exist. It is further shown that the presence of such an additional forcing eliminates the phase shift in horizontal wind oscillations which previous calculations predict to occur at 28 km altitude. Observations show no such phase shift, and thus support the reality of the additional forcing. We also show that the convergence of moisture by the tide cannot directly account for the observed oscillation in rainfall. The origin of the semidiurnal oscillation in rainfall thus remains unaccounted for.

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