Comments on "Cumulus Friction: Estimated Influence on the Tropical Mean Meridional Circulation"

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17 April 1980 and 7 July 1980

The note by Thompson and Hartman (1979, hereafter referred to as TH) purports to show that the influence of cumulus friction, as parameterized by Schneider and Lindzen (1976, hereafter referred to as SL) would be of negligible importance in maintaining the tropical mean meridional circulation. More careful calculations show this conclusion is not justified by existing data.

One objection to TH is that in applying Eq. (1) of their paper, they assumed $u_c = 0$. As discussed in SL, u_c is the zonal velocity of cloud air, and it was argued that u_c in the tropics can reasonably be expected to be close to the zonal velocity u_b of the cloud base air. u_b differs significantly from $u_c = 0$, and the value of u_c is of particular importance in the detrainment layer 100–200 mb. In the detrainment layer, the parameterization of SL represents a drag force due to the mixing of surface air with environmental air at a rate proportional to the cloud-mass flux. Some support for treating horizontal momentum as a conserved quantity in deep convection is provided by the recent analysis of an Oklahoma squall line by Ogura and Liou (1980). The notion that the vertical transport of horizontal momentum by deep cumulus convection is quasi-conservative is arguable; however, TH are examining the consequences of such a hypothesis.

The method chosen by TH for evaluating the effects of cumulus friction is to assume that the zonally averaged zonal wind field is known, and then to deduce the mean meridional velocities consistent with these zonal winds and a specific forcing. The analysis is carried out for the 100–200 mb layer with the forcing representing cumulus momentum flux divergences in the detrainment layer.

For comparison with TH's results, we will calculate the meridional velocity v consistent with the more correct parameterization of the cumulus friction, and compare these v's with the observed 200 mb v, which dominates the observed mass fluxes. We will use the formula

$$F_c = \frac{g}{\Delta p}\{M_c([\bar{u}] - [\bar{u}_b])\}_{200}, \qquad (1)$$

with u_b taken as the zonal velocity at a level near cloud base. TH's formula for computing the mass flux was used in these calculations:

$$[\bar{v}] = \left(\frac{\partial}{\partial y}[\bar{u}] - f\right)^{-1}\Big([\bar{F}_c] - \frac{\partial}{\partial y}\{[(\overline{u'v'})] + [\overline{u^*v^*}])\}, \qquad (2)$$

where the brackets indicate a zonal average, f is the Coriolis parameter, an overbar is a time average over some interval (taken as a season below), an asterisk is the deviation from the zonal average, and the prime indicates the deviation from the time average. Additional forcing terms

$$-[\bar{\omega}]\frac{\partial[\bar{u}]}{\partial \rho} - \frac{\partial}{\partial \rho}([\overline{\omega^*u^*}] + [\overline{\omega'u'}])$$

have been ignored. $[\bar{\omega}]\partial[\bar{u}]/\partial\rho$ is negligible in the 100–200 mb layer as $[\bar{\omega}]$ is small there. Vertical eddy flux divergences are not measured, although they are parameterized by "cumulus friction." The cumulus forced and eddy forced v's add linearly from (2).

The use of (2) to determine the total meridional mass flux, as done by TH, is incorrect except in those regions where

$$\left|[\bar{v}]\left(f - \frac{\partial[\bar{u}]}{\partial y}\right)\right| \gg \left|[\bar{\omega}]\frac{\partial[\bar{u}]}{\partial p}\right| .$$

This inequality may be expected to hold near the top and bottom of the Hadley circulation, and at the latitude of maximum meridional mass flux. In middle layers, the vertical and meridional momentum advections are of the same magnitude, and inclusion of vertical momentum advection by the mean motions will significantly enhance the Hadley cell mass flux in the descending branch. As an illustration, the "observed" $[\bar{\omega}]\partial[\bar{u}]/\partial p$ ($[\bar{\omega}]$ is a derived quantity) at 15°N in northern winter "forces" $[\bar{v}] = 0.54$ m s^{-1} at 300 mb, $[\bar{v}] < 0.1$ m s^{-1} at 100 and 200 mb, and $[\bar{v}] = 0.5$ m s^{-1} at 400 mb, using the Oort and Rasmusson (1971) analysis. That is, two-thirds of the total meridional mass flux at 15°N in northern winter is "forced" inviscidly by $[\bar{\omega}]\partial[\bar{u}]/\partial p$ from 100–400 mb. We use quotes above for "forces" and "forced" because, as discussed below, it is not formally correct to use

0022-4928/80/122803-04$05.00

TABLE 1. Zonal mean precipitation $\bar{p}$ (mm day^{-1}), ratio $\bar{p}_{\max}/\bar{p}_{\mathrm{TH}}$, 200 mb $[\bar{u}]$ (m s^{-1}), $[\bar{u}_b]$ (m s^{-1}), $([\bar{u}] - [u_b]/[\bar{u}]$, $\partial[\bar{u}]/\partial y$ ($\times 10^{-5}$ s^{-1}), cumulus friction ($\times 10^{-5}$ m s^{-2}), calculated v_{200} driven by cumulus friction (m s^{-1}), $-\partial/\partial y([\overline{u'v'}] + [\overline{u^*v^*}])$ ($\times 10^{-5}$ m s^{-2}), calculated v_{200} driven by eddy momentum flux divergences (m s^{-1}), and observed v_{200} (m s^{-1}) for northern winter.

Latitude	$\bar{p}$	$\bar{p}_{\max}/\bar{p}_{\mathrm{TH}}$	$[\bar{u}]$	$[\bar{u}_b]$	$\frac{[\bar{u}] - [u_b]}{[\bar{u}]}$	$\frac{\partial[\bar{u}]}{\partial y}$	Cumulus friction	$(v_{200})_{\mathrm{cf}}$	Eddy flux divergence	$(v_{200})_{\mathrm{ef}}$	$(v_{200})_{\mathrm{obs}}$
25°N	1.2	—	34.0	−0.8	—	—	—	—	—	—	—
20°N	1.1	3.2	23.9	−3.3	1.1	1.85	−1.6	0.5	−3.5	1.1	1.1
15°N	1.4	—	13.6	−5.0	1.4	1.72	−1.3	0.6	−2.7	1.2	1.7
10°N	2.2	1.8	5.0	−5.1	2.0	1.34	−1.1	1.0	−1.2	1.1	2.6
5°N	3.9	—	−1.1	−3.5	−2.2	0.84	−0.5	1.1	0.15	−0.3	3.1
0°	3.9	1.7	−4.2	−1.9	0.55	0.25	0.5	1.9	0.6	2.4	2.6
5°S	4.1	—	−3.9	−1.1	0.72	−0.21	0.6	0.6	−0.01	0	0.6
10°S	4.3	—	−1.9	−1.2	—	—		—	—	—	—

$[\bar{\omega}]\partial[\bar{u}]/\partial p$ as a forcing term in this type of diagnostic calculation. The above calculation shows only that vertical momentum advection must be included to correctly diagnose the distribution of the meridional mass flux, and illustrates the inconsistency of TH's mass flux calculations. $[\bar{\omega}]$ and $[\bar{v}]$ must be calculated simultaneously in the diagnostic procedure, as they are related by mass continuity. Eq. (2) may be used to find the mass flux at the latitude where $[\bar{\omega}] \approx 0$ and the mass flux attains its maximum value, although this latitude is not known *a priori*.

The 200 mb meridional velocities were computed for winter and summer, using zonally averaged data from Oort and Rasmusson (1971) for the Northern Hemisphere and the Southern Hemisphere to 10°S. Zonally averaged precipitation was taken from Schutz and Gates (1972a, 1972b). u_b was evaluated at 950 mb. The results are presented in Table 1 for northern winter and Table 2 for northern summer. No attempt was made to evaluate the correlation resulting from zonal asymmetries in the precipitation and zonal winds, although this "standing eddy" effect is certainly important. Also shown in Tables 1 and 2 is $([\bar{u}] - [u_b])/[\bar{u}]$, which is a measure of the relative difference between using $[\bar{u}_c] = [\bar{u}_b]$ and $[\bar{u}_c] = 0$ in the formula for the cumulus friction.

The 200 mb horizontal eddy momentum flux divergences calculated from the Oort and Rasmusson data and the 200 mb meridional velocities from (2) consistent with $[\bar{u}]$ are also presented in Table 1. TH claim that these v's should agree reasonably well with the observed meridional circulation.

In contrast to the results of TH, the 200 mb v's forced by cumulus detrainment agree reasonably well in magnitude and sign with the observed values. The v's calculated from the observed $\partial(\overline{u'v'} + \overline{u^*v^*})/\partial y$ are comparable to the v's consistent with cumulus detrainment in northern winter, except near the equator, where cumulus friction gives a more realistic simulation. In northern summer, the cumulus driven meridional circulation provides a better simulation than the eddy flux driven circulation. We conclude that the Hadley circulation $[\bar{v}]$ forced in upper levels by cumulus friction alone is consistent with the observed Hadley circulation 200 mb $[\bar{v}]$. There is, however, one anomalous value calculated for $[\bar{v}]_{200}$ at 5°S in the northern summer. Cloud friction is consistent with $[\bar{v}]_{200} \approx 0.5$ m s^{-1}, while the observed $[\bar{v}]_{200} \approx -2.4$ m s^{-1}. As 5°S is near the latitude where $f - \partial[\bar{u}]/\partial y$ changes sign, the discrepancy could be due to relatively small observational errors in $[\bar{u}]$ (O(1 m s^{-1})) in this data-sparse region. The problem could also be due to contribution of the standing eddy fluxes, although measurements of these fluxes are also likely to contain large errors. We note that TH consider the data unreliable between 5°S and 5°N. Zonal mean precipitation estimates (Newell *et al.*, 1974, Fig. 7.14) for the region under consideration,

TABLE 2. As in Table 1 except for northern summer.

Latitude	$\bar{p}$	$\bar{p}_{\max}/\bar{p}_{\mathrm{TH}}$	$[\bar{u}]$	$[\bar{u}_b]$	$\frac{[\bar{u}] - [u_b]}{[\bar{u}]}$	$\frac{\partial[\bar{u}]}{\partial y}$	Cumulus friction	$(v_{200})_{\mathrm{cf}}$	Eddy flux divergence	$(v_{200})_{\mathrm{ef}}$	$(v_{200})_{\mathrm{obs}}$
25°N	2.2	—	1.0	−1.9	—	—	—	—	—	—	—
20°N	2.5	1.6	−2.8	−2.1	0.25	0.6	~0	0	−1.1	0.2	−0.3
15°N	3.5	—	−5.5	−1.6	0.71	0.5	0.69	−0.2	−0.7	0.2	−0.2
10°N	5.1	1.2	−8.1	−0.8	0.9	0.4	1.92	−0.9	0.1	−0.05	−0.8
5°N	4.9	—	−9.8	−1.0	0.9	0.1	2.26	−1.8	0.7	−0.5	−1.7
0°	3.1	2.2	−9.0	−2.0	0.78	−0.46	1.13	−2.4	0.3	−0.6	−2.2
5°S	2.2	—	−4.7	−3.3	0.3	−0.97	0.16	0.53	−0.4	−1.3	−2.4
10°S	1.8	—	1.7	−4.6	—	—	—	—	—	—	—

to which the 200 mb cumulus friction $[\bar{v}]_{\mathrm{cf}}$ are proportional, vary widely, with the values chosen by TH always the smallest. Tables 1 and 2 present the ratios of the largest precipitation estimates to the values used by TH ($\bar{p}_{\max}/\bar{p}_{\mathrm{TH}}$) for the equator, 10°N and 20°N. The $(v_{200})_{\mathrm{cf}}$ in Tables 1 and 2 may be regarded as conservative estimates.

As mentioned above, Eq. (2) may be used to estimate the total Hadley cell mass flux by application at the latitude where $[\bar{\omega}] \approx 0$. Using data presented in Oort and Rasmusson (1970, 1971) and Newell *et al.* (1972, 1974), this quantity is computed for northern winter, and an attempt is made to estimate the range of values that are consistent with the various data analyses. Oort and Rasmusson (1970) find the position and strength of the monthly mean Hadley circulation, with notation (month, latitude, mass flux $\times 10^{10}$ kg s^{-1}), to be (December, 7, 18), (January, 5, 19) and (February, 2, 23). The mean strength of the winter (DJF) Hadley cell is then 20×10^{10} kg s^{-1} and the strength-weighted mean center of the DJF Hadley cell is 4.5°N. Newell *et al.* (1972, Fig. 3.19) place the center of the Hadley cell near 8°N. We use 5°N as the center of the Hadley cell, for convenience in finite differencing, as $[\bar{u}]$ data are tabulated by Oort and Rasmusson (1971) at 0°, 5 and 10°N, while Newell *et al.* (1972) tabulate $[\bar{u}]$ at 0 and 10°N. DJF precipitation estimates (Newell *et al.*, 1974, Fig. 7.14) of various authors range from 3.9 to 5.9 mm day^{-1} at 5°N. Oort and Rasmusson (1971) give $[\bar{u}]_{200}$ of -4.2 m s^{-1} at 0° and 5.0 m s^{-1} at 10°N, while Newell *et al.* (1972) give $[\bar{u}]_{200}$ as -1.1 m s^{-1} at 0° and 7.6 m s^{-1} at 10°N. The smallest mass flux consistent with the data is obtained using 3.9 mm day^{-1} precipitation, $[\bar{u}]_{200} = -1.1$ m s^{-1} at 0° and $[\bar{u}]_{200} = 5.0$ m s^{-1} at 10°N. The largest mass flux is found using the values 5.9 mm day^{-1} precipitation, $[\bar{u}]_{200} = -4.2$ m s^{-1} at 0° and $[\bar{u}]_{200} = 7.6$ m s^{-1} at 10°N. The calculated mass flux ranges from 2.1×10^{10} kg s^{-1} to 16.7 $\times 10^{10}$ kg s^{-1}, or from 16 to 84% of the observed mass flux. Thus, all that can be said concerning the maintenance of the zonally averaged tropical general circulation is that the data are not inconsistent with a significant contribution from cumulus friction.

The question as to whether the 200 mb meridional circulation is driven by cumulus friction or eddy flux divergences may be obscured by observational aliasing. Cumulus detrainment of surface air with surface momentum in the 100–200 mb layer will appear as a $\overline{u^*v^*}$ or a $\overline{u'v'}$ generally in the same direction as the observed eddy momentum flux. This cloud forcing would appear as an eddy flux contribution because the active convective clouds cover only a small fractional area and the area mean wind is approximately the wind in the cloud environment. The sign of u' due to cumulus detrainment will be the same as the sign of $-(u_{200} - u_b)$, i.e., westerly shear with height gives $u' < 0$ since the surface air velocity is easterly relative to u_{200}). Similarly, the sign of v' will be the sign of $-(v_{200} - v_b)$ [in a cellular circulation, the sign of v' will be the sign of $-v_{200}$ since surface air has a meridional velocity opposite in sign to the upper level branch]. Applying these arguments to the Oort and Rasmusson northern winter zonally averaged flow yields $\overline{u'v'} < 0$ for latitudes 10°S, 5°S and the equator, $\overline{u'v'} > 0$ for latitudes 5–25°N. The observed 200 mb $\overline{u'v'}$ changes sign between 15 and 20°N. In summer cumulus momentum transports would appear as $\overline{u'v'} > 0$, 5°S to 25°N, and $\overline{u'v'} < 0$, 10°S. The observed value of $\overline{u'v'}$ is >0 at 10°S to 25°N.

From a mechanistic point of view, it seems more instructive to consider the zonal momentum equation as a formula determining the change of angular momentum of a parcel along trajectories. The effect of frictional forces in the poleward branch of the Hadley cell is generally to decrease the parcel angular momentum from its conserved value, rather than to force a cellular overturning. The strength of the meridional circulation is determined primarily by the atmospheric heat source/sink distribution (Schneider, 1977), almost independent of the frictional forcing.

An inviscid upper branch of the Hadley circulation would be reflected by $f - \partial[u]/\partial y \approx 0$ (i.e., angular momentum conservation). The meridional circulation that must exist to allow geostrophy (the thermal wind relation) with angular momentum conserving upper branch winds and small surface winds produces very small horizontal temperature gradients due to the smallness of f in the tropics (Schneider, 1977). Frictional forces that reduce the vertical shear of the zonal wind to zero imply only a small modification in the strength of the Hadley circulation, as the change in $\partial T/\partial y$ required to maintain the thermal wind balance is small.

The meridional circulation of the same meridional extent as the inviscid circulation required to produce $\partial T/\partial y = 0$ is determined by the heat sources and sinks through the thermodynamic equation and is ~20% stronger than the meridional circulation needed to maintain the temperature distribution consistent with angular momentum conservation for the observed tropical precipitation and radiative cooling rates. Thus, frictional forces do not "drive" the Hadley circulation to a first approximation, but rather act to decrease the zonal winds from their angular momentum conserving maximum values.

Diagnostic calculations of the meridional circulation such as the one presented here, or the calculation of TH are very sensitive to the value of $f - \partial[\bar{u}]/\partial y$ in the tropics, where both f and $\partial[\bar{u}]/\partial y$ are of the same order. The meridional circulation diagnosed from the heat balance equation, using observed heat sources and sinks and temperatures should, by the above argument, be insensitive to small observational errors. This meridional circulation may then be used to accurately diagnose frictional forces through the zonal wind equation (2). We then may restate our results as showing that cumulus friction can be playing an important role in the tropics in reducing the magnitude of the zonal jet from its angular momentum conserving value.

Acknowledgments. This research was supported by NASA Grant NGL-22007-228 and NSF Grant ATM-78-23330.

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