——, 1972b: Air motions in the tropical stratosphere deduced from satellite tracking of horizontally floating balloons. *J. Atmos. Sci.*, **29**, 570–582.
Longuet-Higgins, M. S., 1969: On the transport of mass by time-varying ocean currents. *Deep-Sea Res.*, **16**, 431–447.
Murgatroyd, R. J., 1969: The structure and dynamics of the stratosphere. *The Global Circulation of the Atmosphere*, London, Roy. Meteor. Soc., 159–195.
Newell, R. E., D. G. Vincent, T. G. Dopplick, D. Ferruzza and J. W. Kidson, 1969: The energy balance of the global atmosphere. *The Global Circulation of the Atmosphere*, London, Roy. Meteor. Soc., 42–90.
Phillips, O. M., 1966: *The Dynamics of the Upper Ocean.* Cambridge University Press, 261 pp.
Solot, S. B., and J. K. Angell, 1969a: Temperate latitude 200-mb zonal winds from GHOST balloon flights in the Southern Hemisphere. *J. Atmos. Sci.*, **26**, 574–579.
——, and ——, 1969b: Mean meridional air flow implied by 200-mb GHOST balloon flights in temperate latitudes of the Southern Hemisphere. *J. Atmos. Sci.*, **26**, 1299–1305.

An Analytic Formula for Heating Due to Ozone Absorption

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From the tropopause to about 80 km, the major source of radiative heating in the atmosphere is the absorption of sunlight by ozone. This absorption occurs in the Hartley (2000–3000 Å), Huggins (>3000 Å), and Chappuis (5000–7000 Å) bands (Craig, 1965, p. 168; also Craig, 1950) and is only slightly temperature and pressure dependent. Consequently, to a high degree of approximation one may write

$$I_\nu = I_{\nu\infty}\exp(-\kappa_\nu u), \tag{1}$$

$$Q = \int_\nu \kappa_\nu I_\nu n\, d\lambda, \tag{2}$$

where:

Q heating (ergs sec^{-1} cm^{-3})

u optical depth (cm NTP) $\left[= \int_\infty^z ndz/\cos\theta\right]^\dagger$

† This expression is appropriate only for a flat atmosphere and must be modified to take sphericity into account.

θ zenith angle
n ozone density (cm NTP cm^{-1})
κ_ν absorption coefficient (cm NTP)$^{-1}$
$I_{\nu\infty}$ insolation (incident solar intensity) (ergs cm^{-2} sec^{-1} Å^{-1})
I_ν intensity (ergs cm^{-2} sec^{-1} Å^{-1})
λ wavelength (Å)
z height (cm)

Distributions of $I_{\nu\infty}$ and κ_ν are shown in Figs. 1–3. The quantity $Q/n \equiv \eta$ is solely a function of u, and for the choice of spectral data shown in Figs. 1–3 leads to the distribution of η with u shown in Fig. 4. As noted in Lindzen and Goody (1965) there are two regions of the atmosphere where η may be approximated by a constant:

1) Above ~45 km where with $u < 2\times10^{-3}$ cm NTP we are in a region which is almost transparent for all ozone bands. Hence, ozone band radiation is not significantly attenuated and the heating is largely due to ozone's strongest band, the Hartley band.

2) Below ~30 km where with 0.3 cm NTP$>u>$0.15 cm NTP, most radiation in the Hartley and Huggins bands has been absorbed but the atmosphere is still almost transparent for the Chappuis which dominates the heating.[2]

The question now arises as to whether a simple expression exists which describes both regions of "constant" η as well as the transition between them. Moreover, we wish a convenient way of relating parameters in the expression to spectral data. Now, in the regions of constant η, η depends only on average values of spectral intensities and absorption coefficients for the Hartley and Chappuis bands. Thus, a simple solution might be to use average values for I_ν and κ_ν, appropriate to the Hartley and Chappuis bands, in (1) and (2), modelling both bands as simple Chapman layers (Craig, 1965, pp. 147–150). One then obtains for η the expression

$$\eta = I_H\kappa_H\Delta\lambda_H\exp(-\kappa_H u) + I_c\kappa_c\Delta\lambda_c\exp(-\kappa_c u), \tag{3}$$

where I_H, κ_H and I_c, κ_c are average I_ν, κ_ν for Hartley and Chappuis bands and $\Delta\lambda_H$, $\Delta\lambda_c$ are the bandwidths taken for these bands. For the range of u's relevant to the earth's atmosphere, κ_H, I_H, κ_c and I_c can be chosen to accurately model the regions of constant η; Eq. (3) will then also describe the transition between the two regions. Unfortunately, the accuracy of such an approxi-

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[2] Note that Fig. 4 is misleading on this matter since it includes values of u which greatly exceed the atmosphere's total ozone content; such values could be important, however, in time-dependent calculations where θ may approach $\pi/2$.

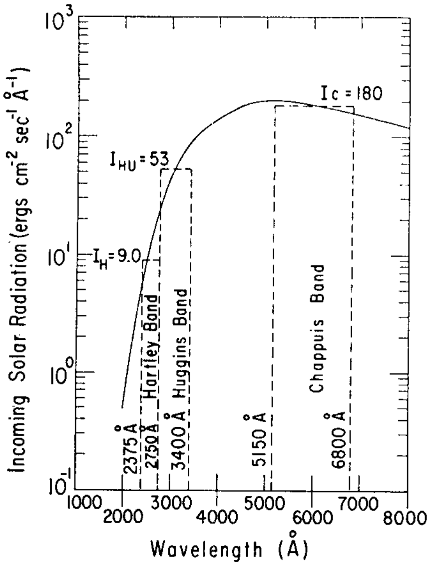

FIG. 1. The spectral disbribution of incoming solar radiation (solid line, after Craig, 1950) and the average values of intensity and band limits used in our modelling procedure.

mation proved poor in the transition zone because of the importance of the Huggins bands in that zone. Modelling the Huggins bands proved reasonably simple. From Fig. 2 we see that κ_ν is almost a linear function on a semi-log plot. Thus, we may write

$$\kappa_\nu = \kappa_{Hu} e^{-M\lambda}, \tag{4}$$

where κ_{Hu}, M are constants of the straight line on the semi-log plot. Taking for I_ν an average over the Huggins

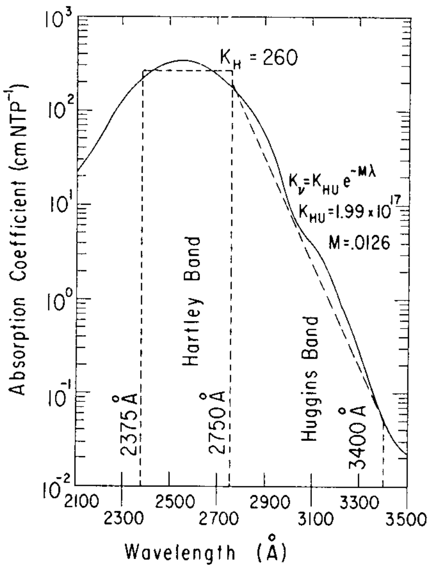

FIG. 2. The absorption by ozone as a function of wavelength for the Hartley and Huggins bands (solid curve, from Craig, 1950) and the approximations (dashed curves) to the absorption used in our modelling procedure.

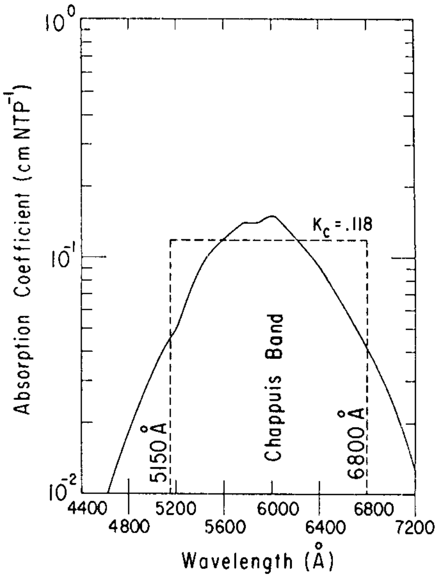

FIG. 3. Same as Fig. 2 except for the Chappuis band.

bands, I_{Hu}, we can easily integrate (2) obtaining

$$\eta_{Hu} = \frac{I_{Hu}}{Mu}\{\exp[-u\kappa_{Hu}\exp(-m\lambda_{\text{long}})] - \exp[-u\kappa_{Hu}\exp(-m\lambda_{\text{short}})]\}, \tag{5}$$

where λ_{long} and λ_{short} are the wavelength limits used for the Huggins bands. Thus, an improved approximation to η consists in the sum of (3) and (5), namely

$$\eta = I_H\kappa_H\Delta\lambda_H \exp(-\kappa_H u) + I_c\kappa_c\Delta\lambda_c \exp(-\kappa_c u) + \frac{I_{Hu}}{Mu}\{\exp[-u\kappa_{Hu}\exp(-m\lambda_{\text{long}})] - \exp[-u\kappa_{Hu}\exp(-m\lambda_{\text{short}})]\}. \tag{6}$$

Expression (6) has, in principle, a substantial number

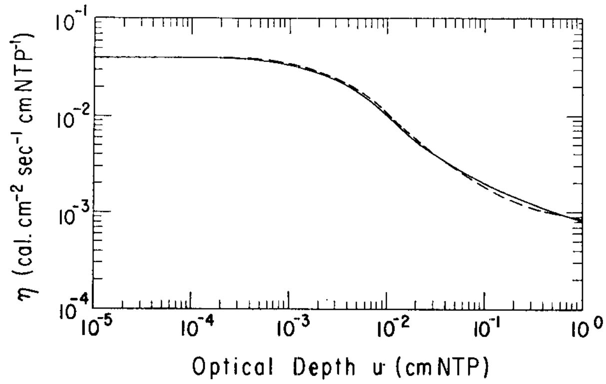

FIG. 4. Specific heating η due to ozone as a function of optical depth u. The solid curve represents Craig's (1951) calculation, the dashed curve our analytic formula.

of adjustable parameters: $\Delta\lambda_H$, $\Delta\lambda_{Hu}$, $\Delta\lambda_c$ (the widths of the Hartley, Huggins and Chappuis bands), I_H, I_c, I_{Hu}, and κ_H and κ_c. If one merely estimates these parameters on the basis of Figs. 1–3, we obtain a uniform 20% error bound in approximating Fig. 4. However, with a little experimentation we were able to choose these parameters so as to obtain the approximation shown in Fig. 4 which, in general, has errors less than 5%. The parameters chosen are shown in Figs. 1–3. The following formulas should prove useful when dealing with more recent or better data than that shown in Figs. 1–3:

Hartley bands:

$$I_H = 9 \text{ ergs cm}^{-2}\text{ sec}^{-1}\text{ Å}^{-1}$$
$$\kappa_H = 260 \text{ cm NTP}^{-1} = 80\% \quad (\kappa_\nu \text{ maximum for Hartley}) \qquad (7)$$
$$\Delta\lambda_H \quad 375 \text{ Å}$$

Chappuis bands:

$$I_c = 180 \text{ ergs cm}^{-2}\text{ sec}^{-1}\text{ Å}^{-1}$$
$$\kappa_c = 0.118 \text{ cm NTP}^{-1} = 80\% \quad (\kappa_\nu \text{ maximum for Chappuis}) \qquad (8)$$
$$\Delta\lambda_c = 1650 \text{ Å}$$

Huggins bands:

$$\kappa_{Hu} = 1.99\times10^{17}$$
$$M = 0.0126$$
$$I_{Hu} = 53 \text{ ergs cm}^{-2}\text{ sec}^{-1}\text{ Å}^{-1} \qquad (9)$$
$$\lambda_{\text{short}} = 2750 \text{ Å}$$
$$\lambda_{\text{long}} = 3400 \text{ Å}$$

Although our formulas for the Huggins bands are not explicitly in a form which would permit adaptation to other data, the method of choosing the parameters, discussed earlier in this note, is obviously adaptable. We chose to model Craig's (1951) somewhat outdated calculation simply because complete spectral information was conveniently available for his calculation. This case is sufficient to demonstrate that ozone heating can be accurately modelled by two Chapman layers plus a modified Chapman layer for the Huggins bands, which is our main result.

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REFERENCES

Craig, R. A., 1950: The observations and photochemistry of atmospheric ozone and their meteorological significance. *Meteor. Monogr.*, 1, No. 2, 50 pp.

——, 1951: Radiative temperature changes in the ozone layer. *Compendium of Meteorology*, T. F. Malone, Ed., Boston, Amer. Meteor. Soc., 292–302.

——, 1965: *The Upper Atmosphere: Meteorology and Physics*. New York, Academic Press, 509 pp.

Lindzen, R., and R. Goody, 1965: Radiative and photochemical processes in mesospheric dynamics: Part I. Models for radiative and photochemical processes. *J. Atmos. Sci.*, **22**, 341–348.

The Altitude of the Scattering Layer near the Mesopause over the Summer Poles

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ABSTRACT

The time and latitude variation in the radiance and altitude of the dense scattering layers observed over the summer poles is reported using OGO 6 airglow photometer data. The average altitude was 84.3 km with a tendency for higher values on the night side than on the day side of the polar cap. The average radiance increased by a factor of 5 between day 163 and day 180 (1969), but decreased thereafter.

Recently we reported the observation of a scattering layer that develops over the summer pole near the mesopause beginning about 12 days before the solstice (Donahue *et al.*, 1972). The discovery of these layers of which noctilucent clouds appear to be extensions was made by a narrow-field, horizon-scanning airglow photometer aboard the OGO-6 satellite. Above 75° latitude the layers appear to contain between 4 and 8×10^6 particles cm^{-2} in a layer less than 5 km thick, if the particles are spherical ice crystals of 1300 Å radius. This is about 50 times the optical depth of noctilucent clouds. Here we wish to report briefly a few other characteristics of these layers.

We have determined how the altitude of the layer varied with (northern) latitude for a large number of passes by the satellite between day 162 and day 186 in