

Multiple Gravity-Wave Breaking Levels

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ABSTRACT

It is noted that gravity waves for which $|\bar{u} - c|$ (\bar{u} = mean flow speed, c = wave phase speed) has a sharp minimum in the upper troposphere or lower stratosphere will have decaying amplitudes above this level despite exponentially decreasing mean density. Eventually this decay ceases and growth resumes. Thus, if a gravity wave breaks below the level of $|\bar{u} - c|_{\min}$, it will cease breaking above this level. Breaking will, however, resume at some higher level. This second breaking level is a lower bound for the level of breaking in the mesosphere since waves too weak to break where $|\bar{u} - c| = |\bar{u} - c|_{\min}$ will break at still higher levels in the mesosphere. Explicit calculations show the "second" breaking levels to be close to observed levels of mesospheric gravity wave breaking. Evidence is also cited for wave breaking in the lower atmosphere, and for the importance of this breaking in the momentum budget of the lower atmosphere.

1. Introduction

In recent papers, the role of breaking gravity waves in stressing and mixing the mesosphere has been highlighted (Lindzen, 1981, 1984; Holton, 1982). The subject is not without controversy. However, even if accepted, there remain obvious questions as to what determines wave scales and amplitudes, and relatedly, breaking levels. One purpose of this note is to suggest a partial solution to the last question—that of breaking levels. We will determine a nontrivial lower bound for mesospheric breaking levels which, for winter at least, is surprisingly close to observed breaking levels.

The essence of our approach is to note that the amplitude of waves generated in the troposphere is limited by the breaking condition at the level where $|\bar{u} - c|$ is a minimum; this level, itself, typically occurs in or just above the troposphere. Moreover, if $|\bar{u} - c|_{\min}$ is sufficiently small (in a sense to be defined in Section 2) then breaking will cease above the level of minimum $|\bar{u} - c|$ because for a considerable distance above this level, wave amplitudes will actually decrease despite decreasing mean density. Eventually amplitudes will begin increasing again and a second breaking level will be established. The calculation of this second level is one of the main purposes of this note. Relevant details are given in Section 2 though the basic mathematical development is taken from Lindzen (1981, 1984) and for the most part will not be repeated here.

In Section 3 we will consider the relation of the present calculations to observations. In particular we will consider long-standing observations of mountain wave breaking in the neighborhood of the tropopause (Lilly, 1972) and the role of such waves in the

momentum budget of the lower atmosphere as well as the mesosphere.

Results and limitations are summarized in Section 4.

2. Mathematical development

The starting point for our analysis will be Eq. (8) of Lindzen (1981) [or Eq. (4) of Lindzen (1984)] for the distribution in space and time of the perturbation contribution to the vertical derivative of temperature by a plane internal gravity wave:

$$\frac{d\delta T}{dz} \approx A\Gamma^{1/2}\bar{T}^{-1/2}\lambda^{3/2}e^{i\int^z \lambda dz} e^{z/2H} e^{ik(x-ct)}, \quad (1)$$

where

δT = perturbation temperature,

z = altitude,

A = amplitude factor,

\bar{T} = basic temperature,

$$\Gamma = \frac{d\bar{T}}{dz} + \frac{g}{c_p},$$

$$\lambda = \text{vertical wavenumber} \approx \left| \frac{N}{\bar{u} - c} \right| \left(1 + \frac{l^2}{k^2} \right)^{1/2},$$

\bar{u} = mean zonal flow,

$$N^2 = \frac{\partial}{\partial z} \Gamma,$$

l = meridional wavenumber,

k = zonal wavenumber,

c = zonal phase speed.

One could of course consider wave packets instead of plane waves as well as other extensions. However, the simplicity and clarity of (1) is desirable at this point.

For present purposes, only the amplitude of (1) is needed; i.e.

$$\begin{aligned} \left| \frac{d\delta T}{dz} \right| &\approx A\Gamma^{1/2}\bar{T}^{-1/2}\lambda^{3/2}e^{z/2H}, \\ &\approx A\Gamma^{1/2}\bar{T}^{-1/2} \frac{N^{3/2}}{(\bar{u}-c)^{3/2}} \left(1 + \frac{l^2}{k^2}\right)^{3/4} e^{z/2H}, \end{aligned} \quad (2)$$

and the condition for breaking is simply

$$\left| \frac{d\delta T}{dz} \right| = \Gamma. \quad (3)$$

The crucial point to note about (2) is that in regions where $|\bar{u}-c|$ is increasing rapidly with height, it may be possible to overcome the tendency of amplitudes to increase associated with the $e^{z/2H}$ factor. Quite obviously this will depend on both the mean shear and the value of $|\bar{u}-c|_{\min}$.

A simplified analysis that focuses on these parameters shows this dependence more clearly. Let us, for the moment, ignore the effect of varying \bar{T} and Γ and assume that $|\bar{u}-c|$ increases linearly above its minimum value; i.e.

$$|\bar{u}-c| = |\bar{u}-c|_{\min} + m(z-z_{\min})$$

where

$$z_{\min} = \text{altitude}$$

where

$$|\bar{u}-c| = |\bar{u}-c|_{\min}.$$

Then from (1) we have

$$\left| \frac{d\delta T}{dz} \right| \propto \frac{e^{(z-z_{\min})/2H}}{[|\bar{u}-c|_{\min} + m(z-z_{\min})]^{3/2}}. \quad (4)$$

Now let

$$\frac{z-z_{\min}}{2H} = \zeta, \quad (5)$$

$$\delta = \frac{|\bar{u}-c|_{\min}}{2mH}. \quad (6)$$

Equation (4) therefore becomes

$$\left| \frac{d\delta T}{dz} \right| \propto \frac{e^{\zeta}}{(\delta + \zeta)^{3/2}} \equiv f(\zeta). \quad (7)$$

Next differentiate f with respect to ζ in order to see the tendency of $|d\delta T/dz|$ above $z = z_{\min}$:

$$\frac{df}{d\zeta} = \frac{e^{\zeta}}{(\delta + \zeta)^{3/2}} \left(1 - \frac{3}{2} \frac{1}{\delta + \zeta}\right). \quad (8)$$

We see, immediately, that f (and, hence $|d\delta T/dz|$) will decrease above z_{\min} provided that $3/2\delta > 1$. As we will soon see, this condition is readily achieved under realistic conditions.

Note that δ depends on both $|(c-\bar{u})|_{\min}$ and the shear m ; more precisely, it depends on the fractional

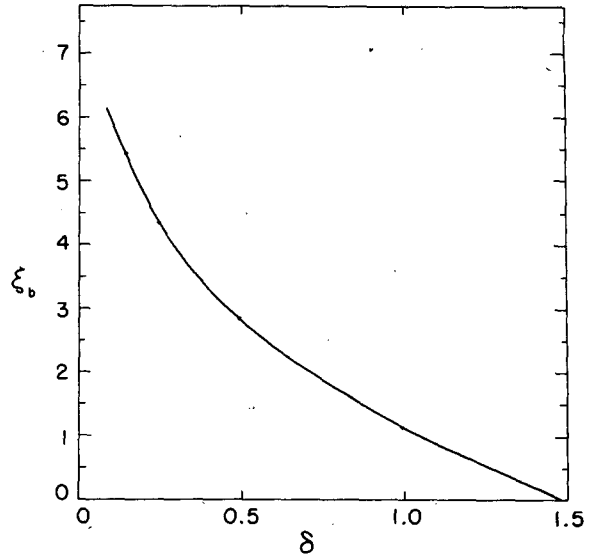


FIG. 1. The term ζ_b (the breaking height z_b , minus the level of minimum $|\bar{u}-c|$ divided by twice the scale height) vs $\delta[|\bar{u}-c|_{\min}/(2H(d\bar{u}/dz))]$. See text for details.

change in $(c-\bar{u})$. Thus, even with constant shear, $df/d\zeta$ eventually turns positive [viz. (8)] and there will be a level where condition (3) (breaking) is again obtained. In terms of (7) $f(0) = \delta^{-3/2}$. Let ζ_b be that value of ζ where $f(\zeta_b) = \delta^{-3/2}$ again.

Figure 1 shows ζ_b as a function of δ .¹ Obviously, the smaller δ is the higher the second (mesospheric) breaking level. Note that for a gravity wave originating in the troposphere, ζ_b is a lower bound for the "mesospheric" breaking altitude because (for a given choice of c) no wave amplitude can exceed the breaking amplitude at z_{\min} . (Moreover, all waves whose amplitudes exceed the minimum value needed for breaking at z_{\min} should, in the absence of damping, reach ζ_b .)

Using Fig. 3 in Lindzen (1981) for winter zonal winds and assuming we are dealing with orographically-generated waves for which $c = 0$, we see that $|\bar{u}-c|_{\min} = \bar{u}_{\min} = 10 \text{ m s}^{-1}$ at $z = z_{\min} = 24 \text{ km}$. A characteristic shear above z_{\min} is $2.3 \text{ m s}^{-1} \text{ km}^{-1}$ and a characteristic scale height is $H = 7 \text{ km}$. With these choices we find $\delta = 0.31$ and from Fig. 1 we see that $\zeta_b = 3.9$ corresponding to $z_b = z_{\min} + 2H\zeta_b = 79 \text{ km}$. This is, in fact, greater than the height at which breaking is believed to be observed. To be sure, it is inappropriate to use a constant shear and a constant Γ for the whole region above z_m . Using Fig. 3 of Lindzen (1981) for wind and the 1966 U.S. Standard Atmosphere for \bar{T} we have used Eq. (2) to calculate $|d\delta T/dz|$ as a function of height. The result is shown in Fig. 2. We also plot Γ as a function of height and

¹ Whenever the parameter ζ and δ are referred to, we are using the idealized constant shear-constant Γ profiles. Otherwise we are using "realistic" profiles.

assume $|d\delta T/dz| = \Gamma$ at $z = z_{\min}$. $|d\delta T/dz|$ intersects Γ again at $z = 65$ km, somewhat lower than our previous estimate—consistent with the fact that \bar{u} ceases to increase above 53 km but below z_b . For comparison purposes we include a plot of f scaled so that it equals 9.80 km^{-1} at z_{\min} . Note that wave growth resumes above $z \approx 50$ km.

The situation in summer is somewhat more ambiguous. Mean winds change sign near $z = 17$ km, so that waves with $c = 0$ cannot reach the mesosphere. Lindzen (1981) hypothesized that phase speeds somewhat in excess of \bar{u}_{\max} (i.e., 15 m s^{-1} at $z = 12$ km) would dominate in the mesosphere. However more concrete information is still not available. What is clear is that if gravity waves originate within the troposphere, then only those with $c > \bar{u}_{\max}$ are likely to reach the upper mesosphere. This still leaves the magnitude of δ to be used in (7) uncertain. From Fig. 3 in Lindzen (1981) we estimate that $m = -2.3 \text{ m s}^{-1} \text{ km}^{-1}$; for H we keep 7 km ; $\bar{U}_{\max} \approx 15 \text{ m s}^{-1}$. For $c = 17 \text{ m s}^{-1}$, $a = 2 \text{ m s}^{-1}$ and $\delta = 0.115$, leading to $\zeta_b = 5.9$. For $c = 25 \text{ m s}^{-1}$, $a = 10 \text{ m s}^{-1}$ and $\delta = 0.575$, leading to $\zeta_b = 2.5$. Finally, we use the more realistic values of \bar{u} in Fig. 3 for Lindzen (1981) to plot $|d\delta T/dz|$ using Eq. (2). A range of choices for c are tried. Figure 3 shows $|d\delta T/dz|$ for $|\bar{u} - c|_{\min} = 2 \text{ m s}^{-1}$; Γ and f are also shown. Figure 4 shows z_b versus $|\bar{u} - c|_{\min}$. Note that Fig. 4 differs significantly from Fig. 1 for large $|\bar{u} - c|_{\min}$ or δ ; this is because the real Γ is not independent of z but rather increases

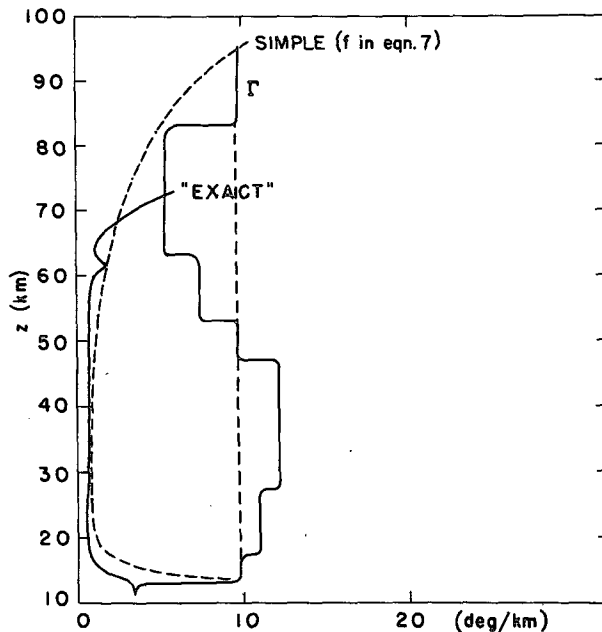


FIG. 3. As in Fig. 2, but for July conditions.

sharply above 12 km. It is clear that results depend strongly on the choice of c . Observational estimates in Lindzen (1981) of $z_b \approx 70$ km suggest $c \approx 18 \text{ m s}^{-1}$, provided that such waves do break at $z_{\max} \approx 12$ km. Evidence of such breaking is not, however, unambiguously available. We shall discuss this further in Section 3.

Finally, it is important to note that Eq. (3) does not include the damping effects of infrared cooling. These effects have been studied by Schoeberl *et al.*

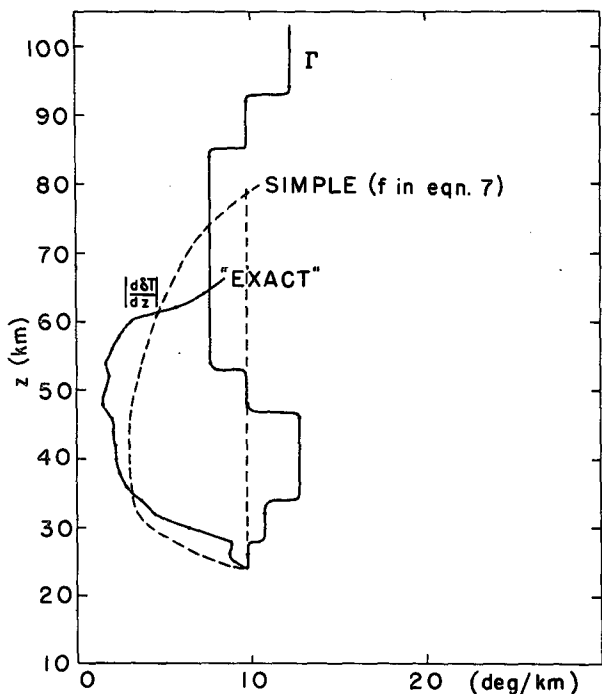


FIG. 2. $|d\delta T/dz|$ vs z for January conditions. Also shown are Γ and the results of a simplified calculation of $|d\delta T/dz|$. See text for details.

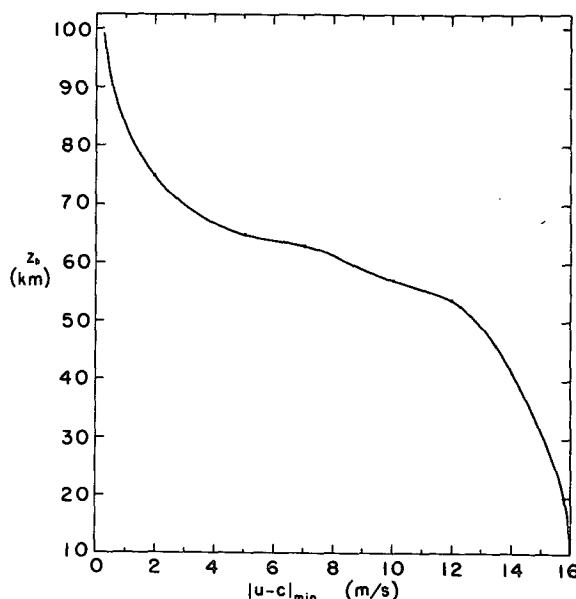


FIG. 4. The level of second breaking vs $|\bar{u} - c|_{\min}$ for July conditions.

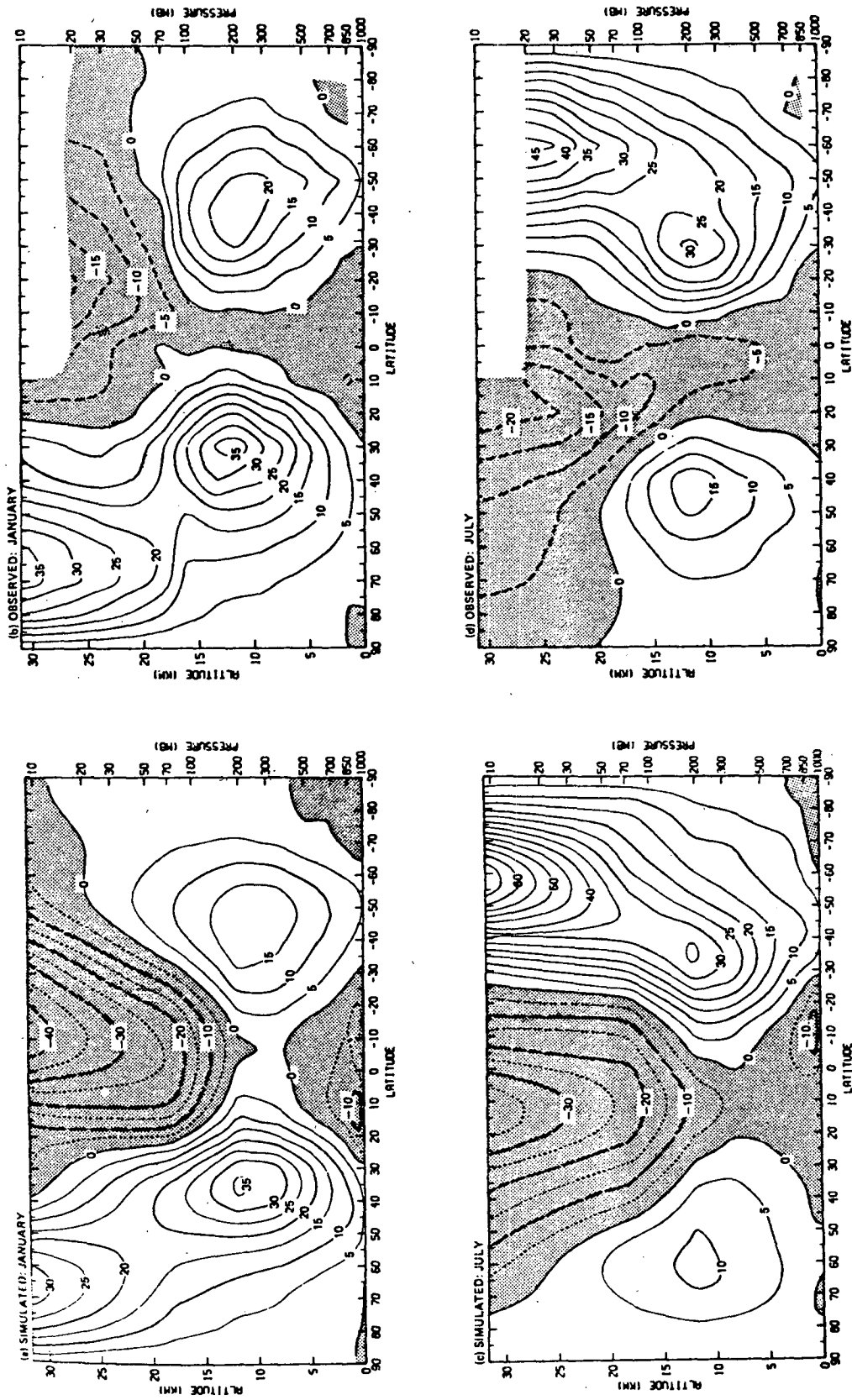


FIG. 5. Zonally-averaged zonal wind ($m s^{-1}$) as simulated by the NCAR community climate model (a) January and (c) July. The observations for the (b) December-February and (d) June-August seasons are from Newell *et al.* (1972). Regions of easterly flow are stippled. Figure taken from Pitcher *et al.* (1983). For winter, compare specifically results near $+35^{\circ}$ in January and near -30° in July. For summer note results near -45° in January and near $+55^{\circ}$ in July.

(1983), Schoeberl (1984) and Fels (1984). Such effects will at the least elevate the estimates for z_b .

3. Wave breaking in the troposphere and lower stratosphere

The preceding sections were concerned with the second level of breaking of internal gravity waves which were hypothesized to have originally broken in the troposphere. The question of whether there actually is wave breaking in the troposphere was not addressed. Indeed, it is irrelevant if we consider the "second" breaking level simply as a lower bound for the mesospheric breaking level.

However, if such breaking were to occur, it might be a significant factor in the momentum budget of the troposphere—albeit not as significant as it appears to be in the mesosphere. The question has, in fact, been addressed by Lilly (1972) and others. Lilly showed that mountain waves are associated with very significant momentum fluxes and the divergence of these fluxes in the lower stratosphere are suggestive of breaking. As Lilly noted, these momentum fluxes are consistent with estimated mountain "torques" at the surface (White, 1949) and should act as a drag on upper tropospheric jets. It was already clear in the 1960s that general circulation models produced negative shears above the jet maximum which were too weak compared with observations. This situation is clearly displayed in Fig. 5, taken from the recent work of Pitcher *et al.* (1983) with the NCAR community climate model. It seems possible that this model error is associated with the absence of lee waves in the model. (Models also typically predict winter polar tropopause temperatures that are colder than observed. In view of the thermal wind relation, these two errors may be related.) Also discernible in Fig. 5 is the fact that model jet maxima are weaker in summer than are observed maxima. Such an error is at least consistent with the possibility that there are internal gravity waves with phase speeds $\sim 20 \text{ m s}^{-1}$ which break in the neighborhood of the jet maxima: such waves would tend to accelerate \bar{u} toward c . Such waves would, moreover, tend to break at the minimum of $|\bar{u} - c|$ so that they would not accelerate the flow above the jet maximum. It should be added that since $|\bar{u} - c|$ is small, normal damping can result in significant momentum deposition—even without breaking.

4. Remarks

This note suggests the interesting possibility that gravity waves that break in the mesosphere are the upward extensions of waves that broke earlier in the upper troposphere/lower stratosphere. Moreover, it has been noted that just as breaking gravity waves appear to play a major role in the momentum budget of the mesopause region (causing a reduction in winds—and, via the thermal wind relation, a reversal

in the pole-to-pole temperature gradient), they may also contribute to the momentum budget of the tropopause region. To be sure, there are, in the tropopause region, other sources of the mean drag: planetary-scale stationary waves and cumulus friction for example. Thus, internal gravity waves may simply add to other effects. However, the work cited in Section 3 suggests at least a finite role.

It should be noted that even if waves do not break in the tropopause region, the assumption that they do break leads to a lower bound on the height of the breaking mesosphere. Moreover, even if breaking does not occur in the tropopause region, there will generally be substantial momentum deposition wherever $|\bar{u} - c|$ has a sharp minimum. The point, as already noted, is that for small values of $|\bar{u} - c|$ the vertical group velocity is much reduced and hence such damping as may be present becomes much more effective.

Before ending this note, one caveat should be mentioned. The basic physics of multiple breaking is straightforward. However, the detailed results in Fig. 2 and 3 obviously depend on the profiles chosen for u and static stability. As is evident from Fig. 5, these vary substantially with latitude. Thus our results should simply be taken as representative. For any specific profile, it is a trivial matter to simply evaluate Eq. (2).

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