Axially Symmetric Steady-State Models of the Basic State for Instability and Climate Studies. Part I. Linearized Calculations

EDWIN K. SCHNEIDER

Department of Meteorology, Massachusetts Institute of Technology, Cambridge 02139

RICHARD S. LINDZEN

Division of Engineering and Applied Physics, Harvard University, Cambridge, 02138

(Manuscript received 23 February 1976, in revised form 7 October 1976)

ABSTRACT

A linearized numerical model of the steady axially symmetric motions on a sphere is developed and applied to the earth’s atmosphere. The motions are driven by radiative cooling, conduction of heat from the surface and applied heat sources. The applied heat sources are meant to represent the zonally averaged effect on the large scale of cumulus convection. Frictional effects included are small-scale turbulent mixing in the vertical, represented by eddy viscosity, and “cumulus friction.” The model results show that cumulus heating and friction drive a meridional circulation comparable to the observed. Detrainment of tail cumulus leads to a horizontal temperature gradient reversal near the tropical tropopause, as is observed. Sea surface temperature gradients are shown to drive a significant meridional circulation below 800 mb. If the cumulus precipitation is proportional to the moisture convergence in the lower atmospheric layers, the moisture convergence produced by the mass circulation driven by sea surface temperature gradients will produce an ITCZ near the latitude of maximum sea surface temperature. Preference for a certain latitude of frictional convergence due to upper level heating (CISK) could alter this conclusion, but it is shown that the most unstable CISK perturbation occurs at the equator.

1. Introduction

An idealized steady-state model, which we believe to be relevant to the understanding of the general circulation of the earth’s atmosphere, is analyzed beginning in this paper and continuing in Schneider (1977). The model is one of a compressible atmosphere on a rotating sphere driven by internal heat sources, meant to represent the effect of the release of latent heat of condensation during cumulus convection, and radiation. Transports of heat and momentum by large-scale eddies, such as those eddies associated with baroclinic instability and topographic forcing, are suppressed. The model is therefore not to be regarded as a sort of two-dimensional approximation to a general circulation model but rather as an instrument to gain understanding of the atmospheric circulation by elimination of some of the more complex behavior.

In particular, this model can be viewed as the calculation of a basic state for stability studies, historically an idealized longitude-independent procedure. Most studies of baroclinic instability assume an analytically tractable, zonally symmetric basic state to perturb. The results obtained from such studies form the basis for much of the current theory of the general circulation and of climate. However, it has been shown that the behavior and structure of baroclinic-barotropic instabilities are sensitive to the form of the basic zonal flow (Brown, 1969; Gall, 1976a,b), so that it is important to choose the basic state correctly. The results found here form a consistent basic state for an a priori discussion of the earth’s general circulation.

Symmetric models have played a large role historically in the understanding of the general circulation of the earth’s atmosphere and many early explanations were symmetric. For a review of some of the more important theories, the reader is referred to Lorenz (1967, pp. 59–78). None of the theories presented in Lorenz are explicitly calculated solutions to the equations of motion. More recently, however, the symmetric circulation has been relegated to a secondary role in the explanation of the general circulation. This shift in emphasis occurred in part because the internal heat source considered in driving the symmetric circulation models was radiative only, and the results did not qualitatively resemble the earth’s atmosphere. Thus, researchers were led to the conclusion that the transports which determined the dynamical structure of the earth’s atmosphere were due to large-scale “eddies” produced by instability of the radiatively driven, symmetric basic state. Simple models such as those of Eady (1950) and Charney (1959), where transports by the meridional circulation were basically ignored and eddy
transports were all-important, were introduced. The results of Charney’s study corresponded well with some features of the mean atmospheric circulation, such as the zonal wind structure, but did not simulate the tropical mean meridional motion well. It is the contention of this work that an important physical process, namely, the heating of the atmosphere by the release of latent heat in deep convection, a mechanism which occurs primarily in the tropics, has been neglected, and that this process will affect the form of the basic symmetric state in an important manner. The presence of this deep internal heat source, together with the spherical geometry of the earth, are also significant effects in the atmosphere that are not analogously found in the dishpan experiments starting with Fultz (1951).

One of the more interesting questions that is implicitly raised by this model is whether “eddies” are absolutely crucial in producing the time and zonally averaged general circulation, or whether the effect of “eddies” is merely to produce quantitative modifications to the basic state. The eddies we are speaking of are not necessarily those due to baroclinic instability. Topographic distortions appear as either stationary or transient eddies in the data, depending on the length of the averaging period, but topographic eddies are not necessarily distinct entities from the mean meridional transports. The relevant observations that the model results are to be compared with are those of the annual mean zonally averaged circulation. The results simulate the observations well in some respects (particularly the tropical temperature and motion fields) and depart significantly from the observations in other respects (particularly the large zonal wind magnitude in the subtropical jet). Accounting for the differences between model results and observations, we must resort to explanations relying on asymmetric and transient motions. However, in the case of the rather large discrepancy in the magnitude of the subtropical jet, for example, explanations may be possible other than those resorting to a significant role for the traditionally important transient eddies due to baroclinic instability. Distortion of the jet due to transient eddies produced by barotropic instability, a possibility because of the large horizontal shear of the zonal winds, could make the results appear much more realistic, as could the effects of stationary eddies due to longitudinal topographic asymmetry.

Where observations and model results are similar, it is important to know whether the physical processes modeled and those acting in reality are the same. This question is, of course, difficult to answer. An example of the modeling of a physical process which yields results in a similar two-dimensional model that are counter to our intuition, and yet when incorporated into a three-dimensional general circulation model gives a reasonable simulation of the zonally averaged dynamical fields, is the use of the nonlinear horizontal viscosity (Smagorinsky, 1963). The use of this formulation of horizontal viscosity in the wet or dry zonally symmetric model of Hunt (1973) produces a westerly wind maximum of 110 m s⁻¹ at the equator near the top of the model, and easterlies appear at the surface in middle and high latitudes, while surface winds in the tropics are westerly (even though the dry tropical meridional circulation is a weak direct, or Hadley, cell). These results bear no resemblance to the observed atmosphere. When this viscosity is incorporated into the GFDL general circulation model (Manabe et al., 1965), however, the time and zonally averaged zonal and meridional winds provide a reasonable simulation of the observed fields. If the nonlinear horizontal viscosity does not model a real physical process, and yet is capable of creating such large differences from what our intuition tells us should happen in a situation that is simpler and more readily understandable than a GCM, can the GCM be called correct, and can the conclusions drawn from it and comparisons of other models with it increase our understanding of the atmosphere?

In order to illuminate the manner in which this and the following paper are organized, we will briefly discuss the manner of calculation of the basic state. The energy sources for the symmetric motions are heat sources and sinks due to both radiation and the release of latent heat by condensation of water vapor. The radiative heating is modeled by a simple Newtonian-cooling type linear law. This parameterization is quite crude, but is expected to be qualitatively correct for the bulk of the troposphere. Modeling of the latent heat release, however, is a difficult problem, since the parameterization of heating by cumulus convection is involved. One approach employed is to use zonally and annually averaged precipitation data, shown in Fig. 1 (after Lorenz, 1967), to specify an atmospheric heating function. In other calculations a simple cumulus parameterization is used, where the net condensation heating in a column is proportional to the water vapor convergence (evaporative plus dynamical) below a specified height. Understanding of results found using the simple cumulus parameterization leads inevitably to questions concerning CISK, the theory of the ITCZ (Charney, 1971), and the tropical boundary layer [as discussed in Schneider and Lindzen (1976b)]. Many of these questions are approachable through a linearized version of the model. If the heat source derived from the internally consistent cumulus parameterization scheme differs qualitatively from the observed heat source used in the simpler calculations, then the observed heat source implicitly includes “eddies” (assuming of course that the cumulus parameterization is correct). The monsoon stationary (or transient, depending on definition) eddy could provide a convenient explanation for the differences between the parameterized and observed latent heat release.

Another process that must be considered in the calcu-
lation of the basic state is vertical momentum as well as heat transport by cumulus—a necessary consequence of cumulus dynamics (Ooyama, 1971; Holton and Colton, 1972; Houze, 1973; Riehl and Soltwisch, 1974; Schneider and Lindzen, 1976a). The cumulus friction parameterization of Schneider and Lindzen (1976a) is used here. The cumulus momentum transport depends on the distribution of the cumulus heat source, and hence may implicitly involve eddies (particularly those due to topographic effects) when the heat source derived from observations is used. But since the cumulus momentum transport is a real effect, its parameterization must be included, and is not a disposable parameter. We attempt to evaluate the significance of cumulus momentum transport in some of the cases and calculations described below.

In this paper the model equations are described, and solutions for linearized versions of the equations are discussed. These solutions are in many cases similar to the solutions of the nonlinear equations, and illustrate interesting results. The linearized solutions presented here should also clarify for the reader the specifically nonlinear features discussed in the second part of this study.

2. Formulation of model

The equations appropriate to the study of the mean axisymmetric motions of the atmosphere are the time-independent Navier-Stokes equations on a rotating sphere averaged around latitude circles. The initial assumptions will be made that the vertical momentum balance is hydrostatic and that the vertical variation of density may be taken as that in an isothermal atmosphere, where it is not important in driving the motions by buoyancy forces (quasi-Boussinesq). The vertical coordinate used is log pressure. Due to the use of this vertical coordinate, the density only enters the parameterized viscous terms. Then the equations to be studied are

\[ \frac{\partial \phi}{\partial \xi} = \frac{RT}{H_0}, \]

\[ \frac{1}{a \cos \theta} \frac{\partial}{\partial \theta} \left( \frac{\partial (a \cos \theta)}{\partial \theta} + \frac{\partial}{\partial \xi} \right) \phi + \frac{\partial}{\partial \xi} \left( e^{iH_0 w} - (e^{-iH_0 w}) \right) = 0, \]

\[ \frac{1}{a \cos \theta} \frac{\partial}{\partial \theta} \left( \frac{\partial (a \cos \theta)}{\partial \theta} + \frac{\partial}{\partial \xi} \right) u + \frac{v}{a \cos \theta} \frac{\partial \phi}{\partial \xi} = -2 \Omega \sin \theta \frac{\partial \phi}{\partial \theta} + F_z, \]

\[ \frac{v}{a \cos \theta} \frac{\partial \phi}{\partial \xi} + \frac{u^2 \tan \theta}{a \tan \theta} + \frac{\partial}{\partial \xi} \left( - \frac{1}{a \tan \theta} \frac{\partial \phi}{\partial \theta} \right) = - \frac{1}{a \tan \theta} F_v, \]

\[ \frac{v}{a \cos \theta} \frac{\partial T}{\partial \xi} + \frac{\partial T}{\partial \theta} \left( \frac{\partial T}{\partial \theta} - \frac{T}{T_0} \right) = \frac{Q}{c_p}. \]

Here \( \xi = -H_0 \ln(p/p_0) \) where \( p \) is pressure and \( p_0 \) the surface pressure, taken independent of latitude; \( H_0 \) is a scale height defined by \( H_0 = R T_0 / g \), \( R \) being a gas constant appropriate to the composition of the troposphere, \( T_0 \) a reference temperature and \( g \) the acceleration of gravity; \( \Gamma = g/c_p \) is the adiabatic lapse rate (in height coordinates) at \( T = T_0 \), where \( c_p \) is the specific heat of air at constant pressure; \( \theta \) is the latitude (positive north of the equator), \( a \) the radius of the earth and \( \Omega \) the earth’s rotation rate; \( \phi = \varphi \) is the geopotential, \( u \) the zonal velocity relative to the surface, \( v \) the meridional velocity, and \( w = \partial \xi / dt \) the vertical velocity with respect to log-pressure coordinates. The frictional forces in the longitudinal and latitudinal directions are represented for the moment by \( F_z \) and \( F_v \), while the heating rate per unit mass is called \( Q \). The model does not include a water budget equation explicitly; we will return to this matter in Part II.

Part of the contribution to \( F_z \) and \( F_v \) and \( Q \) is considered to arise from small-scale turbulent mixing. This mixing is parameterized by a vertically varying dynamic eddy coefficient. That is, the turbulent flux
divergences are represented by
\[
F_{x} = -\frac{1}{\rho} \frac{\partial \mu}{\partial \xi} = -\frac{\mu}{\rho \partial \xi} - \frac{\partial \mu}{\partial \xi},
\]
\[
F_{y} = -\frac{1}{\rho} \frac{\partial \nu}{\partial \xi} = -\frac{\nu}{\rho \partial \xi} + \frac{\partial \nu}{\partial \xi},
\]
\[
Q = \frac{1}{\rho} \frac{\partial T}{\partial \xi} - \frac{T}{c_p \partial \xi}
\]
where \(\mu\) and \(K\) are functions of \(\xi\). A Prandtl number of unity, \(K=\mu\), is assumed, and \(T/T_0 \approx 1\) is used for the diffusion of heat. The approximation is also made that \(\rho = \rho_0 e^{-\xi/\lambda_t}\), which should be adequate for the diffusive terms. Due to the use of \(\xi\) coordinates, the density does not appear explicitly in the nonparameterized terms in the equations of motion. Horizontal eddy diffusion is not included.

The radiative heating is parameterized by a local cooling proportional to the temperature deviation from radiative equilibrium. Thus
\[
\frac{Q_{net}}{c_p} = \frac{T_e(\theta, \xi) - T}{\tau(\xi)},
\]
where \(T_e(\theta, \xi)\) is a radiative equilibrium temperature and \(\tau(\xi)\) a radiative relaxation time, both chosen empirically. This approximation is a linearization about radiative equilibrium of the cooling-to-space approximation for radiative heating, which has been shown by Rodgers and Walsham (1966) to be quite accurate for the earth's troposphere, except near the ground. The radiative equilibrium temperature is chosen to have a surface temperature corresponding to the mean latitudinal surface temperature distribution assumed at the surface (the lower boundary is meant to represent an infinite heat capacity ocean), an interior with the adiabatic lapse rate, and an isothermal upper atmosphere. The isothermal upper atmosphere does not affect the tropospheric results significantly. The approximately adiabatic lower region is found by Manabe and Moller (1961) in their study of the radiative equilibrium of the earth's atmosphere with fixed surface temperature. From the radiative heating calculations of Rodgers and Walsham, using the above radiative equilibrium temperatures, a radiative time constant of about 20 days (approximately constant through the troposphere) was found.

Horizontal eddy viscosity is not included since the only evidence that it is significant is that it is needed for the stability of time-marching numerical models.

The only other kinds of "turbulent" flux divergence terms included in the model are those due to cumulus convection. A heat source term which is supposed to model the distribution of heating due to deep cumulus convection is specified in some calculations. This heat source is specified rather than solved for as a function of the large-scale variables due to the controversial aspects of choosing any one of the existing parameterization schemes. Momentum flux divergence due to cumulus friction is specified as derived in Schneider and Lindzen (1967a):
\[
F_{x} = \frac{1}{\rho} \frac{\partial}{\partial \xi} \left[ M_c(u, \theta, \xi) \left[ u - u_c(\theta, \xi) \right] \right].
\]
Here \(M_c(\theta, \xi)\) is the mass flux in the cumulus hot towers derived from the specified heating by assuming the heating is of the form
\[
M_{con} = \frac{M_c}{\rho} \frac{\partial T}{\partial \xi} + \frac{T}{T_0}
\]
and \(u_c(\theta, \xi)\) is a zonal velocity appropriate to the air in the clouds. Schneider and Lindzen (1967a) showed that choosing \(u_c(\theta, \xi)\) equal to \(u\) at cloud base with no height variation is a reasonable first approximation. A derivation of the mass flux form of cumulus heating may be found in Arakawa and Shubert (1974).

Flux divergences explicitly due to cyclones and anticyclones are neglected, as it is one of the purposes of this work to study the axially symmetric circulation without such effects included.

Eq. (2) is used to define a streamfunction \(\psi\) such that
\[
\begin{align*}
\psi &= \frac{e^{H \theta} \varphi}{\cos \theta} \\
\omega &= \frac{e^{H \theta} \varphi}{a \cos \theta}
\end{align*}
\]
A coordinate transformation is made in the horizontal, defining \(y = a \sin \theta\). Eqs. (1), (4), (6) and (10) are combined to form a vorticity equation (essentially the thermal wind equation).

The reduced system is then nondimensionalized by assigning the proportionality \(\sim H_0, \gamma \sim a, T, T_0 \sim (\Delta T)_0 \sim \Delta T, \mu \sim \gamma, u, u_c \sim U, \psi \sim \Psi, \gamma \sim \Delta T/H_0, \gamma \sim \lambda \sim \lambda_0\) and \(M_c \sim \rho \Psi/\gamma\), \((\Delta T)_0\) a vertical temperature contrast imposed by the boundary condition, and \(\gamma\) is the eddy viscosity at the surface. The nondimensional parameters \(E = \gamma/(2aH_0)\) [the Ekman number], \(R_0 = U/2a\) [the Rossby number] and \(\lambda = 1/(2aE)\) [a radiative-diffusive number] appear. Letting \(U = \Delta T/(2a)\) and \(\Psi = EUH_0\), we obtain the following system of nonlinear equations:
\[
\begin{align*}
\frac{1}{(1-y^2)^{1/2}} \frac{\partial \psi}{\partial y} - \frac{\partial u}{\partial y} &= \frac{\psi}{(1-y^2)^{1/2}} \frac{\partial \psi}{\partial y} - \frac{\partial u}{\partial y} \\
\frac{\partial u}{\partial \xi} &= \frac{\partial u}{\partial \xi} + \frac{\partial}{\partial \xi} \left[ M_c(u - u_c) \right]
\end{align*}
\]
\[ E^2 \rho_0 \frac{\partial}{\partial \xi} \left( e^T \frac{\partial}{\partial \xi} \left( \frac{1}{(1-y^2)^l} \frac{\partial \psi}{\partial \xi} \right) \right) \]
\[ -e^T \frac{\partial}{\partial y} \left( \frac{1}{(1-y^2)^l} \frac{\partial \psi}{\partial \xi} \right) \]
\[ + \frac{\partial u}{\partial \xi} \frac{\partial \psi}{\partial y} = - \frac{1}{(1-y^2)^l} \frac{\partial T}{\partial y} \]
\[ + \frac{E^2}{(1-y^2)^l} \frac{\partial}{\partial \xi} \left[ e^T \left( \frac{\partial}{\partial \xi} \left( \frac{\partial \psi}{\partial \xi} \right) \right) \right] \]  
(12)

\[ \frac{\partial}{\partial \xi} \left( \frac{\partial^2 T}{\partial \xi^2} + \Gamma(T/T_0) \right) = \frac{\partial}{\partial \xi} \left( \frac{\partial ^2 \psi}{\partial \xi^2} + \Gamma \right) \]
\[ + \lambda e^T \left( \frac{\partial \psi}{\partial y} \right) \frac{T}{\tau} + \rho \left( \frac{\partial M_e}{\partial \xi} \right) \left( \frac{T}{T_0} \right) \]  
(13)

In application to the earth's atmosphere, \( \nu \approx 5 \times 10^4 \)
\( \text{cm}^2 \text{s}^{-1} \), \( T_0 \approx 300 \text{ K} \), \( \Delta T \approx 10^5 \text{ K} \), \( \nu \approx 4.4 \times 10^{-5} \text{K} \), \( U \approx 40 \text{ m s}^{-1} \), \( Ro \approx 0.05 \) and \( \lambda \approx 1 \) for \( t \approx 20 \) days. Then in Eq. (12) the terms of \( O(E^2 \rho_0) \) and \( O(\rho \rho_0) \) are neglected, although the diffusive term is retained, as it may be \( O(1) \) near the surface. The equation

\[ \frac{\partial u}{\partial \xi} = - \frac{1}{(1-y^2)^l} \frac{\partial T}{\partial y} + \frac{E^2}{(1-y^2)^l} \frac{\partial}{\partial \xi} \left[ e^T \left( \frac{\partial}{\partial \xi} \left( \frac{\partial \psi}{\partial \xi} \right) \right) \right] \]  
(14)

is obtained as an approximate vorticity equation. Eq. (14) becomes the thermal wind equation in regions where the frictional term (proportional to \( E^2 \)) becomes negligible. The neglect of the \( O(\rho \rho_0) \) term turns out not to be completely \( a \ posteriori \) justifiable for some calculations; however, its inclusion would not introduce qualitative changes in the results. Cumulus friction does not appear in (14) as its effect has been found to be negligible there. For tropical applications the term containing \( \partial T/\partial y \) in (13) may be neglected in comparison with the term containing \( \frac{\partial \psi}{\partial y} \left( \frac{\partial T}{\partial \xi} + \Gamma \right) \) as over the horizontal scale of the Hadley circulation, horizontal temperature differences are negligible compared to vertical differences of potential temperature \( \sim 3 \text{ K} \) compared to \( \sim 30 \text{ K} \). The \( \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial \xi} \) term may become important in higher latitudes, but as the Hadley circulation is \( a \ posteriori \) confined to the tropics, and this is primarily a study of the tropical atmosphere, that term will be neglected.

The static stability \( \left[ \frac{\partial T}{\partial \xi} + \Gamma \right] \) will be considered a function of \( \xi \) only and will be chosen \( a \ priori \) and denoted as \( \sigma \) in the vertical adiabatic compression term. The thermodynamic energy equation then becomes

\[ - \frac{\partial \psi}{\partial \xi} \frac{\partial}{\partial \xi} \left( \frac{\partial T}{\partial y} + \mu \frac{\partial}{\partial \xi} \left( e^T \right) \right) \frac{T}{\tau} + \rho \frac{M_e}{\xi} \]
\[ + \lambda e^T \frac{T}{\tau} + \rho \frac{M_e}{\xi} \]  
(15)

The angular momentum equation (11) is linearized by neglecting the nonlinear advective terms of \( O(\rho \rho_0) \). The details of the effects of the inclusion of these terms will be discussed in Part II and may also be found in Schneider (1975). The linear model then consists of (14), (15) and

\[ \frac{\partial}{\partial \xi} \left( \frac{u_\xi}{\xi} \right) + \frac{\partial}{\partial \xi} \left( M_e(y, \xi) \frac{u_\xi}{\xi} \right) \]
\[ + \lambda e^T \frac{T}{\tau} = 0. \]  
(16)

The cumulus friction term also appears to be \( O(\rho \rho_0) \). However, if the heat source, and consequently \( M_e \), is specified, then \( Q_e \approx \rho \rho_0 \) or \( M_e \approx 1/\rho \rho_0 \) for fixed stability and heating. The cumulus friction term is actually \( O(1) \); it will not go to zero as \( \rho \rho_0 \to 0 \) if the heating rate and stability are externally specified.

It is not, in any event, usually valid to neglect the advective terms in the angular momentum equation in the tropics. The resulting zonal winds will then, in many circumstances, develop large horizontal gradients near the equator and thus be inertially unstable.

The solutions without momentum advection are presented to illustrate the interaction of the other physical processes that are still retained in the equations. The linearized solutions may be regarded as approximate solutions, however, in the limit of infinitesimal amplitude of the heating function and the surface temperature gradients.

Neglect of the advective terms also forces the condition that there can be no vertically integrated horizontal transport of angular momentum. Then the surface zonal winds can exert no surface stress and must be zero in the linear model, as there are no mechanisms now for horizontal transfer of momentum. This will not be true in Part II.

\subsection*{a. Boundary conditions}

The above equations [(14), (15) and (16)] are eighth order in \( \xi \). The constraint that the solutions be symmetric in \( u \) and \( T \) and antisymmetric in \( \psi \) about the equator will be imposed in most cases for efficiency.

The conditions of no mass flux normal to the surface and at the "top" of the atmosphere are imposed. Additionally, it will be assumed that there is no motion across the poles. Therefore,

\[ \psi = 0 \text{ at } \xi = 0, \tau; \quad y = \pm 1. \]  
(17)

The top will also be assumed to be a stress-free surface,
i.e.,
\[
\begin{aligned}
\frac{\partial u}{\partial \xi} - \frac{\partial v}{\partial \xi} &= 0 \\
\frac{\partial u}{\partial \xi} &= 0 \\
\frac{\partial v}{\partial \xi} &= 0
\end{aligned}
\]

at \( \xi = \xi_r \).

(18)

\[ T = T_0(y) \quad \text{at} \quad \xi = 0. \]

(19)

The constant temperature stratosphere assures that there will in general be no significant boundary layer at the top, as a radiative equilibrium-thermal wind solution will satisfy the upper boundary condition on \( u \). At \( \xi = 0 \) a fixed surface temperature is imposed:

Mathematically,

\[
\begin{aligned}
\frac{\mu}{\rho_0} \frac{\partial u}{\partial \xi} &= C_D u |v| \\
\frac{\mu}{\rho_0} \frac{\partial v}{\partial \xi} &= C_D v |v|
\end{aligned}
\]

at \( \xi = 0, \)

(20)

where \( C_D \) is a drag coefficient. This condition is linearized by assuming \( |v| = 5 \text{ m s}^{-1} \). \( C_D \) is taken as \( 10^{-4} \).

The consequence is that \( u = \partial u / \partial \xi = 0 \) at \( \xi = 0 \) from (16).

Also, \( u = 0 \) at \( y = \pm 1 \) since \( \psi = 0 \) there. A consequence of assuming symmetry about the equator from (16) is that \( u = 0 \) at \( y = 0 \).

Eqs. (14), (15) and (16) with boundary conditions (17)–(20) were written in finite differences and solved numerically by Gaussian elimination using a technique described by Lindzen and Kuo (1969).

b. Choice of fixed parameters

The dynamic viscosity is chosen to represent the behavior of small-scale turbulent vertical mixing implied by the behavior of the tropical static stability. The well-mixed nearly adiabatic tropical “mixed layer” implies strong vertical stirring in the lowest kilometer. The free atmosphere seems to have only a slight amount of small-scale vertical mixing, and there is a partially mixed cloud layer between the top of the mixed layer and the trade inversion. Observational analyses (Betts, 1973) show that the bulk of trade cumulus mass flux detains near the bottom of the trade cumulus layer. The free atmosphere is assumed to begin at 1.1 km (near the bottom of the trade cumulus layer) in terms of turbulent transfer. The eddy viscosity \( v = \mu / \rho \) is then given as \( 5 \times 10^4 \text{ cm}^2 \text{ s}^{-1} \) for \( \xi \lesssim 0.6 \text{ km} \) (cloud base), decreasing linearly in height to a value of \( 10^8 \text{ cm}^2 \text{ s}^{-1} \) at \( \xi = 1.1 \text{ km} \), and equal to \( 10^6 \text{ cm}^2 \text{ s}^{-1} \) above 1.1 km.

The radiative decay time \( \tau(\xi) \) in Eq. (7) is chosen to be about 30 days below 300 mb, decreasing to about 15 days at 200 mb and remaining constant above 200 mb. The Ekman number \( E \) is then 0.00437, and the Rossby number \( R_o \) is about 0.05 (see Section 2 for definitions).

The static stability is chosen to represent an isothermal stratosphere and an atmosphere with the mean tropical lapse rate, 3.2 K km\(^{-1} \) in dimensional \( \xi \) coordinates (Newell et al. 1972), in the troposphere above the boundary layer. The static stability in the mixed layer, below 0.6 km, is assumed to be 10\% of that in the free troposphere and there is a transition layer between 0.6 and 2 km (assumed to be the height of the trade wind inversion), where the static stability has a value 15\% smaller than the mean tropical tropospheric lapse rate.

The radiative equilibrium temperature \( T_e(y, \xi) \) is taken to be equal to the assumed boundary temperature at the surface and to decrease vertically at the adiabatic lapse rate to the temperature of the upper boundary. The radiative equilibrium temperature is constant above the height where it reaches the stratospheric temperature. In terms of the nondimensional variables

\[
T_e(y, \xi) = 0.5 \{ [T_0(y) - \Gamma \xi] + [T_0(y) - \Gamma \xi] \}. \quad (21)
\]

The surface temperature is \( T_0(y) \) and \( \Gamma \) is the dimensionless adiabatic lapse rate approximated for \( T/T_0 = 1 \). The stratospheric temperature is scaled to be zero. Then the radiative equilibrium temperature equals the surface temperature at \( \xi = 0 \), and the surface is assumed to have infinite heat capacity. The radiative equilibrium temperature field for \( T_e(y) = 1 - 0.4y \) is approximately that shown in Fig. 3c. A constant upper level radiative equilibrium temperature is chosen partially for convenience. If the atmospheric radiative equilibrium could be modeled by a single absorbing constituent with a semi-gray absorption coefficient in the infrared and no UV absorption (an approximation to water vapor), Goody (1964) shows that the upper levels of the atmosphere would approach a constant temperature with height. The latitudinal variation of the upper level radiative equilibrium temperature (the “skin temperature”) depends only on the latitudinal variation of the infrared radiative flux. By assuming a constant skin temperature with latitude, we are thus assuming that the infrared radiative flux is constant with latitude in radiative equilibrium for a semi-gray atmosphere. This model is consistent with possible reality in having a constant skin temperature with latitude and latitudinal variation in the surface temperature if the total optical depth of the water vapor varies correspondingly. That is, there should be more water vapor in a column at the equator than at the poles.

The constant stratospheric radiative equilibrium temperature was actually chosen for dynamical reasons.
It was desired to study the motions of the lower atmosphere and to find steady-state nonlinear solutions which would be in radiative equilibrium in the stratosphere. [These solutions will be discussed in Part II (Schneider, 1977) and are presented in Schneider (1975)]. If there were any latitudinal variation in the skin temperature, infinite zonal winds would result as the top of the atmosphere is approached through the thermal wind relationship. Meridional motions of sufficient strength to give $\partial T/\partial y \to 0$ as $\xi \to \infty$ or oscillations in sign of $\partial T/\partial y$ with height would then necessarily result in the steady-state solution. That is, the solution could not be in radiative equilibrium in the stratosphere. The necessity of dynamical control of the upper atmosphere temperature field is removed by assuming $\partial T/\partial y \to 0$ fast enough as $\xi \to \infty$ (or at the top for the finite-difference model). Also, the crudeness of the radiative law makes it somewhat pointless to try to model the upper atmospheric motions, where chemistry and dynamics can have an important effect on the composition. On the other hand, the tropospheric distribution of water vapor is strongly connected to the surface temperature, which is assumed, so that there is some justification for decoupling the tropospheric radiation and dynamics.

The parameter $u_r(y,\xi)$ is taken as identically zero in the linear models; $u_r$ is supposed to model the zonal velocity in the clouds, taken as approximately that of the surface air (Schneider and Lindzen, 1976a). Since $u=0$ at the surface in the linear model, $u_r$ is also close to zero. The vertical variation of the heat source due to cumulus convection is taken as that of approximately non-entraining clouds (i.e., $\partial M_c/\partial \xi = 0$), so that $\partial u_r/\partial \xi$ may be expected to be close to zero.

The heat source per unit volume is assumed to increase from zero linearly with height between $\xi = 0.11$ and $\xi = 0.2$ (the region about 1–2 km from the surface), to be constant from $\xi = 0.2$ to $\xi = 1.6$ (200 mb), and to decrease linearly in $\xi$ to zero at $\xi = 2.3$ (100 mb). This vertical distribution of the heating approximates a non-entraining cumulonimbus which detrains between 100 and 200 mb. Vertical distributions of net cumulus heating found by Yanai et al. (1973) and Ogura and Cho (1973) do not differ qualitatively with the above distribution.

Unless otherwise stated the grid used has spacing in the horizontal of $\Delta y = 0.1$ and $\Delta \xi = 0.063$ in the vertical. The top is taken at $\xi_T = 2.6$ and the surface is assumed to be at $\xi = 0$. Increasing the horizontal resolution did not significantly alter the linearized solutions. The grid used has the staggered arrangement shown in Fig. 2.

The height of the top ($\xi_T = 2.6$) is quite close to the top of the heat source ($\xi = 2.3$). The top was taken at this height to obtain maximum vertical resolution in the troposphere. The top was also placed at $\xi_T = 3$ and $\xi_T = 4$, and no significant changes in the solutions from $\xi_T = 2.3$ were found.

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**Fig. 2. Arrangement of dependent variables on finite-difference grid used for numerical calculations.**

3. Case studies

Four groups of calculations were performed. In the first group no upper level heat source was included and the circulation, driven by surface temperature distributions which are symmetric about the equator, was found. The second group contains those cases symmetric about the equator in which a heat source with a horizontal scale of about 20° latitude forces the motions. This heat source distribution approximates our idea of what the symmetric component of the annual mean latitudinal distribution of cumulus-produced precipitation should look like. Cases were run to study the effects of the inclusion of cumulus friction, changing the mixed layer stability, and the relative effects of upper level heating and surface temperature gradients in producing boundary layer mass convergence. The third group of calculations employs a narrow heat source of scale about 3° latitude to examine the latitude dependence of the CISK mechanism as an explanation of the preferred latitude of the ITCZ over a homogeneous ocean. The sensitivity of CISK to the vertical distribution of the heating is also looked at. Finally, a calculation in a pole-to-pole domain using the "observed" annual mean cumulus heating function and cumulus friction was performed for comparison with the mean annual Hadley circulation. The cases studied are summarized in Table 1.

**a. Dry calculation**

Two calculations in which the motions are driven by latitudinal variation in surface and radiative equilibrium temperatures only were performed.
to zero above. Upward motion of about 1 mm s\(^{-1}\) is found at 3\(^\circ\), the closest grid point to the equator, while there is sinking elsewhere, reaching a maximum velocity

\[
T_u(y) = 1 - 0.4y^2, \quad (22)
\]

which will be referred to as T1. The surface temperature is maximum at the equator, with a dimensional equator-to-pole temperature difference of about 40 K. Contours of the zonal wind, streamfunction and temperature for this case are shown in Figs. 3a–3c, respectively. The temperature distribution is very close to the radiative equilibrium temperature, and the zonal winds are approximately those resulting from radiative equilibrium thermal winds. The maximum zonal winds of about 30 m s\(^{-1}\) occur at 20\(^\circ\) latitude, and there is no concentrated jet structure in the \(u\) field, but rather a broad maximum. The zonal wind is westerly everywhere. The stream function plot shows that the meridional circulation is a single direct cell confined between the surface and 800 mb and the equator 30\(^\circ\) latitude, with a net mass flux of about 2\times10^{15} g s\(^{-1}\). The meridional winds have a magnitude of about 0.5 m s\(^{-1}\), with equatorward flow in the tropics below 900 mb and poleward flow between 800 and 900 mb. In middle and upper latitudes the depth of the equatorward flow (the zero stress surface for the zonal wind) decreases toward the pole, as predicted by Ekman theory. There is, however, no increase in the depth of the boundary layer between 10\(^\circ\) latitude and the equator (subject to the resolution limits). This is in part due to the strong decrease of eddy viscosity with height, and in part to the assumed stability of the atmosphere as discussed in Schneider and Lindzen (1976b). The vertical motions are very weak (<1 mm s\(^{-1}\) everywhere) with rising motion equatorward of 8\(^\circ\) latitude and sinking motion poleward.

The detailed structure of the vertical motions is shown in Fig. 4, which gives the vertical velocity as a function of height at approximately 3\(^\circ\), 9\(^\circ\), 15\(^\circ\) and 21\(^\circ\) latitude. The vertical velocity has a maximum amplitude at about 1 km, decreasing in amplitude rapidly

---

**Table 1. Description of cases run with linearized models.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Surface temperature</th>
<th>Heating</th>
<th>Cumulus friction</th>
<th>(M_u(y))</th>
<th>(M_e(y))</th>
<th>(y_{max})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>T1</td>
<td>no</td>
<td>no</td>
<td>Q1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>T2</td>
<td>no</td>
<td>no</td>
<td>Q1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>T3</td>
<td>yes</td>
<td>no</td>
<td>Q2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>T3</td>
<td>yes</td>
<td>yes</td>
<td>Q2</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>T4</td>
<td>yes</td>
<td>yes</td>
<td>Q2</td>
<td>0.15</td>
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</tr>
<tr>
<td>VIa</td>
<td>T5</td>
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<td>yes</td>
<td>Q2</td>
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<td></td>
</tr>
<tr>
<td>VIb</td>
<td>T6</td>
<td>yes</td>
<td>yes</td>
<td>Q2</td>
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<td></td>
</tr>
<tr>
<td>VIc</td>
<td>T7</td>
<td>yes</td>
<td>yes</td>
<td>Q2</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>T8</td>
<td>yes</td>
<td>yes</td>
<td>Q2</td>
<td>observed</td>
<td></td>
</tr>
</tbody>
</table>

In case I the surface temperature distribution is assumed to be

---

**Fig. 3.** Contours of solutions to case I: (a) zonal wind, contour interval 10 m s\(^{-1}\); (b) streamfunction, interval 10\(^\mu\) g s\(^{-1}\); (c) temperature, interval 10 K.
of about \(-0.2\) mm s\(^{-1}\) at 15°. The maximum vertical motion in a continuous version of this model would be at the equator (Schneider and Lindzen, 1976b).

The circulation forced by surface temperature gradients, then, is confined to the surface layer where small-scale turbulent mixing is important, with the temperature being approximately in radiative equilibrium. The calculation cannot be considered self-consistent, since the given static stability is much larger than that produced by the circulation. However, the results will prove useful in understanding the relative roles of surface temperature gradients and upper level heating in producing boundary layer moisture convergence and the ITCZ.

Case II is the “dry” circulation produced by a surface temperature distribution where the temperature at 3° latitude is 0.5 K less than the surface temperature at 9° latitude. The surface distribution is the same as T1.
poleward of 9° N and 9° S, but decreases by 0.5 K from 9° to 3° latitude (3° latitude is the closest temperature grid point to the equator in this calculation). This distribution is T2 in Table 1. The zonal winds are approximately the same as those in case I except for a region of weak easterlies equatorward of 6° latitude and below 400 mb. There is now a two-celled boundary layer circulation, both cells of strength about 10^9 g s^{-2}, with surface branches converging at about 9° latitude. The structure of the vertical velocity is shown in Fig. 5. The low-level vertical velocity is now downward at 3°, 15° and 21° and upward at 9°. The vertical structure of the w field is similar to case I, and the surface-temperature-gradient-induced vertical velocity at 9° and 1.8 km is ~1.2 mm s^{-1}.

The vertical velocity at 1.8 km for cases I and II is compared in Fig. 6. For case I the largest upward motion (w=0.33 mm s^{-1}) occurs at 3° latitude, while the largest sinking motion (w=−0.1 mm s^{-1}), occurs at 9°. For case II there is rising motion of w=0.3 mm s^{-1} at 9° latitude, with sinking elsewhere. The maximum convergence at 1.8 km induced by surface temperature gradients coincides with the surface temperature maximum.

b. Circulation driven by broad upper level heating

The calculations described in this section have in common the features that they are symmetric about the equator and are driven by an imposed heat source with a latitudinal distribution of

\[ Q(y) = \exp \left[ -\left( \frac{|y| - 0.1}{0.3} \right)^2 \right] \]  (23)

denoted as Q1. The maximum heating, at latitudes y=±0.1, is chosen to have a value equivalent to the latent heat of 190 cm year^{-1} precipitation. This heating distribution is meant to approximate the symmetric component of the annual mean latitudinal distribution of precipitation produced by cumulus convection as estimated from Fig. 1. The heating is maximum at 6° latitude and has Gaussian shape with an e-folding length of 18° latitude. Cases were chosen to study the effects of cumulus friction and the relative effects of upper level heating and surface temperature gradients in inducing boundary layer mass (moisture) convergence.

In case III the surface temperature and radiative equilibrium temperature distributions are the same as in case I, corresponding to surface temperature distribution T1. An upper level heat source, with no cumulus friction, was added. The resulting zonal wind and temperature fields are shown in Fig. 7. The upper level temperature now has a rather sharp latitudinal gradient poleward of the maximum heating. The region of sharpest latitudinal temperature gradients slopes poleward with height, being at about 13° latitude for the
260 K isotherm, about 30° for the 210 K isotherm, and about 45° for the 200 K isotherm. A temperature inversion develops equatorward of 10° latitude between 900 and 750 mb. The temperature distribution found in this calculation implies a static stability closer to that imposed in the adiabatic compression term in the heat equation equatorward of 25° latitude than the static stability calculated in the dry models. Poleward of 45° latitude the temperature is close to radiative equilibrium. The zonal winds reflect the large latitudinal temperature gradients, reaching a maximum of 280 m s\(^{-1}\) at 20° latitude. This is much larger than angular momentum conservation would allow; thus the linear model for this case is not a good approximation to the final solution of the steady nonlinear problem, which is obtained in Schneider (1975), and will be presented in Part II (Schneider, 1977). Vertical velocities large enough to modify the temperature field so that the resulting zonal wind could satisfy the thermal wind equation and the correct angular momentum conservation law would have to develop in the nonlinear case, and these will be much larger than those calculated for the linear case. The streamfunction is approximately that found without the upper level heating, as are the v and w fields. The vertical velocities obtained are almost exactly those found in case I below 2 km, except for somewhat more divergence north of 20°. Larger vertical velocities than in case I are found above 2 km, but they are less than 0.5 mm s\(^{-1}\).

Case IV differs from case III only in that cumulus friction is now included. The changes in the solution are significant. Figs. 8a–8c show contours of constant zonal wind, streamfunction and temperature. The latitudinal temperature gradients are nearly zero equatorward of 20° latitude. The temperature increases toward the pole at constant pressure above the 200 mb level, equatorward of 30° latitude. This reversal of the sign of the horizontal temperature gradient is observed in the tropical upper troposphere. Newell (1964) has suggested that momentum flux divergence due to the large-scale baroclinic eddies is responsible for the tropical upper level temperature gradient reversal. An axially symmetric linear model due to Leovy (1964) produces a similar temperature gradient reversal (at the mesopause) by virtue of representing frictional effects by a constant coefficient drag law. In this model the latitudinal variation of the heat source and radiative equilibrium temperature produces a westerly shear with height of the zonal winds in the lower and middle troposphere. The region between 100 and 200 mb is modeled as a layer of cumulus detrainment, with the detrained air having zonal velocity near zero. Thus the detrainment produces a drag on the environmental zonal winds, causing easterly shear with height between 100 and 200 mb. As the zonal winds are essentially in thermal wind balance above the boundary layer, an increase of temperature with latitude results from the cumulus detrainment. The temperatures below 300 mb are in radiative equilibrium poleward of 55° latitude, but the 200 K isotherm approaches the

---

**Fig. 8.** Contours of solution to case IV: (a) zonal wind, interval 15 m s\(^{-1}\); (b) streamfunction, interval 10\(^{10}\) g s\(^{-1}\); (c) temperature, interval 10 K.
In the linearized model, the streamfunction is proportional to the vertical flux of zonal momentum, from (16). If the interior viscosity is negligible,

\[ \psi = -(1-y^2) \frac{1}{y} \text{Ro}M_s(u-u_c). \] (16a)

The presence of internal cumulus heating causes horizontal temperature gradients and thus vertical shear in the zonal winds. The meridional circulation produced by the cumulus friction in (16a) attempts to decrease the magnitude of the horizontal temperature gradients in order to decrease the magnitude of \((u-u_c)\).

In case V, the surface temperature is now assumed to be a constant 300 K (a distribution denoted as T3), thus removing the meridional circulation forced by surface temperature gradients, and making the radiative equilibrium thermal wind solid body rotation (\(u=0\)). Otherwise all parameters are the same as in case IV. The purpose here is to see what effect the "observed" heating has on the boundary layer. The effect on the zonal wind as compared to case IV is a slight lessening of the magnitude of the zonal jet, due to the elimination of the radiative equilibrium temperature gradient. The Hadley cell is also 7% weaker than in case IV, and the surface cell disappears entirely. Practically no boundary layer mass flux is induced by upper level heating due to cumulus friction. It appears, then, that the zonally and time-averaged upper level heating does not induce frictional convergence of moisture in the boundary layer. This is not surprising in terms of CISK theory (Charney, 1971), as the horizontal scale of the upper level heating is quite large.

Vertical profiles of \(w\) for case V are presented in Fig. 9. Vertical velocities are negligible below 2 km; however, above 2 km the vertical velocities for cases IV and V are approximately the same. A good estimate of the solutions for case IV may be obtained by superposition of the results from cases I and V.

In both cases IV and V the vertical velocity between 800 and 200 mb increases approximately exponentially, and decreases abruptly to zero between 200 and 100 mb, the assumed cloud detrainment layer.

The important result from comparison of cases IV and V is that in the linearized model the effect of latitudinally broad upper level heating in inducing boundary layer convergence is negligible compared to the convergence induced by the weak tropical surface temperature gradients. When the low-level convergence is controlled by sea surface temperature patterns, a significant feedback between the convective heating and the boundary layer convergence should not be expected. However, we have not shown here that this is the case, as the cumulus heating has been specified for convenience, rather than calculated as a function of the large-scale fields. We will introduce a crude cumulus heating parameterization in Part II of this study for further consideration of the question of how
boundary layer moisture convergence induced by sea surface temperature distribution and upper level heating interact to determine the position and width of the ITCZ. Also the observed mean annual Hadley circulation mass flux from the linear mode with cumulus friction (case IV) provides a good zero-order approximation to the observed annual mean, which the case without cumulus friction (case II) does not do.

Other calculations were also done with heating Q1. No significant changes were introduced to case IV by making the boundary layer stability the same as that of the free atmosphere. A surface temperature distribution T2 with other model parameters the same as in case IV produces essentially a superposition of the solution from cases II and V.

c. Determination of the latitude dependence of CISK

The f-plane CISK model of Charney (1971) gives the result that latitudes away from the equator are preferred for instability, as the efficiency of Ekman pumping goes to zero at the equator. This feature is a consequence of the fact that the local Rossby radius of deformation (proportional to $f^{-1}$) becomes infinite at equator. It is shown in Schneider and Lindzen (1976b) that the correct latitudinal radius of deformation for the spherical case approaches a constant value of the geometric mean of the radius of the earth and the Rossby radius of deformation at the pole as the equator is approached, rather than diverging so that significant differences in the near-equatorial behavior between calculations done in spherical and f-plane geometry may be expected.

The applied heat source in the following calculations (case VI a–g) is of the shape

$$Q(y) = \exp \left[ - \left( \frac{y-y_0}{0.025} \right)^2 \right], \quad (24)$$

denoted as Q2, where the latitude $y_0$ of maximum heating is varied. The heat source is then a Gaussian in $y$ with $\epsilon$-folding width 1.5° latitude. This is narrow compared to the appropriate radius of deformation. The surface temperature is taken to be constant at 300 K, and the effect of assuming the heat source to be centered at different latitudes is examined. Cumulus friction and radiative effects are also included, but it was found that cumulus friction has negligible effect on the convergence below 1.8 km for the narrow heat source. Radiation also is of secondary importance in the boundary layer, where turbulent mixing is large.

CISK is the unstable growth of a perturbation in a conditionally unstable atmosphere, produced by the cooperation of the release of latent heat in the upper levels and frictional convergence of moisture in the boundary layer. In the simplest calculation, for axially symmetric disturbances on a resting f-plane earth, Charney (1971) assumes that the frictional convergence in the boundary layer is proportional to the vorticity of the interior flow (from simple steady Ekman theory), that the upper heating is proportional to the frictional convergence in the boundary layer (if positive), and that the upper level heating is zero if there is Ekman suction. The release of latent heat increases the low-level vorticity and convergence by stretching of vortex tubes, thus leading to instability. Charney’s calculation predicts no growth at the equator, and growth rates increasing with latitude. The largest growth rates occur for the perturbation of the smallest scale. Cases VIa–g evaluate the boundary layer convergence produced by a narrow, steady, upper level heat source applied at different latitudes. These cases may be regarded as evaluating the latitudinal dependence of the proportionality between the upper level heating and the low-level convergence that would have to exist for a narrow CISK perturbation to be neutrally stable. A larger proportionality factor than the one found here would then be expected to lead to instability. If the boundary layer convergence induced at one latitude $y_1$ by an assumed heating centered at that latitude is larger than the convergence at latitude $y_2$ induced by centering the same heating at $y_2$, then the narrow perturbation may be said to be more unstable at latitude $y_1$ than at latitude $y_2$.

Fig. 10 compares the vertical velocities found at 1.8 km in cases VIa–g. This is chosen as a meaningful height at which to calculate the moisture convergence because it is near the height of the tropical trade inversion, which divides the moist lower layers from the dryer upper atmosphere. For reference the magnitude of the assumed heat source is equivalent to 200 cm year$^{-1}$ precipitation. The heating is applied at 0°, 3°, 6°, 9°, 12°, 15°, and 18° latitude. A horizontal mesh distance of $\Delta y = 0.05$ on a pole-to-pole domain was adopted for these calculations. It is seen that the vertical “pumping” velocity at the latitude where the heat source is applied is upward, while it is downward everywhere else. This result is qualitatively the same if the heating is assumed to decrease rapidly enough so that it acts at one grid point only, and this is approximately the behavior of the assumed heating. Thus the heating can be assumed to be a finite-difference eigenfunction for CISK at neutral stability (i.e., the heating is positive where the vertical velocity at the top of the boundary layer is upward, and approximately zero otherwise), and the relative stability of a perturbation as a function of latitude can be found. The perturbation that induces the largest boundary layer convergence is the most unstable. The results displayed in Fig. 10 show that the response of the vertical velocity is greatest at the equator, with approximately the same response (although negligibly less) at 3° latitude, with the response decreasing rapidly and monotonically from 3° to 18°.

Thus it appears that ITCZ-like modes become less
stable as the equator is approached rather than more stable as stated by Charney (1971).

An interesting feature of the response to the narrow heat source is the appearance of a local maximum of sinking motion at 1.8 km at 3°S (the first grid point south of the equator) for each case studied (the heat source being applied at the equator or in the Northern Hemisphere). For the heat sources applied at 9°, 12°, 15° and 18°N there is another local maximum of sinking motion one grid point to the south of the applied heat source.

Also, if the heating is assumed to be zero below 1.5 km, but unchanged above 1.5 km, the effect on the convergence at 1.8 km is negligible, although vertical velocities below 1.5 km are reduced considerably.

There is substantial frictional convergence induced in the boundary layer by the narrow upper level heating, in contrast to the negligible frictional convergence induced by the broad upper level heat source (case V). This is to be expected from Charney's results.

The calculations of case VI show that the latitude dependence of CISK can be expected to produce an ITCZ at the equator in the absence of other mechanisms. In an experiment discussed in Part II, we find that moisture convergence due to atmospheric motion driven by sea surface temperature gradients determines the position of the ITCZ.

d. Pole-to-pole "realistic calculation"

The linearized equations were solved on a pole-to-pole domain with Δy = 0.1 for this calculation (case VII). The circulation is driven by a surface temperature and radiative equilibrium temperature distribution symmetric about the equator

\[ T_{o}(y) = 1 - 0.4y^2 \]  

and by an imposed upper level heating asymmetric about the equator. (The observed annual mean surface temperature distribution is not symmetric about the equator.) The upper level heating has the vertical
distribution of that used in case IV and is distributed in the horizontal from 20°S to 20°N in the same manner and equivalent to the same precipitation rate as the annually and zonally averaged precipitation given in Fig. 1. Poleward of 20° latitude the heating is assumed to go to zero in approximately the same manner as heating Q1, matched onto the observed heating at ∼20°. This horizontal distribution is shown in Fig. 11. The heating is not taken to have the same distribution as the precipitation outside the tropics, as it is presumed that much of the mid- and upper-latitude precipitation is from large-scale rising motion rather than deep cumulus convection. Thus the mid- and upper-latitude zonally averaged heat source can be expected to have a different vertical distribution than that in the tropics and not to be associated with cumulus friction. Of course, some of the mid- and upper-latitude precipitation is convective, and this division should be taken into account in more sophisticated models. Seasonal effects, particularly, need to be considered outside the tropics where the annual mean temperature-moisture field is stable rather than conditionally unstable. Cumulus friction is included in case VII.

As can be seen in Fig. 12, the asymmetry of the upper level heating introduces slight asymmetry in the resulting solutions. The Southern Hemisphere zonal wind maximum is slightly stronger and more poleward than the Northern Hemisphere zonal wind maximum (μ=0 at the equator due to the linearization of the equations). The Northern Hemisphere Hadley cell is slightly stronger than the Southern Hemisphere Hadley cell, both with mass flux on the order of 4×10^18 g s⁻¹. The Southern Hemisphere Hadley cell extends slightly north of the equator. A surface temperature gradient induced circulation appears below 800 mb, similar to the one found in case IV. There is a single convergence zone where the upward branches of the two Hadley cells meet between 10°S and 10°N. The upward velocities in the convergence zone reach 5 mm s⁻¹, while the vertical velocities in the downward branches have about half this magnitude. The boundary layer convergence (Fig. 13) is positive between 6°N and 6°S, reaching its maximum value at 3°N, coincident with the maximum in the upper level heating.

The mass fluxes in the Hadley cells are somewhat less than those in case IV, although the upward vertical velocities are somewhat larger. In case IV the heating had approximately the same magnitude but was symmetric about the equator, so that more net precipitation and larger upper level heating was implied than in the asymmetric case (i.e., the component of the annually and zonally averaged heating symmetric about the equator was somewhat overestimated). Mass fluxes in case VII are about 60% of those observed in the annual average, and the Northern Hemisphere cell is slightly stronger than the southern hemisphere cell in the model. In the observed annual mean tropical circula-

Fig. 12. Contours of solution to case VII: (a) zonal wind, interval 15 m s⁻¹; (b) streamfunction, interval 10^18 g s⁻¹; (c) temperature, interval 10 K.

The observed annual mean tropical circulation, the Southern Hemisphere cell is slightly stronger than the Northern Hemisphere cell. The discrepancy of this feature from the observed circulation could
possibly be due to the assumption of the symmetry of the radiative equilibrium temperatures and surface temperatures about the equator. The zonally and annually averaged surface temperature maximum appears to be around 5°S, and surface temperature gradients also appear to be slightly stronger in the Southern Hemisphere. This effect was not taken into account. It appears that the strength of the model Hadley cell is proportional in some sense to the horizontal gradient of the heating, which is larger in the Northern Hemisphere than in the Southern Hemisphere in the model.

4. Conclusions

The linearized models show that the observed annual mean heating due to cumulus convection, together with cumulus friction, can produce a Hadley circulation similar to the annual mean observed circulation. When there is no cumulus friction, or when the heat source is assumed to be narrow, the meridional motions obtained are not qualitatively a good approximation to the steady meridional circulation that will result when the zonal momentum equation is used in its correct nonlinear form. The inclusion of cumulus friction (consistent with a broad heating distribution) strengthens the meridional circulation and causes easterly shear of the zonal winds with height in the detrainment layer between 100 and 200 mb by a drag force. Temperatures in this layer then increase with latitude since the force balance in the meridional direction turns out to be geostrophic.

The effect of surface temperature gradients is confined to a surface layer due to the assumed vertical variation of the small-scale mixing. Surface temperature gradients in the tropics are effective in producing low-level moisture convergence, which could lead to upper level heating by cumulus convection.

The linearized model was used to perform calculations on the latitude-dependence of CISK in the formation of the ITCZ. It was found that the instability of a latitudinally narrow, axially symmetric perturbation driven by the release of latent heat decreases as the perturbation is moved away from the equator. The details of the vertical distribution of the applied heat source near the surface will be shown in Schneider (1977) to be important in the motions produced in that region.

Acknowledgments. This research was supported by the National Science Foundation under Grants GA 33990X and OCD 7100333. Computing time was provided by the Atomic Energy Commission and the Goddard Institute for Space Studies.

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