Radiative and Photochemical Processes in Mesospheric Dynamics: Part III, Stability of a Zonal Vortex at Mid-Latitudes to Axially Symmetric Disturbances

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ABSTRACT

The stability of a baroclinic, axially symmetric vortex on an f-plane to axially symmetric disturbances is investigated. It is found that with photochemistry and radiative transfer acting, such disturbances are unstable regardless of the value of the Richardson number. The growth rates under conditions relevant to the mesosphere are, however, very small.

1. Introduction

In the literature of theoretical meteorology (see Saltzman, 1962) there are two basic stability problems concerning a baroclinic fluid (i.e., a fluid in which isobaric and isothermal surfaces do not coincide). For both, it is generally assumed that the basic temperature field has a north-south gradient. One problem concerns the stability of this basic field to axially symmetric disturbances; the other concerns the stability to baroclinic waves (large scale waves traveling in the east-west direction). In both, for unstable disturbances, the meridional components of the disturbance flow advect heat in such a manner as to reduce the north-south temperature gradient. In the first problem the meridional flow arises from the need to conserve zonal momentum (i.e., the changing temperature distribution leads to an alteration of the zonal flow; meridional circulations are needed in order to redistribute and conserve zonal momentum); in the second case the disturbance temperature field varies in the x-(east-west) direction and gives rise to a meridional thermal wind.

Studies of these disturbances have in general assumed the motions to be adiabatic. As pointed out in Part I, this imposes a strong constraint on the phase relation between temperature and the meridional advective heating, a matter of considerable importance to the stability properties. The introduction of ozone photochemistry and radiative processes alters this phase relation, and hence, may be expected also to alter the stability properties. This, of course, depends on whether the time scales associated with the particular phenomenon are shorter or longer than the time scales for photochemical and radiative processes. On the basis of the results of Part I it is clear that no large scale hydrodynamic phenomenon will have a time scale shorter than the photochemical time scale at every level under discussion here.

It is the purpose of this Part and of Part IV to develop mathematically simple and physical plausible models of baroclinic instability, and to investigate the effect of incorporating ozone photochemistry and radiative transfer. In this part we will deal with the stability of a zonal vortex to an axially symmetric disturbance.

For adiabatic motions the condition for stability is found to be (see Eady, 1949; Kuo, 1956, among many others)

$$\frac{1}{f}\left(f-\frac{\partial\bar{u}}{\partial y}\right)\frac{\dfrac{g}{\bar{T}}\left(\dfrac{\partial\bar{T}}{\partial z}+\dfrac{g}{c_p}\right)}{\left(\dfrac{\partial\bar{u}}{\partial z}\right)^2}>1, \tag{1}$$

where $\frac{g}{\bar{T}}\left(\frac{\partial\bar{T}}{\partial z}+\frac{g}{c_p}\right)\Big/\left(\frac{\partial\bar{u}}{\partial z}\right)^2$ is the Richardson number and $f=2\Omega$, where Ω= the vertical component of the earth's rotation rate [see Part I (Lindzen and Goody, 1965) for definitions of other symbols]. Condition (1) indicates that the earth's atmosphere is generally stable. It therefore appears that symmetric meridional motions, where they exist, must be driven by external heat or momentum sources. Several studies of such driven systems have been made (Eliassen, 1952; Kuo, 1956; Leovy, 1964 are examples). The sources in these studies are usually taken to be fixed and unaffected by the circulations they drive, which result from the need for advections to maintain a steady state in the presence of these fixed sources and sinks.

In the present study, heat sources, resulting from perturbations in ozone and temperature, are not fixed, but are homogeneous functions of the perturbation velocity. Consistent with our desire to deal with the simplest model exhibiting the behavior in which we are

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interested, we have adopted the following approximations. We assume density variations to be negligible except where they are associated with gravity, i.e., buoyancy terms (Spiegel and Veronis, 1960). This is the Boussinesq approximation. Our equations are linear in the perturbations and the coefficients functions of the basic fields. We shall treat these coefficients as constants; this amounts to treating $\bar{F}$ and $d\bar{F}/dz$ as constants whenever they appear, where $\bar{F}$ is any basic field and is a legitimate procedure for systems where variations from $\bar{F}$ are small. Thus we are restricted to disturbances whose vertical scale is small compared to the scale-heights for the various coefficients.

As a first step we will utilize the above approximations for adiabatic motions in order to show that known results are preserved.

2. Stability with respect to axially symmetric disturbances—adiabatic case

Consider a basic field of the form

$$\bar{u}=\bar{u}_0+\bar{u}_{01}y+\bar{u}_{02}z, \tag{2}$$

$$\bar{T}=\bar{T}_0+\bar{T}_{01}y+\bar{T}_{02}z. \tag{3}$$

Let our system be rotating at a rate $\Omega=\frac{1}{2}f$, with the axis of rotation parallel to the z-axis. We will keep Ω constant, and introduce no curvature effects (the f-plane approximation; Phillips, 1963). Our disturbance equations are

$$\frac{\partial u}{\partial t}-(f-\bar{u}_{01})v+\bar{u}_{02}w=0, \tag{4}$$

$$\frac{\partial v}{\partial t}+fu=-\frac{1}{\bar{\rho}_0}\frac{\partial P}{\partial y}, \tag{5}$$

$$\frac{\partial w}{\partial t}=\frac{g}{\bar{T}_0}\theta-\frac{1}{\bar{\rho}_0}\frac{\partial P}{\partial z}, \tag{6}$$

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{7}$$

and

$$\frac{\partial\theta}{\partial t}+\bar{T}_{01}v+\left(\bar{T}_{02}+\frac{g}{c_p}\right)w=0, \tag{8}$$

where θ, u, v, w, and p are the disturbance temperature, zonal velocity, meridional velocity, vertical velocity and pressure fields and $\frac{\partial}{\partial x}=0$ by assumption. We may reduce Eqs. (4)–(8) to the following equation for w:

$$\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial z^2}+\frac{\partial^2 w}{\partial y^2}\right)+f(f-\bar{u}_{01})\frac{\partial^2 w}{\partial z^2}-2f\bar{u}_{02}\frac{\partial^2 w}{\partial z\partial y}+\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\frac{\partial^2 w}{\partial y^2}=0, \tag{9}$$

where, assuming the geostrophy of the basic field, we have taken $\frac{g}{\bar{T}_0}\bar{T}_{01}=-f\bar{u}_{02}$. Let us assume for (9) a solution of the form $w=\hat{w}(z)e^{\gamma t+i\alpha y}$, where α is real and γ may be complex. (9) becomes

$$\{\gamma^2+f(f-\bar{u}_{01})\}\frac{d^2\hat{w}}{dz^2}+2i\alpha f\bar{u}_{02}\frac{d\hat{w}}{dz}-\left\{\alpha^2(\gamma^2)+\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\right\}\hat{w}=0. \tag{10}$$

Eq. (10) is equivalent to

$$\frac{d^2N}{dz^2}+QN=0, \tag{11}$$

where

$$\hat{w}=e^{-iPz}N,$$

$$P=\frac{2\alpha f\bar{u}_{02}}{\gamma^2+f(f-\bar{u}_{01})},$$

and

$$Q=\frac{\alpha^2}{\gamma^2+f(f-\bar{u}_{01})}\left\{\frac{(f\bar{u}_{02})^2}{\gamma^2+f(f-\bar{u}_{01})}-\left[\gamma^2+\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\right]\right\}.$$

The solution for $\hat{w}$ is

$$\hat{w}=Ae^{-iPz}(e^{iQ^{\frac{1}{2}}z}+Be^{-iQ^{\frac{1}{2}}z}), \tag{12}$$

where A is an arbitrary amplitude and B is determined by a boundary condition.

There are three obvious choices for boundary conditions. The first is $w=0$ at $z=0$ and H, i.e., the fluid is confined by two horizontal plates. In this situation $B=-1$ and $Q^{\frac{1}{2}}=n\pi/H$. Since $n\pi/H$ is real, γ^2 must be real and hence P must be real. $\mu=P\pm Q^{\frac{1}{2}}$ is, therefore, also real.

The second case is that of a semi-infinite domain where $w=0$ at $z=0$, and is bounded at ∞. Again $B=-1$, and real μ satisfies the second boundary condition. However, there is also the possibility of γ complex if $\mathrm{Im}(P)<-|\mathrm{Im}(Q^{\frac{1}{2}})|$, for which we have bounded solutions with μ complex. This case has not been investigated in detail.

The third case is that of no boundaries, boundedness at $z=\pm\infty$ being the only boundary conditions; the result of these conditions is that μ must be real.

We wish to deal with a disturbance whose characteristic height is small compared to the height of the medium. The boundaries are far removed from the region of interest, and their nature is either very complicated or unknown. Under these conditions the last of the above three cases seems to be most appropriate.

The stability conditions for this case are easily arrived at. Knowing that $\hat{w}$ must be of the form $e^{i\mu z}$, with μ real, we substitute this form into (10) and solve for γ^2 to obtain the relation

$$\gamma^2=-\left[1+\left(\frac{\alpha}{\mu}\right)^2\right]^{-1} \times\left\{\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\left(\frac{\alpha}{\mu}\right)^2+2f\bar{u}_{02}\left(\frac{\alpha}{\mu}\right)+f(f-\bar{u}_{01})\right\}, \quad (13)$$

or

$$\gamma^2=-\left[1+\left(\frac{\alpha}{\mu}\right)^2\right]^{-1}F\left(\frac{\alpha}{\mu}\right), \quad (14)$$

where

$$F\left(\frac{\alpha}{\mu}\right)=\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\left(\frac{\alpha}{\mu}\right)^2+2f\bar{u}_{02}\left(\frac{\alpha}{\mu}\right)+f(f-\bar{u}_{01}).$$

Now, $F\left(\frac{\alpha}{\mu}\right)$ is a quadratic in $\left(\frac{\alpha}{\mu}\right)$, and $\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)$ is, in general, positive. Therefore, from the elementary theory of equations [see Hall and Knight (1887) for example] a necessary and sufficient condition for F to be positive is

$$f(f-\bar{u}_{01})\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)>(f\bar{u}_{02})^2, \quad (15)$$

or

$$f(f-\bar{u}_{01})\,\mathrm{Ri}>1,$$

where

$$\mathrm{Ri}=\frac{g}{\bar{T}_0}\frac{\left(\bar{T}_{02}+\frac{g}{c_p}\right)}{(\bar{u}_{02})^2}. \quad (16)$$

But, when F is positive, γ^2 is negative and the system is stable. Hence, (16) is identical with condition (1), and the traditional result is seen to be preserved.

3. Stability with respect to axially symmetric disturbances with photochemistry and radiative transfer

The inclusion of photochemical and radiative processes in the above problem involves replacing Eq. (8) with Eq. (36) of Part I; i.e.,

$$\left\{\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)\right\}\theta =-\left(\frac{\partial}{\partial t}+B\right)\left(v\bar{T}_{01}+w\left(\bar{T}_{02}+\frac{g}{c_p}\right)\right) -\eta(v\bar{\varphi}_{01}+w\bar{\varphi}_{02}), \quad (17)$$

or, when η is constant,

$$\left\{\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)\right\}\theta =-\left(\frac{\partial}{\partial t}+a+B\right)(v\bar{T}_{01}+w\bar{T}_{02})-\left(\frac{\partial}{\partial t}+B\right)\frac{g}{c_p}w, \quad (18)$$

(see Section 2 of Part II). Assuming the geostrophy of the basic field, (18) becomes

$$\left\{\frac{\partial^2}{\partial t^2}+(a+B)\frac{\partial}{\partial t}+(aB+\eta C)\right\}\theta =-\left(\frac{\partial}{\partial t}+a+B\right)\left\{v\left(-\frac{f\bar{T}_0}{g}\bar{u}_{02}\right)+w\bar{T}_{02}\right\} -\left(\frac{\partial}{\partial t}+B\right)\frac{g}{c_p}w. \quad (19)$$

We are imposing boundedness at $z=\pm\infty$ and, since we are also taking our coefficients to be constant, our solutions for all fields will be of the form $\exp(\gamma t+i\alpha y+i\mu z)$, where α and μ are real. Substituting into Eqs. (4)–(7) and (19) we obtain a quartic dispersion relation for γ,

$$\begin{aligned}&\gamma^4\left[1+\left(\frac{\alpha}{\mu}\right)^2\right]+(a+B)\left[1+\left(\frac{\alpha}{\mu}\right)^2\right]\gamma^3\\&+\left\{f\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]+(aB+\eta C)\left[1+\left(\frac{\alpha}{\mu}\right)^2\right]\right.\\&\left.+\left(\frac{\alpha}{\mu}\right)\left[f\bar{u}_{02}+\left(\frac{\alpha}{\mu}\right)\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\right]\right\}\gamma^2\\&+\left\{(a+B)\left[f\left(f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right)\right.\right.\\&\left.+\left(\frac{\alpha}{\mu}\right)\left[f\bar{u}_{02}+\left(\frac{\alpha}{\mu}\right)\frac{g}{\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)\right]\right]\\&\left.+\frac{g}{\bar{T}_0}\frac{g}{c_p}B\left(\frac{\alpha}{\mu}\right)^2\right\}\gamma+f\left(f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right)(aB+\eta C)=0, \quad (20)\end{aligned}$$

where we have set $\bar{u}_{01}=0$ for the sake of simplicity.

Now, if for our basic state we have a zonal wind in geostrophic balance with a photochemical-radiative equilibrium temperature field, then the Richardson number will be very large (as is the observed Richardson number). In this case, it is a straightforward, though somewhat tedious matter, to show that when $\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]>0$, the four roots of Eq. (20) consist of a pair of complex roots with negative real parts corresponding to gravity waves in a rotating system with slight radiative-photochemical damping, and two more roots with negative real parts corresponding to radiative-photochemical relaxations (see Section 6 Part I) modified by motion. When $\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]$ is negative, the two damped gravity-type waves remain; however, one of the two remaining roots is now positive, i.e., we have a new mode of instability.

Clearly, it is this last feature that is of greatest potential interest. It turns out (as we will show *a posteriori*) that for these instabilities the terms $\partial v/\partial t$ and $\partial w/\partial t$ in Eqs. (5) and (6), respectively, are small compared to other terms. Thus the unstable mode is sufficiently slow in development for pressure to remain hydrostatic, and for the zonal velocity to remain geostrophic. Such motions have been previously studied by Eliassen (1952). If we neglect the $\partial v/\partial t$ and $\partial w/\partial t$ terms, then we obtain a quadratic in γ instead of (20):

$$\gamma^2\left\{\left(\frac{\alpha}{\mu}\right)^2\frac{g}{f\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)+2\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}+f\right\}$$
$$+\gamma\left\{(a+B)\left[\left(\frac{\alpha}{\mu}\right)^2\frac{g}{f\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)+2\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}+f\right]\right.$$
$$\left.+a\frac{g}{f\bar{T}_0}\frac{g}{c_p}\left(\frac{\alpha}{\mu}\right)^2\right\}+(aB+\eta C)\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]=0. \quad (21)$$

Let

$$Q=\left(\frac{\alpha}{\mu}\right)^2\frac{g}{f\bar{T}_0}\left(\bar{T}_{02}+\frac{g}{c_p}\right)+2\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}+f,$$

and

$$P=(a+B)Q+a\frac{g}{f\bar{T}_0}\frac{g}{c_p}\left(\frac{\alpha}{\mu}\right)^2.$$

Since we have Ri≫1, the results of Section 2 show that Q and hence P are positive (never zero). Eq. (21) becomes

$$Q\gamma^2+P\gamma+(aB+\eta C)\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]=0. \quad (22)$$

For $\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]=0$, $\gamma=0$ is the physically interesting root of (22), and for $\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]<0$, this root becomes positive, implying instability. Note that this instability depends on the presence of the joint radiative-photochemical restoring term $(aB+\eta C)$ (see Section 6, Part I). We may see this explicitly as follows. Let $\left(\frac{\alpha}{\mu}\right)_c$ be such that $\left[f+\left(\frac{\alpha}{\mu}\right)_c\bar{u}_{02}\right]=0$, and let $\left(\frac{\alpha}{\mu}\right)=\left(\frac{\alpha}{\mu}\right)_c+\left(\frac{\alpha}{\mu}\right)'$. Near $\left(\frac{\alpha}{\mu}\right)=\left(\frac{\alpha}{\mu}\right)_c$

$$\gamma=-P^{-1}(aB+\eta C)\bar{u}_{02}\left(\frac{\alpha}{\mu}\right)',$$
$$=-\frac{(aB+\eta C)}{(a+B)Q-a\dfrac{g}{f\bar{T}_0}\dfrac{g}{c_p}\left(\dfrac{\alpha}{\mu}\right)^2}\bar{u}_{02}\left(\frac{\alpha}{\mu}\right). \quad (23)$$

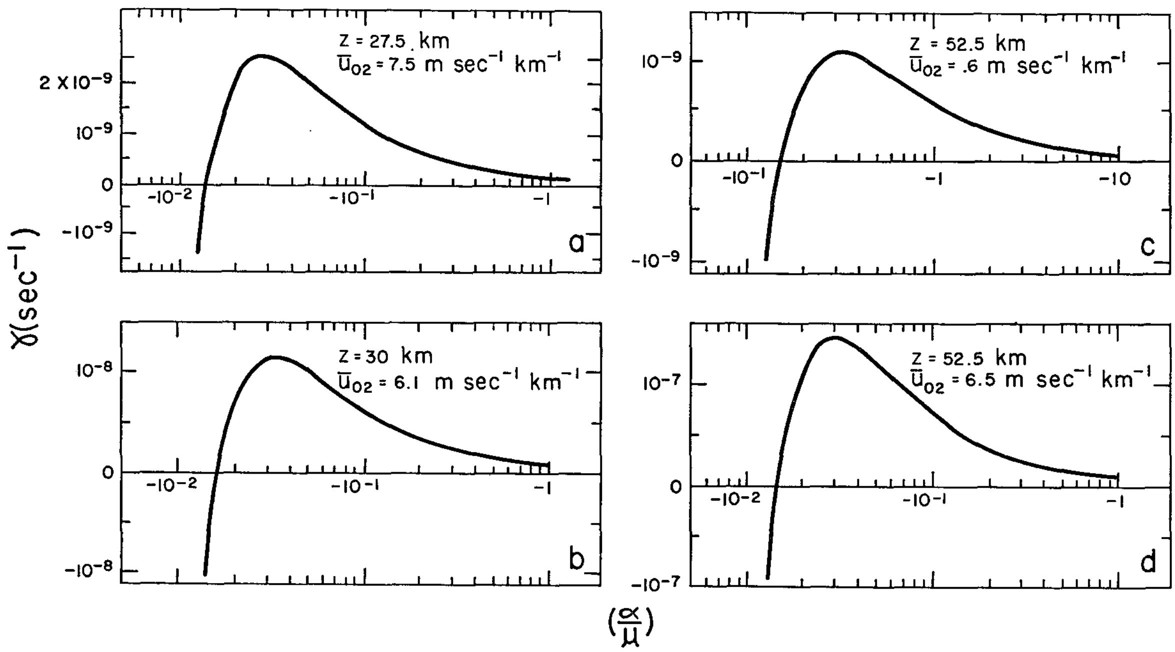

FIG. 1. The growth rate, γ, for axially symmetric disturbances. (α/μ) is the ratio of meridional to vertical wave numbers for the disturbances. Also indicated is the shear used in the calculation. For cases (a)–(c) the shear corresponds to photochemical-radiative, geostrophic equilibrium in Spring. For case (d) the observed winter shear was employed. The dependence of B and C on z is neglected.

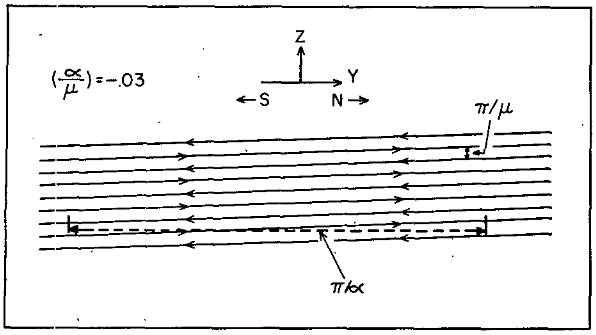

FIG. 2. Meridional circulation for $(\alpha/\mu)=-3\times10^{-2}$.

Eq. (23) suggests that the degree of instability is increased by increasing $\bar{u}_{02}$ and the ratio $(aB+\eta C)/(a+B)$, but the dependence upon $\bar{u}_{02}$ is complicated by the effect of this quantity in the magnitude of Q.

In Fig. 1 we show γ as a function of (α/μ) for photochemical and radiative parameters typical of 27.5, 30, and 52.5 km, computed from Eq. (21). Geostrophic shears for Spring photochemical-radiative equilibrium temperature fields are employed. We have also used the observed Winter shear at 52.5 km (Murgatroyd, 1957). The details of the equilibrium calculations may be found in Part I and in the author's thesis (Lindzen, 1964). In general, the features suggested by Eq. (23) occur in the exact calculations. For Figs. 1a, 1b and 1d the shears are approximately the same, but the ratio $(aB+\eta C)/(a+B)$ is increasing; in Figs. 1c and 1d, $(aB+\eta C)/(a+B)$ is the same, but the shear in 1d is much greater. Note, in particular, the very strong influence of shear.

We see in Fig. 1 that for high shear the maximum growth rate for an unstable disturbance occurs for $\left(\frac{\alpha}{\mu}\right)\simeq-0.03$. Fig. 2 gives the meridional circulation associated with $\left(\frac{\alpha}{\mu}\right)=-0.03$, obtained from the continuity equation (7). The motions are almost horizontal, vertically oscillating streams, rising slightly from south to north.

At this stage we can compute $\partial v/\partial t$ and $\partial w/\partial t$ to see whether they are small compared to fu and $(g/\bar{T}_0)\theta$ respectively, as assumed. The relevant ratios turn out to be [using Eqs. (4)–(8) with $\partial v/\partial t$ and $\partial w/\partial t$ neglected]

$$\frac{\partial v}{\partial t}\Big/fu=\frac{\gamma^2}{f\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]},$$

and

$$\frac{\partial w}{\partial t}\Big/\frac{g\theta}{\bar{T}_0}=\frac{\gamma^2\left(\frac{\alpha}{\mu}\right)^2}{f\left[f+\left(\frac{\alpha}{\mu}\right)\bar{u}_{02}\right]}.$$

Reference to Fig. 1 (noting $f\sim10^{-4}$ sec^{-1}) shows that both these ratios are $\lesssim10^{-6}$, and hence our approximations were valid.

The importance of an instability is, to a certain extent, measured by its growth rate. From Fig. 1 we see that $\gamma\sim(10 \text{ years})^{-1}$ at 27.5 km, $\gamma\sim(2 \text{ years})^{-1}$ at 30 km and $\gamma\sim(2.5 \text{ mo})^{-1}$ at 52.5 km. The growth rates of these zonal disturbances are therefore small, although for $\mu\lesssim1$ km the dissipation rates associated with molecular processes are at least an order of magnitude smaller. In the absence of other motions, the above described motions will certainly be unstable modes, but it will be shown in Part IV that the growth rate for baroclinic waves in mid-latitudes is much larger. Thus, we are faced with the problem of the growth of these instabilities in the presence of the more rapidly growing baroclinic waves, which, mathematically, is an extremely difficult problem. However, since zonal wavelengths for the baroclinic waves are quite long, ($\sim$5000 km, or at least 20 times longer than the most rapidly growing disturbance discussed here) it is possible that, locally, the disturbances might grow on top of the baroclinic waves.

4. Effects of vertical inhomogeneities

In the calculations of Section 3 the coefficients in Eqs. (4), (5), (6), (7), and (19) were taken to be constant. As mentioned in Section 1 this implies a restriction to disturbances whose vertical scale is small compared to the scale-heights for the variation of the various coefficients (usually of the order of 8 km). The major exception to this statement is the scale-height for the variation of the photochemical parameters, B and C; this scale-height is of the order of 2 km below 30 km, although it increases with height. Thus, our restriction to small vertical scales is fairly severe.

The inclusion of the z-variations of B and C leads to an intractable problem. For a preliminary discussion

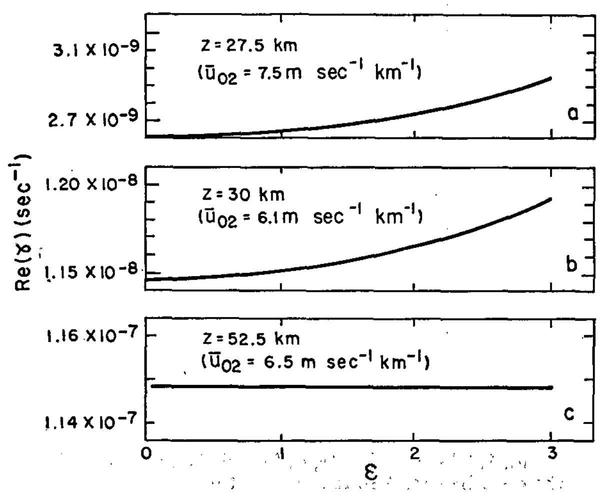

FIG. 3. The growth rate of disturbances as a function of ϵ. Re (γ) is maximized with respect to Re (α/μ).

we therefore treat the z-variations of these quantities as small perturbations and examine their effects to 'first order.' As an expansion parameter, we have used $\epsilon = \frac{1}{\mu}\frac{1}{B}\frac{dB}{dz}$, the ratio of the disturbance scale to the scale-height of B. The results of this calculation do not justify the inclusion here of the laborious details (Lindzen, 1964). In Fig. 3 the real part of γ, maximized with respect to (α/μ), is plotted against ϵ. In general, the effect of ϵ is slight; γ increases slightly with ϵ indicating that larger disturbance scales are slightly more unstable. Even this effect is virtually absent at 52.5 km. Another effect of finite ϵ, which is not shown in Fig. 3, is that the instabilities become overstable, i.e., γ has an imaginary part. The imaginary part of γ reaches a magnitude of 0.74×10^{-8} sec^{-1} at 27.5 km, 0.15×10^{-7} sec^{-1} at 30 km and 0.39×10^{-9} sec^{-1} at 52.5 km—all for $\epsilon=3$.

5. Conclusions

Thus, joint interactions of hydrodynamics, radiative transfer and photochemistry can destabilize a hydrodynamic system that would otherwise be stable. This observation is of some interest in itself, but the question remains, as to whether the particular unstable mode described in this paper is of any meteorological significance.

The circulation shown in Fig. 2 is reminiscent of streaming-type disturbances described by aufm Kampe *et al.* (1962), Sawyer (1961) and others for various regions of the mesosphere. Such streamings have also been noted in the records from rocket soundings at Eglin AFB in Florida in the region 20–60 km (Reed, private communication). In view of the very small growth rates of the instabilities described here, it is difficult to identify them with the observed phenomena. On the other hand, the fact that photochemical and radiative processes dissipate disturbances for some values of (α/μ) while slightly amplifying disturbances with other values of (α/μ) may prove important in selecting the form for disturbances whose energy is generated by still other processes.

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