The Influence of Stable Stratification on the Thermally Driven Tropical Boundary Layer

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ABSTRACT

Simple linearized models of the steady, thermally driven, axially symmetric motion of a stably stratified fluid on a rotating sphere are used to study the near-equator behavior of the boundary layer. It is found that the inclusion of the stratification introduces the natural horizontal length scale L of the geometric mean of the radius of the earth and a global Rossby radius of deformation. The tropical boundary layer is found to have constant finite depth over a length scale $O(L)$ from the equator through pressure boundary layering. The baroclinicity of the fluid is important for the interior flow also for a length scale $O(L)$ from the equator. Therefore, the inclusion of stable stratification is likely to be necessary in simple models describing features of the intertropical convergence zone.

An approximate solution to the boundary layer equations using a method devised by Kuo (1973) are compared to "exact" solutions. Kuo's method is found to introduce significant errors.

1. Introduction

The traditional local constant viscosity theory of the Ekman layer associated with a steady barotropic interior flow predicts a singularity in the boundary layer depth at the equator. There is similar singular behavior of the local boundary layer in an unstratified atmosphere associated with a periodic barotropic interior flow at the critical latitude where the wave frequency equals the Coriolis frequency (Holton *et al.*, 1971). Linearized theoretical models of the steady axisymmetric motion of a fluid in a thin shell on a rotating sphere also encounter the problem of equatorial singularities in both the boundary layer and interior solutions (Charney, 1973; Pedlosky, 1969).

Charney (1973) used a Boussinesq model driven by fixed temperature gradients at the upper and lower boundaries communicated to the fluid by eddy heat diffusion. The heat and momentum vertical eddy coefficients were taken as constant throughout the fluid, while there was no horizontal diffusion. To lowest order the boundary layer was unstratified and the equatorial boundary layer solutions were singular when the boundary temperature varied as $\sin^2$(latitude) or faster near the equator. The interior solution would also have been singular at the equator for a large class of temperature boundary conditions that Charney did not study; namely, those in which the temperature at one boundary varied rapidly enough near the equator, and the temperatures at the boundaries did not vary equally.

Pedlosky (1969) studied a model similar in some respects to Charney's, but different in that stratification was allowed to become important, horizontal mixing was included, and the boundary conditions were chosen to be imposed zonal velocities and heat fluxes at the top and bottom. Pedlosky found that there is a nondimensional horizontal length scale of $O(S^{\frac{1}{4}})$ for a boundary layer centered on the equator (with the zonal velocity geostrophic) needed to remove singularities in the asymptotic zonal and meridional velocities; S is a stratification parameter proportional to a global thermal Rossby number. This analysis did not, however, find the behavior of thermally driven tropical boundary layers parallel to the horizontal boundaries.

Kuo (1973) used a quasi-local model to show that the influence of vertical heat advection in a stably stratified atmosphere removed the singular behavior in the boundary layer near the equator. The traditional assumption that the pressure does not boundary layer was relaxed. The model equations were solved, however, using the approximation that the Coriolis parameter can be considered locally constant, allowing solution by separation of variables; then the

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Coriolis parameter is allowed to vary parametrically in the results. There is a finite-depth "boundary layer" at the equator with, however, a somewhat large depth for earth-like parameters.

The purpose of this work is to study the effect of stratification on the behavior of the tropical boundary layer and interior using a steady axisymmetric model similar to Charney's. Vertical diffusion of zonal and meridional momentum are included so that boundary layering is allowed near the bottom surface. The models are Boussinesq, linear, and assume constant vertical viscosity with height, no horizontal mixing and constant stability. A model with a Newtonian cooling-law-type heating is solved analytically, while one with diffusive heating is solved numerically. No simple behavior is found in either case. The boundary layer is found to have a nearly constant depth over the tropics for parameter values applicable to the earth's atmosphere. The natural horizontal length scale for the tropical motions is found to be the geometric mean of the radius of the earth and the Rossby radius of deformation at the poles, corresponding to the length scale found by Pedlosky. The horizontal length scale for the tropical boundary layer is the same as that of the interior. The inclusion of stratification allows the "boundary layering" of the pressure to become important near the equator through vertical heat advection, and forces the boundary layer solutions to be global.

Kuo's method of approximation in the non-separable diffusive case is shown to lead to some quantitative inaccuracy in the region of interest (near the equator). In particular, the boundary layer is found to be several times too deep there, and vertical motions are concentrated too near the equator.

2. Model equations

The linearized nondimensional equations for steady, axisymmetric, hydrostatic Boussinesq motions in a thin shell on a rotating sphere with constant vertical eddy viscosity are

$$\mathrm{E}^2\frac{\partial^4\psi}{\partial z^4}-y(1-y^2)^{\frac{1}{2}}\frac{\partial u}{\partial z}=(1-y^2)\frac{\partial T}{\partial y} \tag{1}$$

$$\frac{\partial^2 u}{\partial z^2}+\frac{y}{(1-y^2)^{\frac{1}{2}}}\frac{\partial\psi}{\partial z}=0 \tag{2}$$

$$-\mathrm{Ro}S\frac{\partial\psi}{\partial y}=Q. \tag{3}$$

The independent coordinates are $y=\sin(\text{latitude})$ and $z=\text{height}$. The dependent variables are the meridional streamfunction ψ, the zonal velocity u, and the temperature T. The nondimensional parameters entering are $\mathrm{E}=\nu/(2\Omega H^2)$ (the Ekman number), $\mathrm{Ro}=U/(2\Omega a)$ (the Rossby number), and S the static stability, assumed constant and non-dimensionalized by a vertical temperature difference and the height of the fluid; ν is the eddy viscosity, Ω the planetary rotation rate, H the depth of the fluid, a the planetary radius, and U a zonal velocity scale; U is scaled thermally and is, for atmospheric applications, $U=R(\Delta T)_v/2\Omega a$, where R is the gas constant and $(\Delta T)_v$ a vertical temperature deviation. These equations may be derived from Eqs. (3.1)–(3.5) in Charney (1971, pp. 128–129), replacing $\mathrm{E}T_{zz}/\sigma$ in (3.5), the heat conduction term, with Q. Terms of O(Ro) in (3.1) and (3.2), the horizontal momentum equations, are neglected, as is the horizontal heat advection term $\mathrm{Ro}vT_\phi$ in (3.5). The meridional and vertical velocities are assumed proportional to E. Eq. (3.4), the continuity equation, is satisfied by a streamfunction. The static stability T_z is represented by S. Dimensionless velocities are obtained from the streamfunction ψ by the relations

$$v=\frac{1}{(1-y^2)^{\frac{1}{2}}}\frac{\partial\psi}{\partial z}\quad\text{and}\quad w=-\frac{\partial\psi}{\partial y}.$$

The fluid will be assumed to be bounded at $z=0$ and $z=z_T$, where we take $z_T=1$ in the following. The streamfunction ψ must be constant along $z=0$ and $z=z_T$ as there is assumed to be no flow across these boundaries. The fact that u is zero or finite at the poles implies $(\partial\psi/\partial z)|_{y=\pm1}=0$ through (2). We choose then $\psi|_{z=0,1}=0$. The upper boundary conditions on the horizontal velocities are chosen to be no stress:

$$\left.\frac{\partial u}{\partial z}\right|_{z=z_T}=0,$$

$$\left.\frac{\partial v}{\partial z}\right|_{z=z_T}=\frac{1}{1-y^2}\left.\frac{\partial^2\psi}{\partial z^2}\right|_{z=z_T}=0.$$

Vertical integration of (2) from 0 to z_T then implies that $(\partial u/\partial z)|_{z=0}=0$. The values $u=v=0$ at $z=0$ are chosen for the lower boundary conditions on the horizontal velocities.

Integrating (2) vertically from the surface to z results in

$$\frac{\partial u}{\partial z}=-\frac{y}{(1-y^2)^{\frac{1}{2}}}\psi. \tag{4}$$

We may then eliminate $\partial u/\partial z$ from (1) yielding

$$\mathrm{E}^2\frac{\partial^4\psi}{\partial z^4}+y^2\psi=(1-y^2)\frac{\partial T}{\partial y}. \tag{5}$$

Two functional forms of Q in (3), which yield the same asymptotic results if the equations are approximated in terms of a Rossby number expansion (as done by Charney), are used. The results from these studies will be discussed next.

a. Cooling-law heating function

A Newtonian cooling law $Q=(T_e-T)/\tau$ is used in (3), where τ is the nondimensional radiative relaxation time, assumed constant with latitude and height for this calculation, and $T_e=T_e(y,z)$ is a specified radiative equilibrium temperature. The heat equation

$$-\mathrm{Ro}S\frac{\partial\psi}{\partial y}=\frac{T_e-T}{\tau} \tag{3a}$$

and (5) are used, defining $R_s=\mathrm{Ro}S\tau$, to find an equation for ψ:

$$\mathrm{E}^2\frac{\partial^4\psi}{\partial z^4}+y^2\psi-(1-y^2)R_s\frac{\partial^2\psi}{\partial y^2}=(1-y^2)\frac{\partial T_e}{\partial y}. \tag{6}$$

Eq. (6) may be solved by separation of variables. Assuming

$$\psi=\sum f_n(y)g_n(z)$$

and that the f_n are complete and orthogonal [which is demonstrable from (7)],

$$R_s\frac{\partial^2 f_n}{\partial y^2}-\frac{y^2-\lambda_n}{1-y^2}f_n=0, \tag{7}$$

$$\mathrm{E}^2\frac{\partial^4 g_n}{\partial z^4}+\lambda_n g_n=h_n(z), \tag{8}$$

where the $h_n(z)$ arise from the decomposition of the forcing, i.e.,

$$(1-y^2)\frac{\partial T_e}{\partial y}=\sum f_n(y)h_n(z). \tag{9}$$

Boundary conditions on (6) will be assumed to be no vertical or meridional velocity at the lower boundary, no vertical velocity or stress at the upper boundary, and no mass meridional motion across the poles, i.e.,

$$\psi=\begin{cases}\dfrac{\partial\psi}{\partial z}=0 & \text{at } z=0\\[2ex] \dfrac{\partial^2\psi}{\partial z^2}=0 & \text{at } z=1\\[2ex] 0 & \text{at } y=\pm 1\end{cases} \tag{6a}$$

Eq. (7) is known to have a complete orthonormal set of eigenfunctions associated with real eigenvalues, being of the Sturm-Liouville type. The eigenfunctions are either symmetric or antisymmetric about $y=0$ (the equator).

To find the eigenvalues of (7) we write $f_n=(1-y^2)\eta_n$, as Leovy (1964) does for a similar equation, giving

$$R_s(y^2-1)\frac{\partial^2\eta_n}{\partial y^2}+4yR_s\frac{\partial\eta_n}{\partial y}+(2R_s+y^2-\lambda)\eta_n=0, \tag{10}$$

subject to the boundary conditions that η_n is bounded at $y=\pm1$. If ψ is antisymmetric about the equator (u and T symmetric about the equator) η_n may be written

$$\eta_n=a_1^n y+a_3^n y^3+a_5^n y^5+\cdots.$$

Substituting η_n in (10) and equating powers of y, the recursion relations

$$a_3^n-\left(1-\frac{\beta\lambda_n}{2\cdot 3}\right)a_1^n=0, \tag{11}$$

$$a_{2j+1}^n-\left[1-\frac{\beta\lambda_n}{2j(2j+1)}\right]a_{2j-1}-\frac{\beta}{2j(2j+1)}a_{2j-3}=0, \quad j\geqslant 2, \tag{12}$$

are found with $\beta=1/R_s$. These recursion relations are essentially the same as those resulting from a power series solution of Laplace's tidal equation for a constant depth world-ocean. The eigenvalues λ_n may be found by solving the infinite set of linear homogeneous simultaneous equations (11) and (12), the only non-trivial solutions being those in which the determinant of the resulting matrix operator is zero. In practice the power series expansion is truncated, the determinant of a finite square matrix being zero then solved, and the truncation continued to higher orders until the eigenvalues change negligibly (Lindzen, 1970).

For a sample calculation, the equilibrium temperature is chosen to be

$$T_e=(1-z)\left(1-\frac{y^2}{2}\right)$$

with $R_s=0.015$, corresponding to a v scale of about 40 m s^{-1}, a dimensional radiative relaxation time of about 20 days, and a static stability of 3.5°C km^{-1}, or Ro$=0.05$, $S=0.3$ and $\tau=1$. E$=0.00437$ ($\nu=5\times10^5$ cm^2 s^{-1}) is also assumed.

To lowest order a Rossby-Ekman number expansion of (6) for this case would give $T=T_e$, $\psi=-[(1-y^2)/y]\times(1-z)$, so that v and w would be infinite at the equator in the interior, and the Ekman layer depth would diverge like $y^{-\frac{1}{2}}$ as $y\rightarrow 0$.

With the above assumptions for the forcing, the vertical structure equation becomes

$$\mathrm{E}^2\frac{d^4 g_n}{dz^4}+\lambda_n g_n=-\alpha_n(1-z), \tag{13}$$

subject to the boundary conditions

$$g_n(0)=g_n(1)=g_n'(0)=g_n''(1)=0. \tag{14}$$

The α_n's arise from the eigenfunction decomposition of T_e. Then $g_n(z)=-[\alpha_n(1-z)/\lambda_n]+\tilde{g}_n$, which is the result of a boundary layer approximation ($\lambda_n\gg\mathrm{E}^2$) but also could be an exact solution. $\tilde{g}_n$ satisfies

$$E^2\frac{d^4\tilde{g}_n}{dz^4}+\lambda_n\tilde{g}_n=0, \tag{15}$$

with the boundary conditions

$$\left.\begin{aligned} &\tilde{g}_n(0)-\frac{\alpha_n}{\lambda_n}=0\\ &\left.\frac{d\tilde{g}_n}{dz}\right|_{z=0}+\frac{\alpha_n}{\lambda_n}=0 \end{aligned}\right\}. \tag{16}$$

We really only invoke a boundary layer by requiring

$$\tilde{g}_n\to 0 \quad \text{as} \quad \eta_n=z\frac{\lambda_n^{\frac{1}{4}}}{E^{\frac{1}{2}}}\to\infty. \tag{17}$$

The solution to (13) is then

$$g_n\approx\frac{\alpha_n}{\lambda_n}\{-(1-z)+\exp(-z/D_n)\times[\cos(z/D_n)+(1-D_n)\sin(z/D_n)]\},$$

where

$$D_n=(2E/\lambda_n^{\frac{1}{2}})^{\frac{1}{2}}$$

as long as $D_n\ll 1$, which is true *a posteriori* for all λ_n with the assumed parameters (there is a lowest real λ_n and the $\lambda_n\to\infty$ as $n\to\infty$ due to the type of eigenvalue problem).

The three smallest eigenvalues turn out to be

$$\lambda_1\approx 0.36,\ \lambda_2\approx 0.77,\ \lambda_3\approx 1.15.$$

The approximate analytic solutions for ψ, u, v, w are

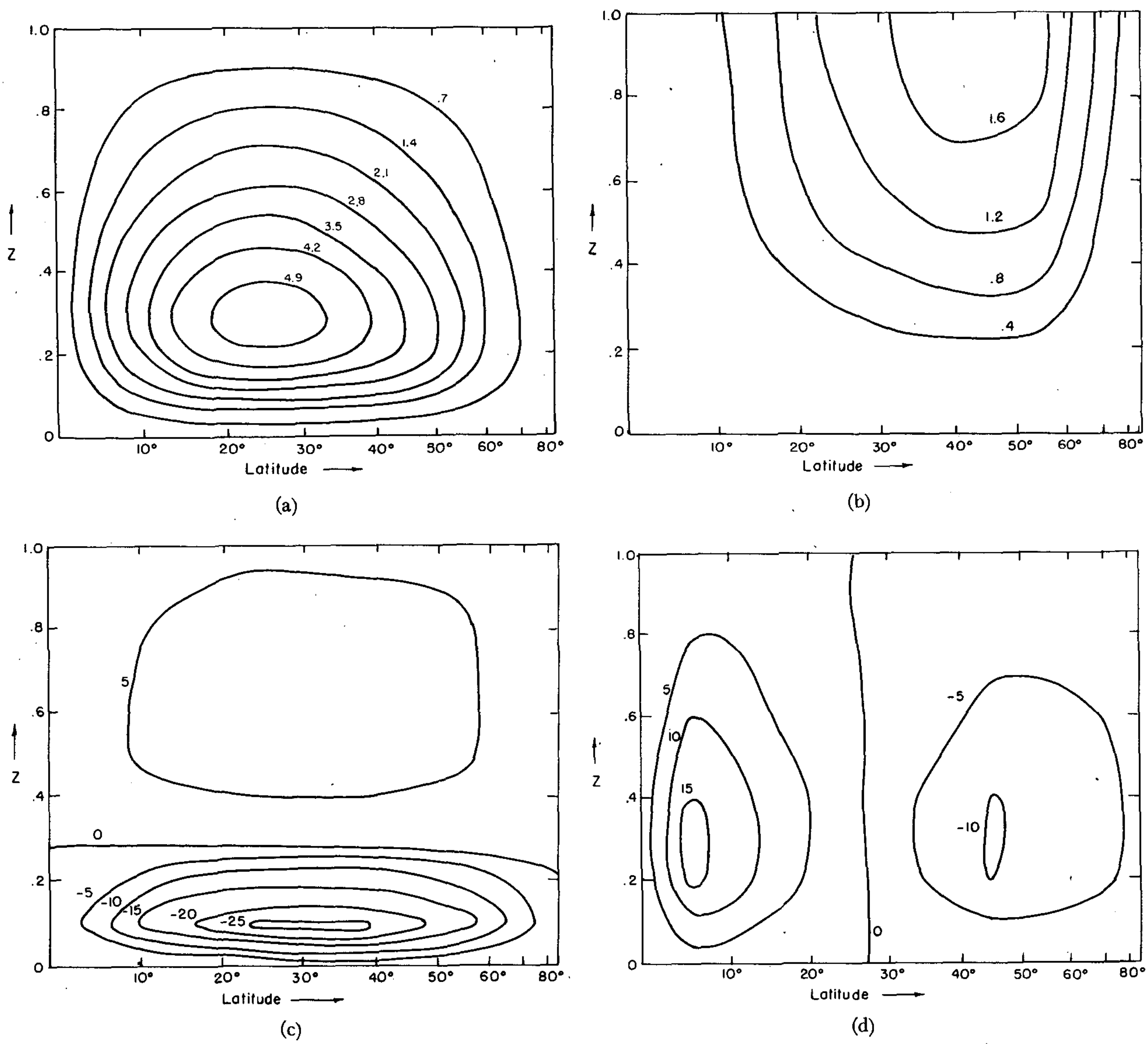

FIG. 1. Contours of the analytic solution of the radiative case: (a) streamfunction (× −1), (b) zonal wind, (c) meridional velocity, and (d) vertical velocity.

shown in Fig. 1. The results from the numerical solution of (1), (2) and (3a) agree well with the analytic results.

Since the modeling of the heating leads to a separable problem, each horizontal eigenmode is associated with a vertical structure that has a constant boundary layer depth at all latitudes. In addition, the gravest mode is associated with the largest boundary layer depth, while each higher mode is associated with a smaller boundary layer depth. Then the boundary layer depth associated with the smooth forcing appears to be almost constant in the solution (the first three eigenmodes are excited by this forcing in the amplitudes 0.30, −0.20 and 0.07, respectively). The boundary layer depth appears somewhat smaller in high latitudes than in low latitudes since the gravest mode's amplitude decays near the pole.

There is a natural length scale in (7) if $R_s \ll 1$ of $y \sim R_s^{1/4}$, and correspondingly, $\lambda_n \sim R_s^{1/2}$. Eq. (7) becomes approximately

$$\frac{d^2 f_n}{dy^2} - (y^2 - \lambda_n) f_n = 0$$

with this scaling. $R_s^{1/4} = (\mathrm{Ro} S_T)^{1/4}$ corresponds approximately to a global Rossby radius of deformation over which the influence of a point source at the equator is distributed. The eigenfunctions of (7) change from oscillitory to exponential behavior at approximately $R_s^{1/4}$ away from $y=0$. In the context of this Boussinesq model, tracing back thorugh the non-dimensionalization, $\mathrm{Ro}S$ corresponds to $(L_{OR})^2/a^2$ where L_{OR} is the classical Rossby radius of deformation at the pole. The length scale appropriate to the problem is then (if $\tau = 1$) of order $(L_{OR} a)^{1/2}$, or the geometric mean of the radius of the earth and the Rossby radius of deformation at the pole. This length scale also appears in the theory of linearized waves on an equatorial β plane as the distance from the equator where (up to a mode-dependent factor) gravity wave solutions change from oscillatory to exponential behavior, and the scale is also appropriate for Rossby waves (Lindzen, 1967).

Pedlosky finds the same length scale to be appropriate for equatorial dynamics in his model.

There are two features of the solution that provide the corrections to the divergent equatorial solutions found in a Charney-type model. First, the stability of the free atmosphere induces a boundary layer in the meridional direction throughout the depth of the fluid $O(R_s^{1/4})$ around the equator above the surface boundary layer. In this region vertical heat advections act to smooth temperature gradients enough for the zonal velocities to remain geostrophic and for all velocities to remain bounded. This is the equatorial boundary layer found by Pedlosky. Also, the baroclinicity of the atmosphere allows the pressure to boundary layer near the ground so that the boundary layer depth remains finite and nearly constant over again the length scale $O(R_s^{1/4})$ from the equator.

b. Conductive heating function.

We now consider $Q = \partial^2 T/\partial z^2$ conductive heating (Prandtl number equals 1). The heat equation is then

$$\frac{\partial^2 T}{\partial z^2} = -\mathrm{Ro}S \frac{\partial \psi}{\partial y}. \tag{3b}$$

Eliminating T between (3b) and (5) we obtain

$$\mathrm{E}^2 \frac{\partial^6 \psi}{\partial z^6} + y^2 \frac{\partial^2 \psi}{\partial z^2} + (1-y^2) R_s \frac{\partial^2 \psi}{\partial y^2} = 0, \tag{18}$$

where now $R_s = \mathrm{Ro}S (= L_{OR}^2/a^2)$. The boundary conditions needed in addition to (6b) are assumed to be fixed latitudinally-varying temperatures at the upper and lower boundaries. $T(y,z_T)=0$ and $T(y,0)=T_0(y)$ will be chosen for simplicity. Then the boundary conditions for ψ derived from (5) are

$$\left.\begin{aligned} \left.\frac{\partial^4 \psi}{\partial z^4}\right|_{z=z_T} &= 0 \\ \left.\frac{\partial^4 \psi}{\partial z^4}\right|_{z=0} &= \frac{1-y^2}{\mathrm{E}^2} \frac{\partial T_0}{\partial y} \end{aligned}\right\}. \tag{18a}$$

The interior solution for the conductive heating problem may be found by ignoring the $\mathrm{E}^2(\partial^4\psi/\partial z^4)$ term in (1). A separable equation for T

$$\frac{\partial^2 T}{\partial z^2} + R_s \frac{\partial}{\partial y}\left(\frac{1-y^2}{y^2} \frac{\partial T}{\partial y}\right) = 0 \tag{19}$$

is then found, assuming $\psi(y,z_T)=0$, which may be solved using the upper ($T=0$) and lower temperature boundary conditions. This interior temperature field and the associated u and ψ fields are bounded and valid everywhere except near the lower boundary, as the lower boundary conditions on the ψ field not satisfied there. The boundary layer equation, however, is (18) and is not separable. Therefore numerical solutions of (1), (2) and (3b) were performed for illustrative cases. The approximate solution of (1), (2) and (3b) found by ignoring vertical heat advection is the same as the approximate solution of (1), (2) and (3a) found using the same method. In particular, the interior and boundary layer solutions are singular at the equator.

Kuo's method of approximating the solutions to (18) reduces in this instance to treating the y^2 terms as parameters and solving the equation by separation of variables. This approach yields the equations

$$R_s(1-y^2)\frac{d^2 f_n}{dy^2} + \lambda_n f_n = 0, \tag{20}$$

$$\mathrm{E}^2 \frac{d^6 g_n}{dz^6} + \eta^2 \frac{d^2 g_n}{dz^2} - \lambda_n g_n = 0, \tag{21}$$

where $\eta = y$ (a constant).

The gravest horizontal eigenfunction f_1 is then

$$f_1 = (12)^{1/2}(y - y^3),$$

with eigenvalue $\lambda_1 = 6R_s$. The lower boundary condition on ψ is

$$E^2 \frac{\partial^4 \psi}{\partial z^4}\bigg|_{z=0} = -\frac{f_1}{(12)^{1/2}}$$

for $T_0 = 1 - y^2/2$.

Then

$$\psi = g_1(z) f_1(y) (12)^{1/2}$$

and

$$\left.\begin{array}{c} E^2 \dfrac{d^4 g_1}{dz^4} = -1 \quad \text{at} \quad z = 0 \\ g_1(0) = g_1(1) = g_1''(1) = g_1''''(1) = g_1'(0) = 0 \end{array}\right\}. \tag{22}$$

Eq. (21) with boundary conditions (22) was solved numerically, the resulting ψ being shown in Fig. 2. It is seen that Kuo's approximation gives a boundary layer depth of about $z_{BL} = 0.5$ south of 6° latitude. This does not appear to be a "boundary layer", but this depth did not change significantly as the lid was raised to $z_T = 2$ and $z_T = 3$. The magnitude of the streamfunction is about -2.4 in nondimensional units, compared to -0.7 for the "radiatively" driven model.

The streamfunction found by numerical solution of (1), (2) and (3b) with the same boundary conditions and parameter values as above are displayed in Fig. 3. Here the boundary layer depth appears much more uniform, $z_{BL} \approx 0.23$ from the equator to approximately 24° latitude [24° latitude $\approx (6R_s)^{1/2}$]. The absolute magnitude of the streamfunction is decreased by 25% from the approximate solution and the center of the gyre is shifted from 6° latitude to 15° latitude. Vertical velocities near the equator are decreased by about a factor of 3. Therefore, Kuo's method of finding approximate solutions to the conductively driven case does not appear to give reliable quantitative information in the tropics, especially concerning the tropical boundary layer. Of course the tropical boundary layer depth found by Kuo's method could be made more realistic by choosing a sufficiently small eddy viscosity, but the point of the calculation is not to model the actual atmospheric boundary layer, but to quantitatively assess Kuo's method of approximation. The length scale $O(R_s^{1/2})$ is important in the conductive case also, with the boundary layer depth remaining nearly constant for approximately this distance from the equator.

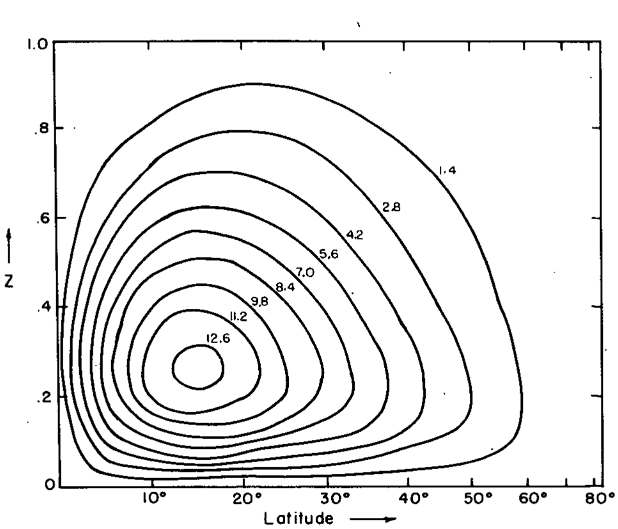

FIG. 2. Streamlines (× −1) for approximation by Kuo's method of diffusive case.

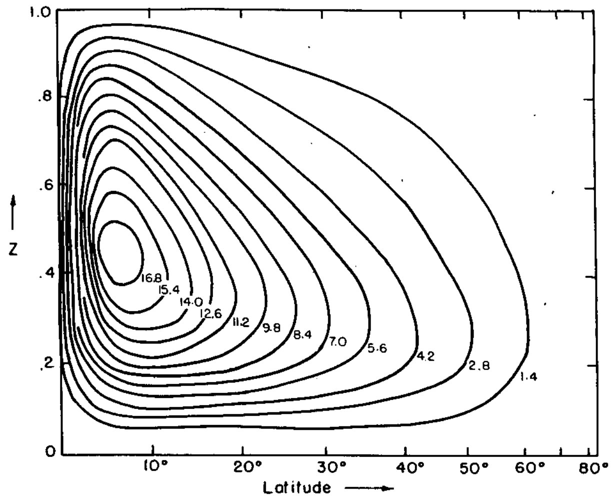

FIG. 3. As in Fig. 2 for numerical solution of the diffusive case.

Calculations were carried out with the temperature forcing antisymmetric about the equator for both the cooling law and conduction law cases. The results are similar to those reported above.

3. Conclusions

Stable stratification removes the equatorial singularities found in linearized models of steady axisymmetric flow in a thin shell on a rotating sphere. The horizontal length scale L of the geometric mean of the radius of the sphere and a global Rossby radius of deformation is important in the equatorial latitudes in the interior and boundary layer of the fluid. The influence of the stable stratification is important over a distance $O(L)$ from the equator. Mid-latitude Ekman-layer-type behavior produces incorrect predictions, as to the gross behavior of the model tropical boundary layer. The depth of the boundary layers in these calculations is nearly constant over a distance $O(L)$ from the equator. The traditional assumption that the pressure does not boundary layer cannot be made near the equator.

Theories of the intertropical convergence zone which depend crucially on the assumption of a barotropic frictional boundary layer, including CISK theories (Charney, 1971) and the critical latitude mechanism of Holton *et al.* (1971), should be reexamined. The CISK mechanism has been studied by Schneider (1975) and will be presented in a subsequent paper.

The use of Kuo's scheme for the parameterization of the tropical boundary layer in a stable, stratified atmosphere is an improvement on the use of Ekman-layer-type parameterization, but his method is likely to produce significant errors.

It should be noted that the primary purpose of this paper has been to analyze the character of steady tropical boundary layers rather than to present a model of the symmetric tropical circulation. The latter problem is dealt with in Schneider (1975) and will also be presented separately. The basic problem, of necessity, involves more realistic modeling of turbulent viscosity, lower boundary conditions and latent heating.

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REFERENCES

Charney, J. G., 1971: Tropical cyclogenesis and the formation of the Intertropical Convergence Zone: *Mathematical Problems in Geophysical Fluid Dynamics*, W. H. Reid, Ed., *Lectures in Applied Mathematics*, Vol. 13, Amer. Math. Soc., 355–368.

——, 1973: Planetary fluid dynamics. *Dynamic Meteorology*, P. Morel, Ed., D. Reidel, 97–352.

Holton, J. R., J. M. Wallace and J. A. Young, 1971: On boundary layer dynamics and the ITCZ. *J. Atmos. Sci.*, **28**, 275–280.

Kuo, H. L., 1973: Planetary boundary layer flow of a stable atmosphere over the globe. *J. Atmos. Sci.*, **30**, 53–65.

Leovy, C., 1964: Simple models of thermally driven mesospheric circulations. *J. Atmos. Sci.*, **21**, 327–341.

Lindzen, R. S., 1967: Planetary waves on beta-planes. *Mon. Wea. Rev.*, **95**, 441–451.

——, 1970: Quantitative theory of atmospheric tides and thermal tides. *Atmospheric Tides*, Gordon and Breach, 200 pp.

Pedlosky, J., 1969: Axially symmetric motion of stratified, rotating fluid in a spherical annulus of narrow gap. *J. Fluid Mech.*, **36**, 401–415.

Schneider, E. K., 1975: The Hadley circulation of the earth's atmosphere. Ph.D. thesis, Harvard University, 285 pp.