On the Consistency of Thermistor Measurements of Upper Air Temperatures

Richard S. Lindzen

National Center for Atmospheric Research, Boulder, Colo.

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Recent measurements of temperatures in the region 30–60 km (Beyers and Miers, 1965) yield the controversial result that daytime temperatures at 45 km may be as much as 15K greater than nighttime temperatures. This result is at great variance with theoretical estimates which predict differences on the order of 4K (Lindzen, 1966). Beyers *et al.* (1966) attempted to determine the temperature variation consistent with observed wind variations. Unfortunately, this calculation was extremely dependent on the precise details of the wind, and hence, inconclusive. They also calculated the adiabatic effects resulting from vertical motions caused by the diurnal radiational heating as computed by Leovy (1964), and found these inadequate to account for the observed day-night temperature differences. These discrepancies suggest that either the present estimates of the diurnal radiative drive or the data are in error. We shall restrict ourselves in this note to the possibility of an error in the data. An interesting point arises in this connection.

The theoretical estimates imply that time of maximum temperature should vary with altitude while Beyers and Mier's results show a maximum at about 1400 hours local time at all altitudes. This suggests the obvious possibility that direct solar irradiation of the thermistor bead package is producing an error. Beyers and Miers, noting this possibility, point out that Wagner (1964), in an analysis of the thermistor's accuracy, estimates the error resulting from solar irradiation of the bead to amount to only 1–2C in the altitude range 40–50 km. They add, as confirmation of Wagner's estimate, that the oscillations of the bead in and out of the chute's shadow (period$\sim$5–7 sec) result in oscillations of the bead temperature of from 1–2C. This, however, is not precisely a confirmation. Wagner's estimate is based upon the relatively steady response of the bead to solar irradiation, while in the data of Beyers and Miers the bead is giving the same amplitude of response in a few seconds.

The remainder of this note concerns itself with a simple analysis of the response of the bead to an oscillating irradiation analogous to that resulting from the swinging of the bead. Let δT be the difference between the temperature of the bead and the ambient medium.

Then

$$\frac{d\delta T}{dt}=Q-\mu\delta T, \tag{1}$$

where μ^{-1} is the response time of the bead to the medium, and

$$Q=\frac{aJA}{C}\tfrac{1}{2}(1+\cos\omega t), \tag{2}$$

where a is the short wave absorptivity of bead, J the solar constant, A the cross-sectional area of bead, C the heat capacity of bead, and $\frac{1}{2}(1+\cos\omega t)$ is an approximate representation of the time variation of the irradiation due to the swinging of the bead. According to Beyers and Miers, $\omega=2\pi/5$–7 sec. For our purposes, therefore, it will be adequate to take $\omega=1\ \text{sec}^{-1}$. If μ is taken to be constant, then Eq. (1) is appropriate to a bead swinging at a constant altitude. In the realistic case where the bead is falling, account must be taken of the altitude variations of μ and the ambient temperature. The effects of these variations, however, are largely extraneous to the present considerations, and we will, therefore, confine ourselves to the simpler case.

Assuming $\delta T=0$ at $t=0$, we have for the solution of (1)

$$\delta T=\frac{\hat{Q}}{\mu}\left\{1-e^{-\mu t}-\frac{\mu^2}{\omega^2+\mu^2}e^{-\mu t}\right\} +\frac{\hat{Q}}{\omega}\frac{1}{\left(1+\dfrac{\mu^2}{\omega^2}\right)^{\frac{1}{2}}}\sin\left(\omega t+\tan^{-1}\frac{\mu}{\omega}\right), \tag{3}$$

where

$$\hat{Q}=\frac{1}{2}\frac{aJA}{C}.$$

The condition $\delta T=0$ at $t=0$ implies that there is no error in the thermistor reading at $t=0$. This is probably not very realistic, and may result in the first term on the right hand side of (3) being substantially too small. It has, however, no effect on the second term.

Wagner's investigation corresponds to (3) when $\omega=0$. The first term shows that the oscillating irradiation of the bead can give rise to a steady difference between bead and ambient temperatures. This may be important in explaining the large day-night temperatures. It is the second term, however, that describes the temperature oscillation reported by Beyers and Miers of amplitude $\mathcal{A}=1\text{C}$.

According to (3)

$$\mathcal{A}=\frac{\hat{Q}}{\omega}\frac{1}{\left(1+\dfrac{\mu^2}{\omega^2}\right)^{\frac{1}{2}}}. \tag{4}$$

From Wagner (1964) we have $A=8.04\times10^{-4}\ \text{cm}^2$, $C=8.03\times10^{-6}$ cal deg^{-1} and $a=0.1$. Also, $J=0.33\times10^{-1}$ cal cm^{-2} sec^{-1}, and from Beyers and Miers, $\omega=1\ \text{sec}^{-1}$. Using these figures,

$$\frac{\hat{Q}}{\omega}=0.165\text{C},$$

and

$$\mathcal{A}=0.165\text{C}\times\frac{1}{\left(1+\left(\dfrac{\mu}{\omega}\right)^2\right)^{\frac{1}{2}}}. \tag{5}$$

From (5) we see that, using Wagner's data, we would expect a temperature oscillation much smaller than that observed by Beyers and Miers even with infinite response time (i.e., $\mu=0$). For shorter response times the situation is even worse. Wagner suggests a response time near 40 km of about 1 sec for which $\mathcal{A}=0.117\text{C}$.

It is difficult at this point to account for the difference between (5) and the White Sands observations. Suffice to say that the White Sands data not only disagree with current theories of the diurnal oscillation, but also appear to display internal inconsistency. One may surmise either that the parameters used by Wagner are incorrect or that some feature has been omitted from the instrument analysis or both. If any of these cases prevail then it is likely that the day-night temperature differences obtained by Beyers and Miers are in error.

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