The Application and Applicability of Terrestrial Atmospheric Tidal Theory to Venus and Mars

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(Manuscript received 20 April 1970)

ABSTRACT

The parameters of conventional atmospheric tidal theory which vary from planet to planet are isolated and evaluated for Earth, Mars and Venus. The assumptions of tidal theory are investigated in order to determine their validity for Mars and Venus.

1. Introduction

All rotating planets are subject to periodic heating. The period will be the solar day (though there will be significant harmonic distortion leading to periodicities which are integral fractions of a solar day as well). The response of the atmosphere to this forcing can be a problem as involved as the study of the general circulation; but for the earth a simplified theory has been moderately successful. Both this theory and terrestrial observations are discussed in detail in a monograph by Chapman and Lindzen (1970). Some more recent theoretical developments to which I shall refer in this paper are to be found in a set of papers (Lindzen, 1970a, b; Lindzen and Blake, 1970). In this paper I will sketch terrestrial tidal theory with special reference to its dependence on parameters which vary from planet to planet. I will also describe in detail those factors which can lead to the breakdown of the assumptions involved in terrestrial tidal theory. The values of the above mentioned parameters and the nature of the factors on Venus and Mars will be discussed. It will become clear that terrestrial tidal theory provides useful information about Mars; its applicability to Venus is much more questionable.

2. Review of terrestrial tidal theory

a. Major assumptions

Chapman and Lindzen (1970) describe many assumptions and/or approximations involved in terrestrial tidal theory. Of these, four prove most important as limitations on the usability of the theory:

1) Tides may be considered as linear perturbations on a mean basic atmospheric state.

2) Horizontal variations in the mean basic temperature and pressure are ignored, as are mean motions.

3) Dissipative mechanisms (eddy and molecular diffusion, radiation, ion drag) are ignored.

4) No account is taken of surface topography. The earth is assumed to be a smooth sphere; longitudinal variations of radiation absorbing gases are ignored.

b. Equations

Subject to the above assumptions (as well as some less important ones), the following are our equations for tidal perturbations:

$$\frac{\partial u}{\partial t} - 2\omega v \cos\theta = -\frac{1}{a}\frac{\partial}{\partial\theta}\left(\frac{\delta p}{\rho_0}\right), \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\omega u \cos\theta = -\frac{1}{a\sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{\delta p}{\rho_0}\right), \tag{2}$$

$$\frac{\partial \delta p}{\partial z} = -g\delta\rho, \tag{3}$$

$$\frac{\partial \delta\rho}{\partial t} + w\frac{d\rho_0}{dz} = -\rho_0\nabla\cdot\mathbf{V}, \tag{4}$$

$$c_v\left(\frac{\partial \delta T}{\partial t} + w\frac{dT_0}{dz}\right) = -gH\nabla\cdot\mathbf{V} + J, \tag{5}$$

$$\frac{\delta p}{p_0} = \frac{\delta T}{T_0} + \frac{\delta\rho}{\rho_0}. \tag{6}$$

In Eqs. (1)–(6) the symbols are defined as follows:

- θ co-latitude
- φ longitude
- z altitude
- a radius of the rotating sphere
- ω rotation of the planet
- u velocity in the θ direction
- v velocity in the φ direction

w vertical velocity

$\left.\begin{matrix}\delta T\\ \delta\rho\\ \delta p\end{matrix}\right\}$ tidal $\left\{\begin{matrix}\text{temperature}\\ \text{density}\\ \text{pressure}\end{matrix}\right\}$

J thermo-tidal heating per unit mass per unit time

$\left.\begin{matrix}T_0\\ \rho_0\\ P_0\end{matrix}\right\}$ basic $\left\{\begin{matrix}\text{temperature}\\ \text{density}\\ \text{pressure}\end{matrix}\right\}$ fields

H RT_0/g

R gas constant for atmosphere

g acceleration of gravity

c_v heat capacity at constant volume

The value of J, on earth, is due to insolation absorption by H_2O (0–18 km) and O_3 (18–72 km). Extreme ultraviolet (EUV) absorption by O_2, N_2 and O is important for the earth's thermosphere. The geometry used is shown in Fig. 1.

In general, we assume a vertical distribution $T_0(z)$ where p_0 and ρ_0 are then related to T_0 (through the hydrostatic pressure relation and the perfect gas law) as

$$p_0=p_0(0)\exp\left[-\int_0^z\frac{dz}{H}\right],\tag{7}$$

$$\rho_0=p_0/(gH).\tag{8}$$

Eqs. (1) and (2) involve northerly and westerly momentum, Eq. (3) is the hydrostatic pressure relation, (4) the equation of continuity, (5) the thermodynamic energy equation, and (6) the linearized perfect gas law.

c. *Mathematical procedure*

Our procedure involves seeking solutions of the form

$$f(\theta,z)e^{i(\sigma t+s\varphi)},\tag{9}$$

where $\sigma=2n\pi(1\text{ solar day})^{-1}$, $n=1, 2, \cdots$, and s (the zonal number) $=0, \pm1, \pm 2, \cdots$. Having assumed the time and longitude dependence given by (9), we reduce our equations to a single equation in the dependent variable

$$y=-\frac{1}{\gamma\sqrt{p_0p_0(0)}}\left(i\sigma\delta p+w\frac{dp_0}{dz}\right),\tag{10}$$

$$=-\frac{1}{\gamma\sqrt{p_0p_0(0)}}\frac{Dp}{Dt},$$

where $\gamma=c_p/c_v$. This particular choice of a variable turns out to simplify mathematical manipulation. In addition, once a solution for y is obtained it proves easy to relate other fields (u, v, w, δT, δT, δp) to y. Next we replace z as our vertical coordinate by

$$x=\int_0^z\frac{dz}{H}.\tag{11}$$

This step, too, is motivated primarily by convenience.

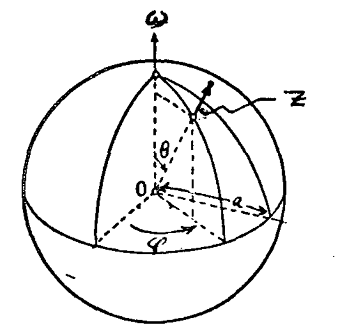

FIG. 1. Coordinates used in tidal theory.

The equation thus obtained for y is separable in its x and θ dependence. Let

$$y=\sum_n y_n(x)\Theta_n(\theta).\tag{12}$$

The equation from which the Θ_n's are obtained is

$$F[\Theta_n]=-\frac{4a^2\omega^2}{gh_n}\Theta_n,\tag{13}$$

where

$$F=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\sin\theta}{f^2-\cos^2\theta}\frac{\partial}{\partial\theta}\right)-\frac{1}{f^2-\cos^2\theta}\left(\frac{s}{f}\frac{f^2+\cos^2\theta}{f^2-\cos^2\theta}+\frac{s^2}{\sin^2\theta}\right),$$

$f=\sigma/2\omega$ and $[(4a^2\omega^2)/gh_n)]^{-1}$ is the separation constant which is usually described in terms of h_n, known as an *equivalent depth*. Eq. (13) is called *Laplace's tidal equation*. If we require that Θ_n be bounded at the poles, then (13) defines an eigenfunction-eigenvalue problem where $\{\Theta_n\}$ for all n is the set of eigenfunctions, known as *Hough functions*, and $\{h_n\}$ for all n is the set of eigenvalues.

How (13) is solved is described in Chapman and Lindzen (1970). $\{\Theta_n\}$ is an orthogonal set of what are, as far as we know, complete functions. Thus, we may expand $J(x, \theta)$ as

$$J=\sum_n J_n(x)\Theta_n(\theta).\tag{14}$$

The equation for the vertical structure of the nth *Hough mode* is then given by

$$\frac{d^2y_n}{dx^2}+\left[\frac{1}{h_n}\left(\kappa H+\frac{dH}{dx}\right)-\frac{1}{4}\right]y_n=\frac{\kappa J_n}{\gamma gh_n}e^{-x/2},\tag{15}$$

where $\kappa=(\gamma-1)/\gamma$. The solution of (15) requires two boundary conditions. The assumption that the earth is a smooth sphere leads to the condition that w, the vertical velocity, is zero at $z=0$. Expressed in terms of

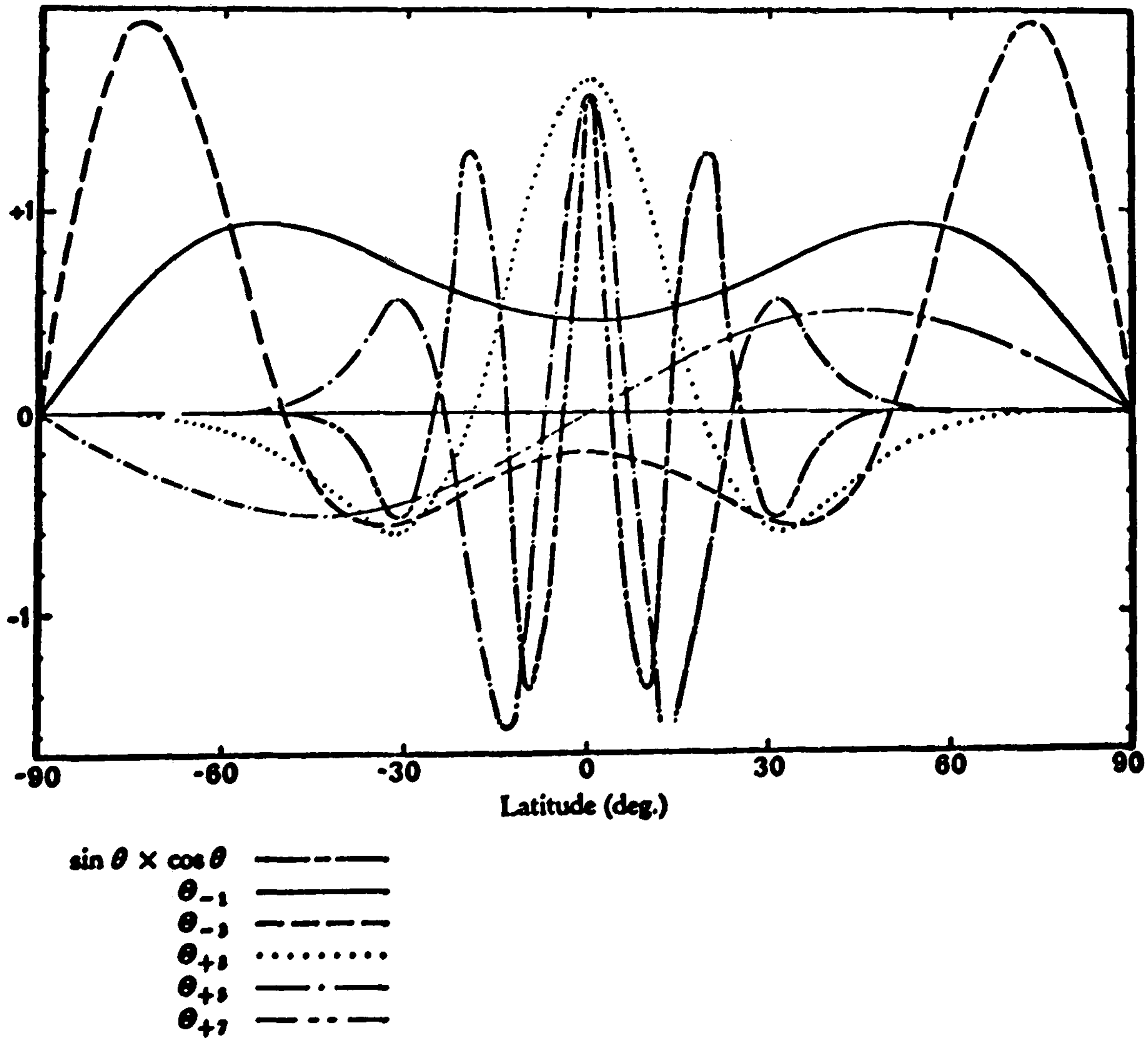

FIG. 2. Symmetric Hough functions for the migrating solar diurnal tide on Earth and Mars. Also shown is sinθ cosθ, the most important odd mode. Negative subscripts are used for modes with negative equivalent depths, positive subscripts for modes with positive equivalent depths (taken from Chapman and Lindzen, 1970).

y_n, this condition becomes

$$\frac{dy_n}{dx}+\left(\frac{H}{h_n}-\frac{1}{2}\right)y_n=0, \quad at \quad x=0. \tag{16}$$

As an upper boundary condition, it is often sufficient to assume y_n remains bounded as $x \to \infty$. However, when the top of our atmosphere is isothermal, (15) may have two solutions, both of which are bounded. They are of the form $e^{\pm i\lambda x}$ where

$$\lambda=\left(\frac{\kappa H}{h_n}-\frac{1}{4}\right)^{\frac{1}{2}}. \tag{17}$$

The solution $e^{i\lambda x}$ is associated with an upgoing wave, $e^{-i\lambda x}$ with a downcoming wave. If we assume a radiation condition, the latter must be rejected.

d. Comments on Laplace's tidal equation

In Section 2a I described the main assumptions of terrestrial tidal theory. To what extent these assumptions are peculiarly suited to earth will be discussed later. Given these assumptions, however, it is essential for our present purposes to note precisely where else in the theory we introduce parameters which vary from planet to planet. If we write the eigenvalue of Laplace's tidal equation as

$$x_n=(gh_n)/(4a^2\omega^2),$$

then the eigenfunctions and eigenvalues depend only on our choices for s the zonal wavenumber, and f $[=\sigma/(2\omega)]$; they are *independent* of the absolute rotation rate, the mass of the planet, the radius of the planet and the thermodynamic properties of the planet's atmosphere.

In Table 1 we show the solar day (which determines σ), the sidereal day (which determines ω), f for semi-diurnal and diurnal oscillations, and calculated values of x_n, also for semi-diurnal and diurnal oscillations, all as determined for Mars, Venus and Earth. We should first note that since the values of f are essentially the same for Earth and Mars, the Hough functions and x_n's are also the same for both planets. The smaller f is, the more important is the role of Coriolis forces. Thus, for Venus, Coriolis forces are relatively unimportant for tides induced by the rotation of the planet within the radiation field of the sun. The same is true for semi-diurnal tides on Earth and Mars. However, the diurnal tides on Earth and Mars are significantly affected by Coriolis forces. Indeed, when $f^2<1$, there exist latitudes where $f^2 = \cos^2\theta$, poleward of which Coriolis forces dominate inertia, and equatorward of which inertia dominates Coriolis forces.[1] Now, thermally driven tides are nothing more than internal gravity waves for which we know the precise period of the excitation. On a rotating plane, internal gravity waves can propagate vertically when $\sigma^2>4\omega^2$ (ω is the rotation rate of the plane); when $\sigma^2<4\omega^2$, however, they cannot propagate vertically. On a sphere $\omega\cos\theta$ replaces ω and for diurnal tides on Earth and Mars, $\sigma^2<4\omega^2\cos^2\theta$, poleward of $\theta=60°$, 120°. This gives rise to the existence of negative equivalent depths. As we shall see in the next section, negative equivalent depths are associated with exponential decay with height rather than vertical propagation. The situation becomes clearer when we look at Fig. 2 where the first few symmetric diurnal Hough functions for Earth and Mars are shown. The Hough functions with negative equivalent depths have their amplitudes concentrated at high latitudes while those with positive equivalent depths have their amplitudes concentrated at low latitudes. For semi-diurnal tides on Earth and Mars, $\sigma^2\geqslant 4\omega^2\cos^2\theta$ everywhere and there are no negative equivalent depths. The semi-diurnal Hough functions for Earth and Mars shown in Fig. 3 are quite similar to associated Legendre polynomials. For Venus neither diurnal nor semi-diurnal tides will have any negative equivalent depths and Hough functions will be similar to the semi-diurnal Hough functions for Earth and Mars.

Hough functions are the appropriate expansion functions for tidal w, δp, $\delta\rho$ and δT fields. The reader can see from Eqs. (1) and (2) that expansion functions for u and v will be different. Expansion functions for u for diurnal tides on Earth and Mars are shown in Fig. 4.

e. Comments on the vertical structure equation

The vertical structure equation (15), and hence the vertical structure of a Hough mode, depend on $T_0(x)$, the various gas parameters (γ, κ, R), h_n, g, and the distribution of excitation, $J_n(x)$. We find that h_n is related to x_n, the eigenvalue of Laplace's tidal equation, as

$$h_n=\frac{4a^2\omega^2}{g}x_n, \tag{18}$$

where h_n, depends on the mass, rotation rate and radius of the planet.

Let

$$\lambda^2=\frac{1}{h_n}\left(\kappa H+\frac{dH}{dx}\right)-\frac{1}{4}. \tag{19}$$

When $\lambda^2>0$, (15) describes the upward propagation of a wave away from the excitation. When $\lambda^2<0$,

[1] Where $f^2=\cos^2\theta$, we appear to have singularities in Laplace's tidal equation; however, they are removable, and solutions are bounded at these points.

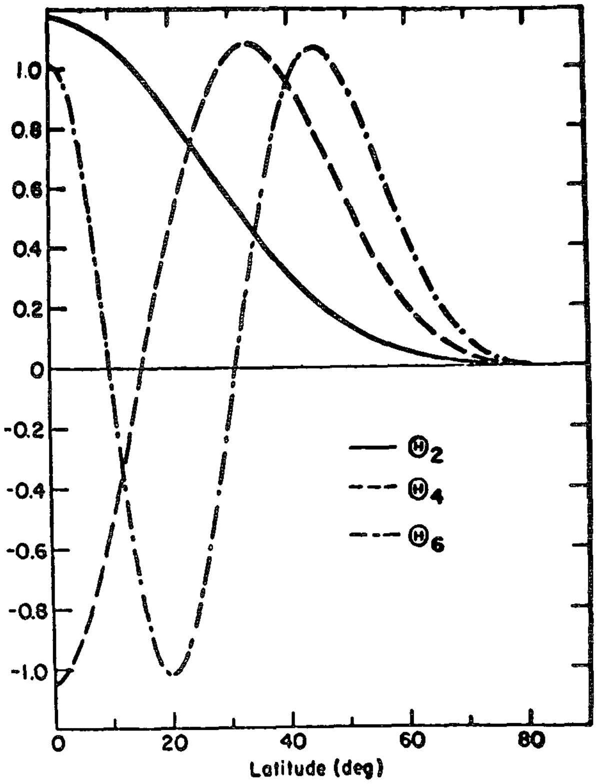

FIG. 3. Symmetric Hough functions for the migrating solar semi-diurnal tide on Earth and Mars (taken from Chapman and Lindzen, 1970).

TABLE 1. Parameters for Mars, Venus and Earth associated with the solution of Laplace's tidal equation.

	Mars	Venus	Earth
Solar day	24 hours	117 days	24 hours
Sidereal day	24 hours	247 days	24 hours
f_{diurnal}	0.5	1.055	0.5
$f_{\text{semi-diurnal}}$	1.0	2.110	1.0
x_n diurnal	$+7.895\times10^{-3}$, -1.38×10^{-1}, etc.	4.7×10^{-1},* etc.	$+7.895\times10^{-3}$, -1.38×10^{-1}, etc.
x_n semi-diurnal	8.84×10^{-2}, etc.	4.9×10^{-1},* etc.	8.84×10^{-2}, etc.

* Estimated by approximate means.

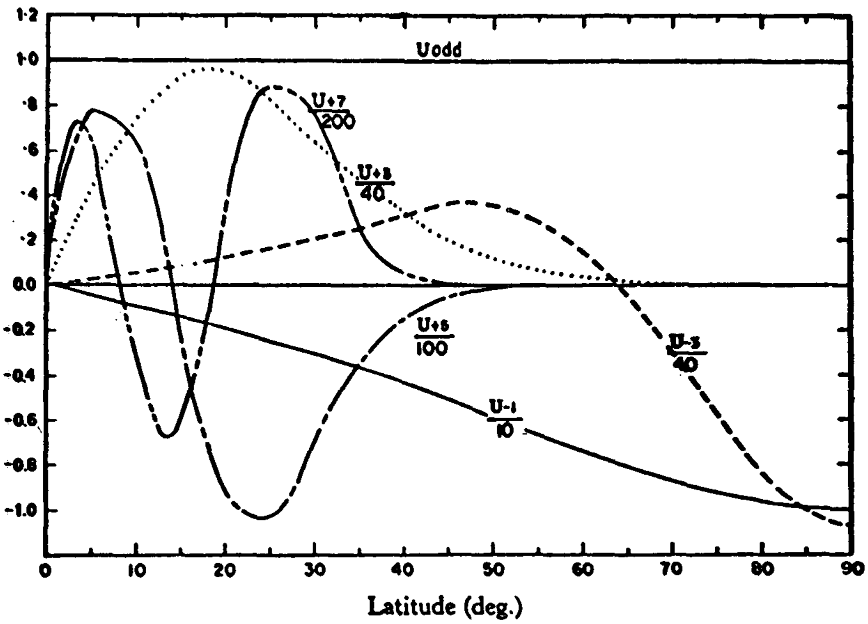

FIG. 4. Expansion functions for the latitude dependence of the solar diurnal component of u, the north-south wind speed component, on Earth and Mars. The functions have been divided by the amounts shown (taken from Chapman and Lindzen, 1970).

(15) describes the exponential decay with height[2] of a disturbance away from the excitation; thus, λ^2 will be negative whenever h_n is negative. When $h_n>0$, λ^2 will be positive if $h_n<\frac{1}{4}(\kappa H+dH/dx)$. When $\lambda^2>0$, the local vertical wavelength of a tidal mode is given by $2\pi H/\lambda$. For the earth's atmosphere which is generally statically stable and where dH/dx changes sign several times with increasing altitude, a good estimate of the average vertical wavelength is obtained by assuming an isothermal atmosphere in which λ is given by (17). We shall make similar estimates for Mars and Venus. More generally, the wavelength is inversely proportional (for short wavelengths) to the square root of the potential temperature gradient. Thus, a lapse rate which is 90% of the adiabatic lapse rate will lead to a threefold increase in vertical wavelength over the isothermal estimate.

TABLE 2. Parameters for Mars, Venus and Earth associated with vertical structure.

	Mars	Venus	Earth
$4a^2\omega^2/g$	78.4 km	0.1486 km	88.4 km
$h_n$diurnal	+ 0.619 km, −10.8 km etc.	+0.07 km etc.	+ 0.698 km, −12.2 km, etc.
$h_n$semi-diurnal	+ 6.93 km, etc.	+0.07 km, etc.	+ 7.85 km, etc.
$\bar{T}$	200K	600K	260K
$\bar{H}=R\bar{T}/g$	10.2 km	12.9	7.5 km
$L_n$diurnal	33.6 km, etc.	12.4 km etc.	28. km, etc.
$L_n$semi-diurnal	200 km, etc.	12.4 km, etc.	250 km, etc.

[2] From Eqs. (10) and (1)–(6), it may be shown that the vertical structures of u, v, w, δT, $\delta p/p_0$, and $\delta\rho/\rho_0$ are similar to that of y—except for a factor proportional to $1/\sqrt{p_0}$ or, equivalently, $e^{x/2}$.

In Table 2 we list various parameters associated with vertical structure for Mars, Venus and Earth. Conditions on Mars and Earth are strikingly similar. On both, the main semi-diurnal mode has a very long vertical wavelength while the main propagating diurnal mode has a wavelength on the order of 30 km. On Venus both the main diurnal and semi-diurnal modes are associated with vertical wavelengths of ~12 km.

The actual response in any given mode will depend on the amount and effectiveness of excitation. The effectiveness of excitation depends on two factors, i.e., i) a given energy of excitation (J/ρ_0) will be more effective as the height at which it is applied increases, and ii) due to interference effects, the effectiveness of a given excitation will depend on the ratio D/L, where D is the thickness of the excitation and L is the vertical wavelength of the mode. Effectiveness increases as D/L increases from 0 to 0.5 and decreases as the ratio increases above 0.5 [see Lindzen (1966) for details].

f. *Atmospheric tides on Earth, Mars and Venus*

Atmospheric tides on Earth form the best (for obvious reasons) examples of the above considerations. For over a century surface pressure data have been analyzed for tides. The semi-diurnal component is regular with an amplitude of ~1 mb at the equator. The diurnal component is much more erratic with an amplitude of only 0.5 mb at the equator. Observations above the ground are more recent and are described in detail in Chapman

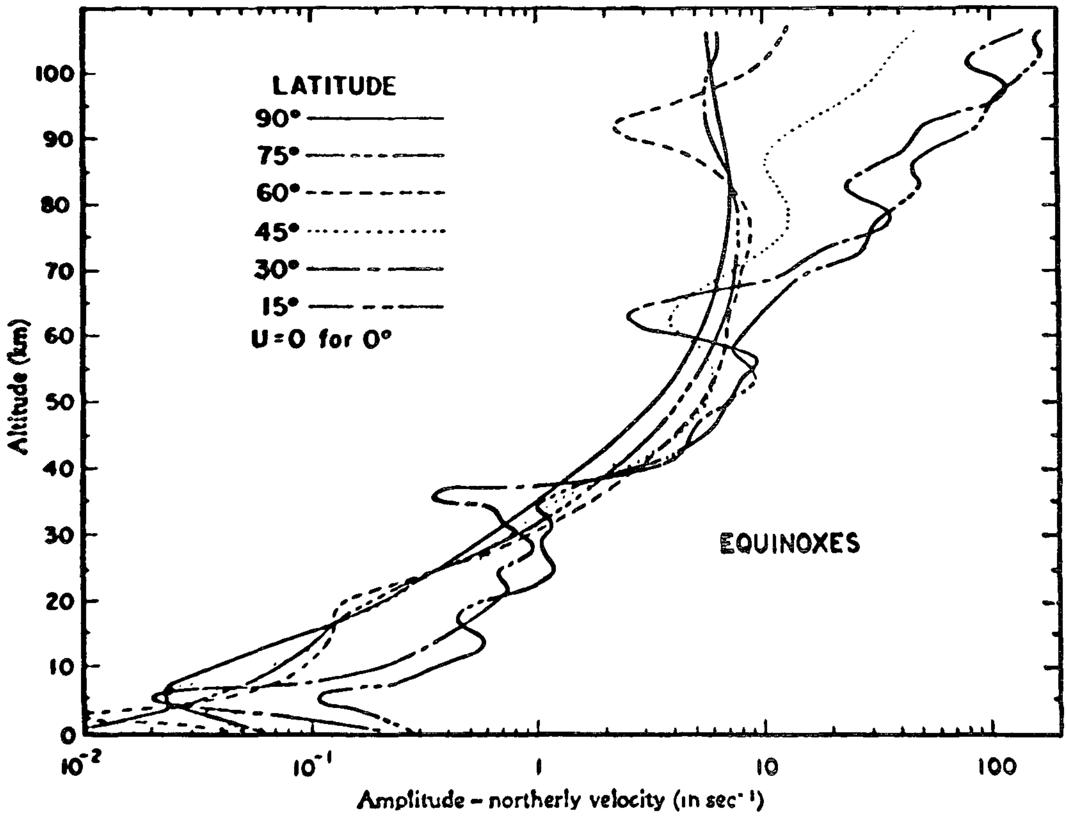

FIG. 5. Theoretical altitude distribution of the amplitude of the solar diurnal component of u on earth. Results are shown at 15° intervals of latitude; the calculations assume an isothermal basic state (taken from Chapman and Lindzen, 1970).

and Lindzen (1970). Only in the last 15 years have the main sources of excitation been identified. They are insolation absorption by water vapor in the first 18 km of the atmosphere and insolation absorption by ozone between about 20 and 80 km. The ozone excitation is effective in exciting the main semi-diurnal mode but ineffective in exciting the main propagating diurnal mode. Water vapor, on the other hand, is effective at exciting the main propagating diurnal mode. The reasons for this may be deduced from the discussion at the end of Section 2e. Calculations of the type we have described have proven successful in predicting the atmospheric response to the above excitations from the ground to ~90 km. Above 90 km some of the assumptions described in Section 2a begin to break down. Also, excitation due to UV absorption by O_2 and EUV absorption by O_2, N_2 and O become locally important. Some examples of theoretical results for the earth's atmosphere are shown in Figs. 5–8. Figs. 5 and 6 show the altitude variation of the amplitude and phase of the diurnal oscillation in the north-south wind speed component u at various latitudes. Figs. 7 and 8 show the same for semi-diurnal oscillations. The behavior of other fields is similar. Notice in Figs. 5 and 6 the clear distinction between high latitudes where the "trapped" Hough modes dominate and low latitudes where the "propagating" Hough modes dominate. No such distinction appears for the semi-diurnal oscillations.

The assumption that terrestrial tidal theory may be applied to Venus and/or Mars is, as we shall see later, a tenuous one. However, making this assumption, what can we say about atmospheric tides on Mars and Venus? The answer is very little without knowledge of the distribution of excitation. *A priori* we may state that semi-diurnal excitation will be ~30% of the magnitude of the diurnal excitation. This is merely a measure of the harmonic distortion in a heating function which is zero at night, and rises from zero at sunrise to a maximum at local "noon," falling to zero again at sundown. Due to the fact that the atmospheres of both Venus and Mars are composed primarily of CO_2, a radiatively intensely active gas, we may expect excitation magnitudes on these planets to be greater than on Earth. Due to the immense thickness of Venus' atmosphere, much excitation there may occur well above the ground.

Since the Hough modes for Mars are very similar to those on Earth, the atmospheric response will depend on the thickness of the excitation. If the effective thickness of the excitation is greater than 50 km then we may presume a predominantly semi-diurnal response, while if the thickness is less than 30 km we may expect a primarily diurnal response. On Venus both the main

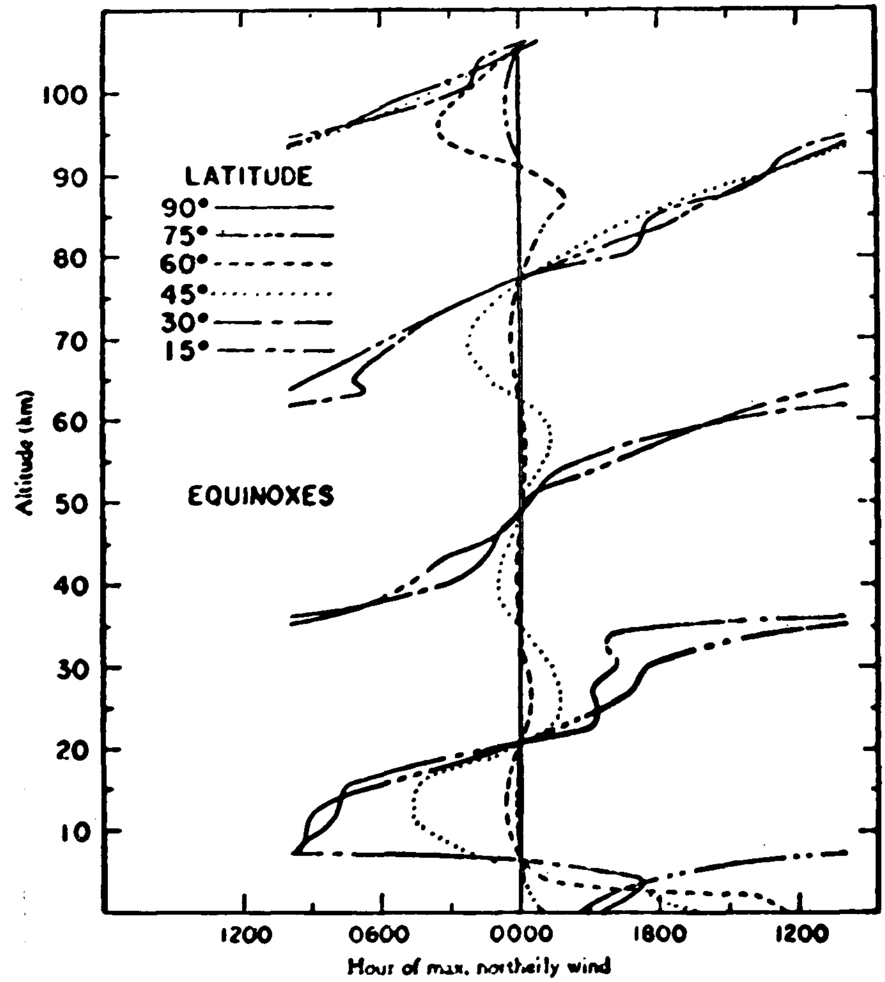

FIG. 6. Same as Fig. 5 except for the phase of the diurnal component of u on earth.

diurnal and semi-diurnal modes have the same vertical structure. We therefore expect a primarily diurnal response. However, the effect of dissipation, which I will discuss later, will be greater for the diurnal than for the semi-diurnal oscillations (simply because of the longer period of the diurnal oscillations); this may serve to reduce the difference between diurnal and semi-diurnal amplitudes on Venus.

3. Assessment of assumptions

In this Section I would like to specify in some detail the conditions under which the four assumptions cited Section 2a will be violated. I will begin with assumption 2.

a. Neglect of horizontal variations in temperature and pressure, and neglect of mean winds

1) We may neglect horizontal variations in temperature when the variations of vertical wavelength [see Eq. (19) for vertical wavenumber] over the characteristic horizontal scale for the particular Hough mode are small compared to the vertical wavelength itself. This condition seems to be roughly satisfied on Earth, and it seems likely to be equally satisfied on Mars and Venus.

2) Mean (zonal) winds may be neglected when wind speeds are small compared to the zonal phase speed of a tide. If U is a characteristic zonal speed, then

$$U \ll c = \frac{2\pi \times \text{planetary radius}}{1 \text{ solar day}}. \tag{20}$$

Now, $c \approx 450$ m sec^{-1} on Earth, 250 m sec^{-1} on Mars and 5 m sec^{-1} on Venus. The neglect of mean zonal wind is a good first-order approximation on Earth, and seems likely to be so on Mars as well. According to recent estimates of wind speeds in Venus' lower atmosphere (Goody and Robinson, 1966; Golitsyn, 1968), this may even be a good approximation there. However, in Venus' upper atmosphere, where winds of $\sim$100 m sec^{-1} have been reported, the consideration of the effects of the mean wind will be essential to the dynamics of the tides.

We turn next to assumption 3.

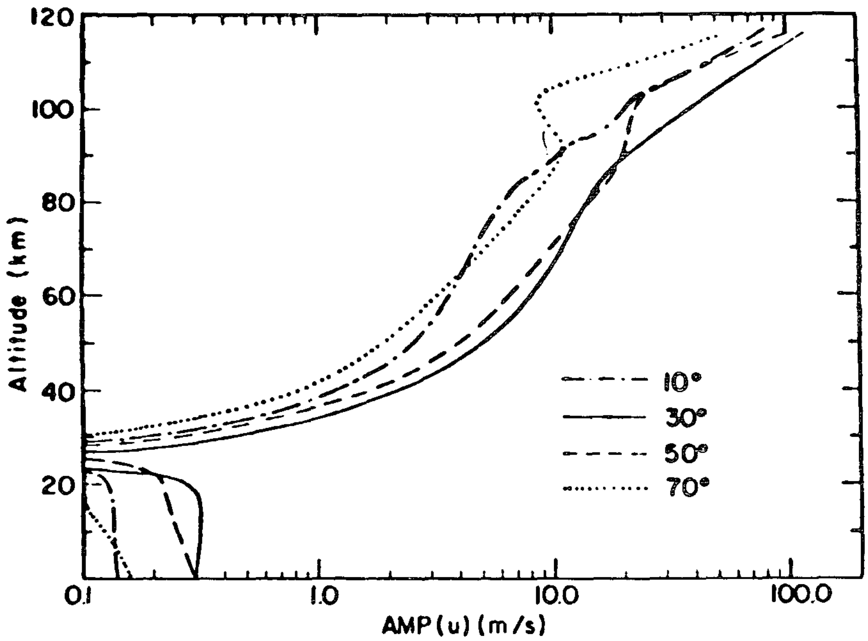

FIG. 7. Theoretical amplitude of the solar semi-diurnal component of u at various latitudes on earth; T_0 has been taken from the equatorial standard atmosphere (taken from Chapman and Lindzen, 1970).

b. Neglect of dissipation

Without specifying the dissipative mechanism let us define τ_{diss} as the time scale for dissipation, and τ_{tide} as the period of a tidal mode. Two rather distinct situations may exist with respect to dissipation.

1) If $\tau_{\text{diss}}<\tau_{\text{tide}}/2\pi$, then the tidal dynamics are fundamentally altered since dissipation is more important than inertia.

2) The presence of any dissipation will tend to reduce the $e^{x/2}$ growth of propagating modes. If, moreover,

$$\frac{\tau_{\text{tide}}}{2\pi}<\tau_{\text{diss}}<\frac{\tau_{\text{tide}}}{(\lambda/2H)},$$

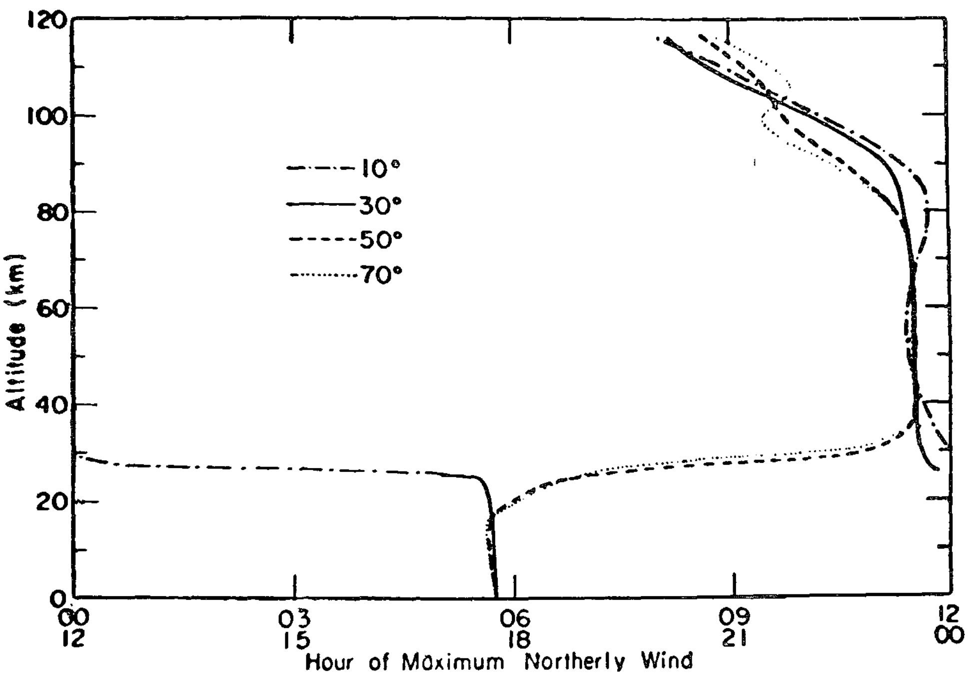

FIG. 8. Same as Fig. 7 except for the phase of the semi-diurnal component of u on earth.

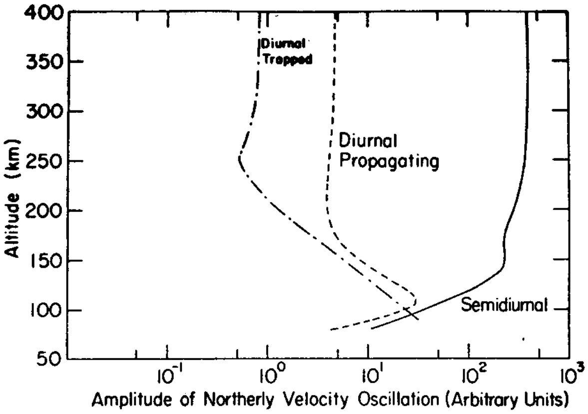

FIG. 9. Theoretical altitude distribution of the amplitude of u for the main semi-diurnal, the main propagating diurnal, and the main trapped diurnal modes on earth. Molecular viscosity and thermal conductivity have been taken into account.

then the vertical wavelength of the tidal mode will be relatively unaffected, but the $e^{x/2}$ growth above the region of exictation will be replaced by actual decay of amplitude.

How do the above considerations affect tides on the various planets? For Earth, condition 1) only arises from molecular diffusion growing in importance as $1/\rho_0$ and from ion drag in the F region (the latter is actually swamped by the former in the Earth's atmosphere—at least insofar as tides are concerned). Radiative cooling contributes modestly to condition 2) but is insufficient to lead to decay. The effects of molecular viscosity and thermal conductivity on terrestrial atmospheric tides has only recently been explored (Yanowitch, 1967; Lindzen 1970a, b; Lindzen and Blake, 1970) and the results may prove illuminating for tides on Mars as well.

Let

$$\tau_{\rm diss}^{-1}=\left(\frac{2\pi}{L}\right)^2\frac{\mu}{\rho_0},$$

where μ is molecular viscosity, and L the vertical wavelength, and let $\chi=\tau_{\rm tide}/\tau_{\rm diss}$. It turns out that the amplitude of the main propagating diurnal mode reaches a maximum when $\chi=1/6(x=16.2$ or $z\approx 108$ km) and decays markedly above this height, ultimately asymptoting to a constant. The amplitude of the main semi-diurnal mode stops growing when $\chi\approx 1/6$ and asymptotes to a constant without decay.

What happens above the level where $\chi\approx 1/6$ is determined by the ratio $\beta=2\pi H^2/L$. When $\beta>2$ at the level where $\chi\approx 1/6$ we get decay; when $\beta<2$ the wave asymptotes to a constant without decay. For $\beta<1$ there is even reflection due to increasing viscosity. Eddy viscosity proves generally unimportant for terrestrial atmospheric tides. In Figs. 9 and 10 we show the results of some calculations of the vertical structure of tidal modes as influenced by molecular viscosity and thermal conductivity. Fig. 9 shows the variation with height of the u component amplitude of the main semi-diurnal, main diurnal propagating, and main diurnal trapped modes. Fig. 10 shows the phases of these modes. Note that increasing viscosity and conductivity cause the exponential decay of trapped modes to cease. Note also that there is no phase variation with height when viscosity and conductivity are dominant.

Present estimates of radiative cooling on Mars (Gierasch and Goody, 1968) suggest that it will contribute importantly to condition 2) but is too small to involve condition 1). As a result of molecular viscosity (and conductivity) on Mars, the amplitude of the main propagating diurnal mode will reach a maximum (assuming a CO_2 atmosphere with 5-mb surface pressure) at $x=13.7\pm 1$. The main semi-diurnal mode will have its maximum amplitude at $x=17.6\pm 1$. Although we are uncertain as to appropriate eddy coefficients on Mars, we can say that eddy viscosities $>10^6$ cm^2 sec^{-1} will significantly affect the diurnal propagating modes (but not the trapped modes). Eddy viscosities $>10^8$ cm^2 sec^{-1} will affect the main semi-diurnal mode as well. In unpublished calculations I have found that the viscous dissipation of the semi-diurnal tide forms a significant thermospheric heat source on Earth. Even in the absence of EUV heating, this can lead to exospheric temperatures of $\sim$600K. Similar effects are to be expected on Mars.

As usual we are relatively ignorant of actual conditions on Venus. However, in view of the high density of its lower atmosphere, we know that molecular viscosities and conductivities will be low. While eddy coefficients are unknown, from results described in Section 2 we may state that $\nu_{\rm eddy}>0.4\times 10^4$ cm^2 sec^{-1} will eliminate the $e^{x/2}$ growth of the diurnal mode and $\nu_{\rm eddy}>0.8\times 10^4$ cm^2 sec^{-1} will do the same for the main semi-diurnal mode. A value of $\nu_{\rm eddy}>2.4\times 10^4$ cm^2 sec^{-1} will completely alter our assumed physical balance for the diurnal propagating mode, while $\nu_{\rm eddy}>4.8\times 10^4$ cm^2 sec^{-1} will do the same for the main semi-diurnal mode. It has been suggested by Schubert and Young (1970) that radiative transfer of heat is, over much of Venus' atmosphere, more important than either eddy or molecular transport of heat and momentum. If this is so, then a correct theory might neglect viscosity in which case a simple extension of conventional terrestrial tidal theory (e.g., Dickinson and Geller, 1968) might prove useful.

Next I shall discuss assumption 1.

c. Neglect of nonlinear effects

Tidal modes are rotationally modified internal gravity waves for which $\mathbf{v}\cdot\nabla\ll\partial/\partial t$ even for considerable amplitudes. Nonlinear effects are most commonly due to two particular occurrences. The first possibility is that

$$\text{i)}\qquad \frac{\delta\rho}{\rho_0}\gtrsim 1$$

someplace, i.e., that the predicted amplitude of the tidal density oscillation exceeds the mean density. A second important possibility is that a given tidal mode might become unstable. This is a nonlinear effect only insofar as the motion resulting from the instability will be nonlinear. Let

$$\left.\begin{aligned} &T=T_0+\delta T\\ &\mathbf{u}=\mathbf{u}' \text{ (tidal horizontal velocity)}\\ &\mathrm{Ri}=\frac{g}{T_0}\left(\frac{\partial T}{\partial z}+\frac{g}{c_p}\right)\bigg/\left(\left|\frac{\partial \mathbf{u}}{\partial z}\right|\right)^2 \end{aligned}\right\}.$$

When

ii) $\mathrm{Ri}<1/4$,

the tide becomes unstable.

Since the amplitudes of $\delta\rho/\rho_0$, u', δT, etc., all grow as $e^{x/2}$ in the absence of dissipation, nonlinearity, as defined by i) above [which appears to be a potentially important factor for the terrestrial semi-diurnal tide (Lindzen and Blake, 1970)] or instability, as defined by ii) [which appears to be an important factor for the terrestrial diurnal tide (Lindzen and Blake)], will eventually occur at some height—for propagating modes at least. The greater the excitation, the less the effect of dissipation, and the more effective the excitation, the lower this height will be.

The explicit situation for the earth's atmosphere is as follows. The diurnal propagating mode is believed to become unstable at about 88 km over the equator where the amplitude of δT is $\sim$40K. This instability must cease above $\sim$108 km where molecular diffusion causes amplitude decay. We associate this height with the tropical ($\pm$35–40° latitude) turbopause. The turbulence generated should yield eddy coefficients on the order of 10^7 cm^2 sec^{-1} which have almost *no* effect on the main semi-diurnal mode. For this mode $\delta\rho/\rho_0$ reaches (theoretically) 0.6 before viscosity and conductivity become dominant. This suggests an important but not dominant role for nonlinearity.

Due to the relatively great excitation expected on Mars, the diurnal propagating mode there may become unstable at very low altitudes (the amplitude of δT need only be $\sim$ 22K for instability to develop since g on Mars is smaller than it is on Earth), giving rise to a "turbopause" at 13.7$\pm$1 scale heights. As on Earth, the turbulence generated by this instability will not significantly attenuate the main semi-diurnal mode which will be dominated by molecular diffusion at 17.6$\pm$1 scale heights. If semi-diurnal excitation is sufficiently great, the semi-diurnal mode will go nonlinear.

Due to the small size of $\partial/\partial t$ on Venus, tides there may well prove nonlinear. However, this will, of course, depend on the excitation. If $\partial/\partial t$ terms prove negligible compared to other terms in the full equations of motion, then the "tide" may be studied (as is commonly done for Venus) in terms of horizontal steady convection where time enters parametrically. However, as we have shown, viscous terms will dominate $\partial/\partial t$ only when $\nu_{\mathrm{eddy}}\gg10^4$ cm^2 sec^{-1}. Parenthetically, it should be noted that $\partial/\partial t$ is larger than the Coriolis parameter on Venus.

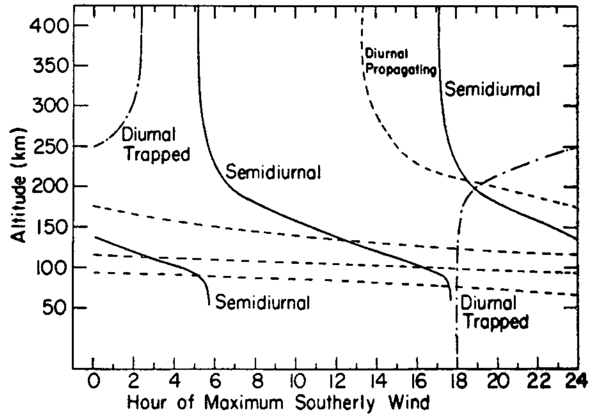

FIG. 10. Same as Fig. 9 except for the phases of the various modes.

d. Neglect of orography

Very little work has been done, thus far, on the effects of orography on atmospheric tides. It is clear, of course, that orography (and surface conditions in general) will affect tidal winds and temperatures near the surface. However, unless the Fourier spectrum of orography has strong components at zonal wavenumbers 1 and 2, the effect of orography on the migrating tidal fields (i.e., those following the sun) well above the surface is likely

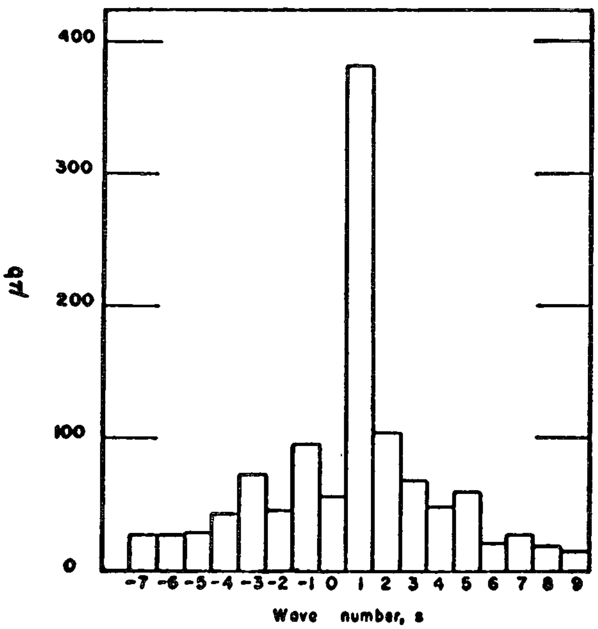

FIG. 11. The amplitudes (averaged over latitudes from 60S to the north pole on earth) of the diurnal pressure waves for different zonal wavenumbers s (taken from Chapman and Lindzen, 1970).

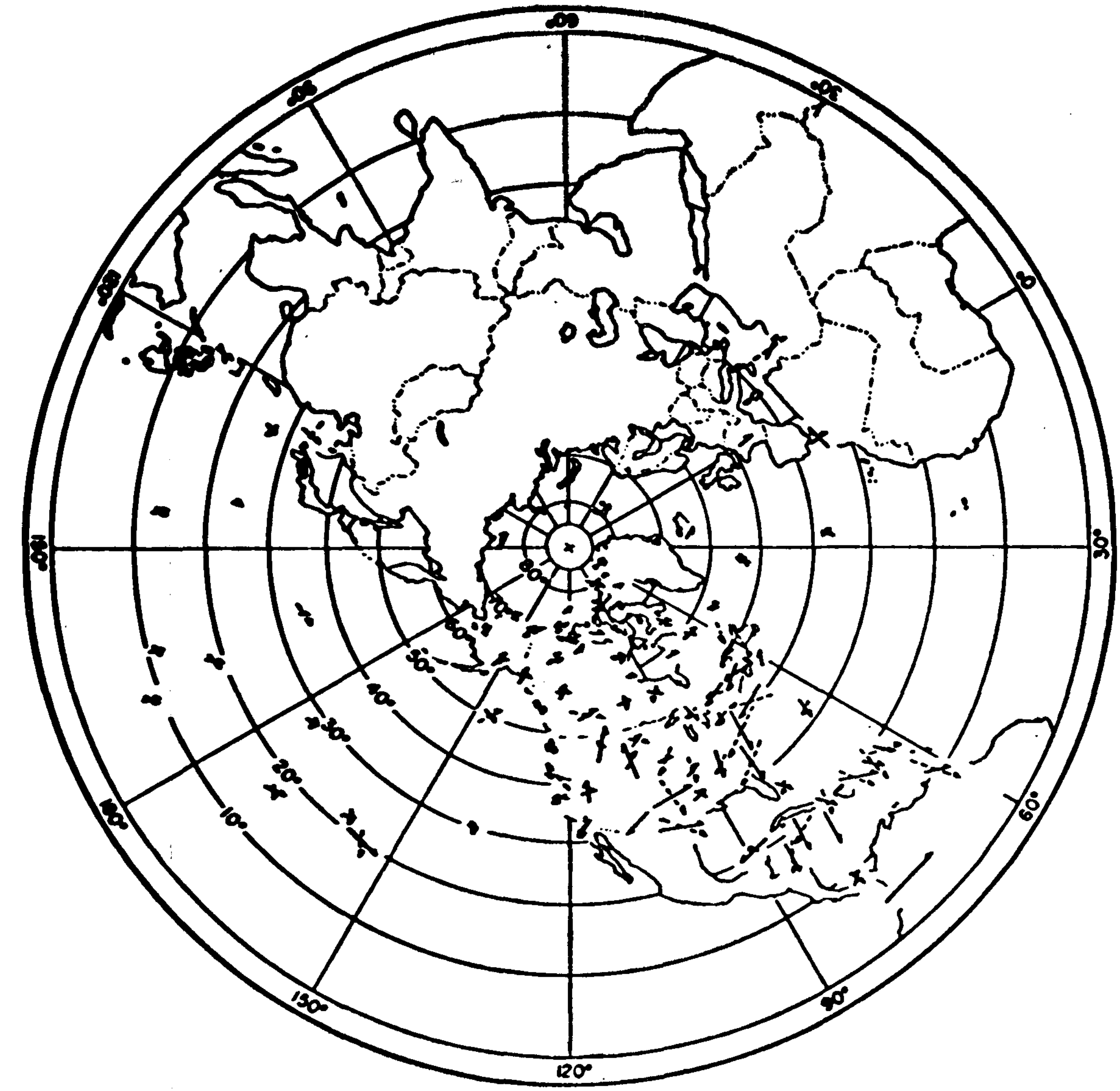

FIG. 12. Annual average wind difference 0000–1200 GMT (solid) and 0300–1500 GMT (dashed) at 700 mb, plotted in vector form (taken from Chapman and Lindzen, 1970). Scale: $\frac{1}{4}$ inch = 1 m sec^{-1}.

to be small; the same applies for the migrating surface pressure wave. Orography can produce waves, with tidal periods, which do not follow the sun. Such waves are typically associated with small positive and negative equivalent depths, and tend to decay rapidly with height (due to dissipation in the case of small positive equivalent depths). How this applies to terrestrial atmospheric tides will be shown later.

In considering the effects of orography on atmospheric tides for different planets, certain theoretical arguments should prove useful. Let $\xi(\theta, \varphi)$ be the surface elevation. If ξ is less than the smaller of H and $L/2$ (H, scale height; L, vertical wavelength of a tidal mode),[3] and if $\nabla\xi \ll 1$, then the orography interacting with the migrating tide will produce an effective vertical velocity at some reference ground level given by

$$w \approx \mathbf{u}_{\text{tidal}} \cdot \nabla\xi.$$

In the absence of orography there will still be a tidal w whose magnitude will be given by

$$w \approx \frac{4a\omega^2}{g} \frac{n_{\text{hor}}}{f^2} u_{\text{hor}},$$

where n_{hor} is a characteristic horizontal wavenumber. For the modes I have been discussing

$$\frac{n_{\text{hor}}}{f^2} \approx O(1),$$

[3] Even this may be significantly violated on Mars.

FIG. 13. Annual average wind differences 0000–1200 GMT at 60 mb, plotted in vector form (taken from Chapman and Lindzen, 1970). Scale: $\frac{1}{4}$ inch = 2 m sec^{-1}.

and $4a\omega^2/g \approx 2\times10^{-2}$ on Mars, 2×10^{-5} on Venus and 1.3×10^{-2} on Earth. If we let max $(\xi)\approx 4$ km then $\nabla\xi\approx 4$ km$/L_{\text{hor}}$, where L_{hor} is a characteristic horizontal scale for the orography. Then $\nabla\xi$ will be comparable with or larger than $4a\omega^2/g$ when

$$L_{\text{hor}} \lesssim 4\text{km} \Big/ \left(\frac{4a\omega^2}{g}\right). \qquad (21)$$

When (21) is satisfied, then the w kinematically induced by orography will be larger than the w's normally associated with the tide. This will occur for $L_{\text{hor}} < 200$ km on Mars, 2×10^5 km on Venus and 300 km on Earth. For Earth and Mars, the scales for which the kinematic effects of orography are important are unlikely to interfere with migrating tides. For Venus, this is no longer true.

Finally, a word is in order on "thermal orography." On earth important effects result from the facts that land and sea have different surface temperature oscillations, that there are temperature oscillations along mountain slopes, and that water vapor varies with longitude. Whether such thermal orography exists on other planets is not clear.

Figs. 11–14 should give the reader some idea of how orography influences the diurnal tide of the earth's atmosphere. Fig. 11 shows the amplitudes of diurnal surface pressure oscillations associated with different

FIG. 14. Annual average wind differences 0000–1200 GMT at 15 mb plotted in vector form (taken from Chapman and Lindzen, 1970). Scale: ¼ inch ≈ 2 m sec^{-1}.

zonal wavenumbers: $s=1$ is the migrating tide; the remainder are due to orographic effects. Clearly, the surface pressure oscillation is primarily due to the migrating tide. In Figs. 12–14 are shown maps of vector wind differences of wind measurements taken 12 hr apart in universal time at different altitudes. The maps should therefore show the flow fields due primarily to diurnal oscillations and if the diurnal oscillations are mainly due to the migrating tide then zonal wavenumber 1 should be evident in these maps. At 700 mb (Fig. 12) the wind fields seem to be largely associated with orography. Diurnal gyres associated with surface features are clearly in evidence. At 60 mb (Fig. 13) there are still tidal gyres, but they are primarily associated with very large-scale surface features. By 15 mb (Fig. 14) orographic effects seem to have decayed and we are left with a predominantly wavenumber 1 flow pattern.

4. Concluding remarks

Conventional terrestrial atmospheric tidal theory appears, not surprisingly, to be a good first order approximation to reality on Earth. It has, therefore, been possible to deal with extensions to the theory as modest modifications of almost adequate calculations. The use of similar calculations for Mars will probably be less adequate if excitation is as strong as currently supposed. Strong excitation will probably lead to the instability of diurnal tides on Mars. However, the use of terrestrial tidal theory still permits us to estimate the height of the

Martian turbopause. Surprisingly, terrestrial tidal theory might be applicable to Venus if eddy coefficients are less than 10^4 cm^2 sec^{-1}, if mean and tidal flows are less than a couple of meters per second, and if there is almost no orography. In this case the tidal response would be mostly diurnal with a rather detailed vertical structure (the vertical wavelengths of the main diurnal and semi-diurnal modes are both about 12 km). However, the "ifs" are numerous and uncertain.

Acknowledgments. I wish to acknowledge the support of the National Science Foundation through Grant GA-1622. I also want to thank the Department of Environmental Sciences of Tel Aviv University for its hospitality. Several of the calculations reported here were done while I was a visitor at that institute.

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[4] Where a given author's work has been reviewed in Chapman and Lindzen (1970), I have tended to omit citations to the original reference in the text. In particular, Fig. 11 is due to Haurwitz and Figs. 12–14 to Wallace and Hartranft.

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