

NOTES AND CORRESPONDENCE

Wave-CISK and Tropical Spectra

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ABSTRACT

An equivalent depth of 10 m for the oscillations of the tropical atmosphere was suggested by wave-CISK calculations. It is here shown that the dispersive properties of such an atmosphere are in agreement with observed power spectra for southerly and westerly winds.

In a lengthy paper Lindzen (1974) investigated the consequences of wave-CISK in a tropical atmosphere. Briefly, oscillations in cumulus heating forced internal waves whose low-level convergence forced the cumulus heating. Cumulus heating was parameterized as being linearly proportional to convergence at the top of the mixed surface layer, provided that the convergence was positive. Friction and other dissipative processes were ignored. It was found that the process proceeded most effectively when the internal waves had a vertical wavelength

$$L \approx 2.5z_c,$$

where z_c is the convective level for surface air. Wave-CISK thus implies that the atmosphere has an equivalent depth of $h \approx 10$ m in the tropics in addition to the normal equivalent depth $h \approx 10$ km. [For a discussion of atmospheric waves and equivalent depths see Lindzen (1967, 1971).] The existence of a 10 m equivalent depth has many implications for tropical meteorology, some of which were discussed in Lindzen (1974). Most notably, the existence of a small equivalent depth makes possible the existence of wave solutions restricted to the tropics. The purpose of this note is to describe the implications of 10 m equivalent depth for tropical wind spectra—a matter which was not described in detail in Lindzen (1974).

In Fig. 1, I show some typical observed spectra for zonal and meridional winds at various tropical stations as evaluated by Chang *et al.* (1970). Clearly, there are many irregular features in these spectra but two features are usually regarded as typical:

1) Most of the power in the u (zonal wind) field appears to occur at long periods (13–50 days). A

similar peak for the v (meridional wind) field is generally absent.

2) Most of the power in the v field is found at shorter periods (3–7 days). There are also secondary peaks in the u field at these periods.

In Fig. 2, I show the dispersive properties of waves for a 10 m equivalent depth where no mean flow is assumed: $n = -1$ refers to Kelvin waves, $n = 0$ (easterly) is a mixed gravity-Rossby wave, while $n = 0$ (westerly) is a gravity wave. Also shown are the properties of higher order gravity and Rossby modes. However, due to the confinement of CISK to the tropics, the $n = 0$ and $n = -1$ modes (which are, themselves, most closely confined to the tropics) should be most prominent. Now, the periods shown in Fig. 2 are those that would be observed for oscillations in an atmosphere otherwise at rest. The phase speed of the Kelvin waves is ~ 10 m sec⁻¹. Curves lying above the Kelvin wave curve in Fig. 2 have phase speeds > 10 m sec⁻¹; curves lying below have phase speeds < 10 m sec⁻¹. In a moving atmosphere, it is argued in Lindzen (1974) that the period shown in Fig. 2 would be measured by an observer moving at a speed characteristic of the mean flow in the region below cloud base. If this speed is comparable to the wave's relative phase speed then the period measured by a stationary observer will, of course, be very different from that shown in Fig. 2. However, the mean speeds are generally considerably less than 10 m sec⁻¹ so that, in fact, the periods shown in Fig. 2 for Kelvin waves and faster waves should be approximately those observed at the ground. Assuming the largest scale waves will be preferentially excited in the mean, we see that the periods of Kelvin waves for wavenumbers 1, 2, 3, ..., are 47 days, 25 days, 17

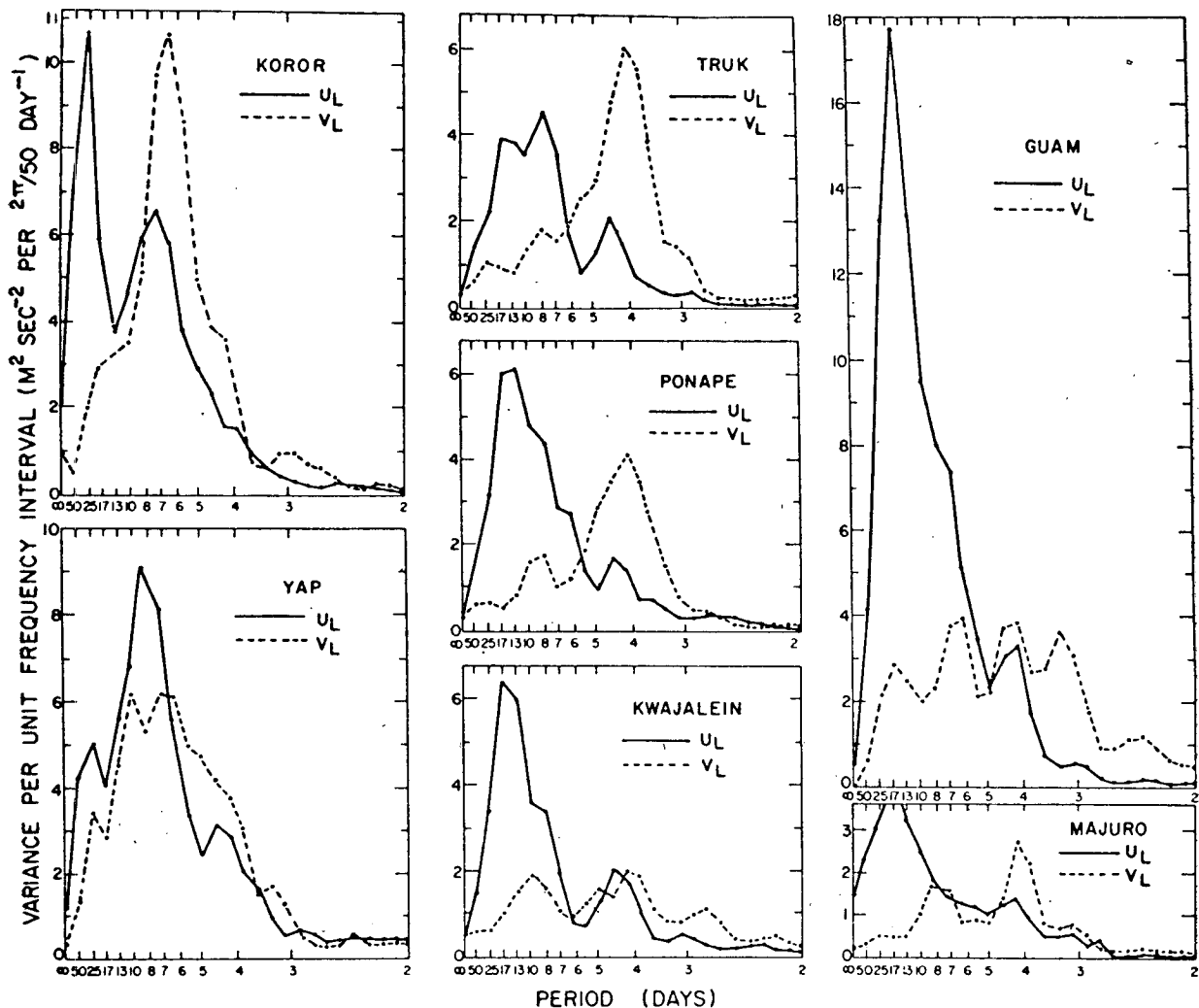


FIG. 1. Power spectra for the vertically averaged (surface to 400 mb) zonal and meridional wind components at selected stations (from Chang *et al.*, 1970). N. B. If these spectra are associated with a 10 m equivalent depth, then the vertical averaging will have diminished apparent power.

days, . . . Moreover, the Kelvin waves involve the u -field but not the v -field. Hence, Kelvin waves in an atmosphere with a 10 m equivalent depth reasonably account for the long-period spectral peaks for the u -field displayed in Fig. 1. Turning to the $n=0$ modes, we see that for $s < 9$, their periods lie between 3–7 days. Moreover, since $n=0$ modes are antisymmetric about the equator, they affect only the v -field at the equator—though a few degrees away from the equator the u - and v -fields are comparably affected. Thus, the $n=0$ modes appear to reasonably account for the shorter period spectral peak in the v -field, and the accompanying secondary spectral peak in the u -field.

The above argues strongly for the existence of a 10 m equivalent depth in the tropical atmosphere. This, in turn, has some implications which I will only mention briefly, leaving the details to separate publication:

- 1) The wave-CISK calculations in Lindzen (1974) omitted viscosity and thermal conductivity. Calculations performed with Mr. Lloyd Shapiro have shown that the inclusion of eddy viscosity in the mixed layer does not strongly affect the inviscid results. However, the existence of an eddy viscosity $> 10^3 \text{ cm}^2 \text{ sec}^{-1}$ above the mixed layer would noticeably stabilize the long-period Kelvin waves leading to an increased equivalent depth for optimal wave-CISK and shorter periods.

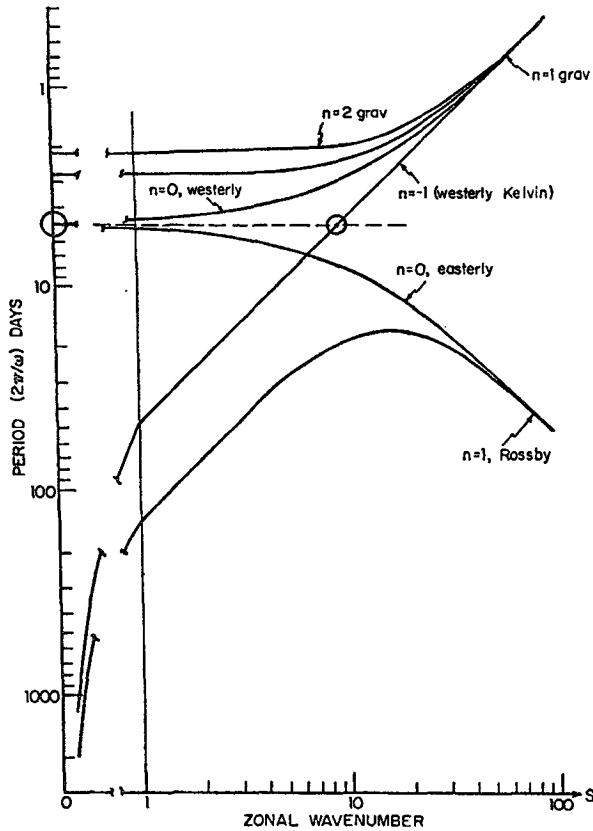


FIG. 2. Wave period as a function of zonal wavenumber for various wave types in an atmosphere with a 10 m equivalent depth (from Lindzen, 1974).

The observation of 45–50 day oscillations by Madden and Julian (1972) strongly suggests $\nu \lesssim 10^8 \text{ cm}^2 \text{ sec}^{-1}$ above the mixed layer.

2) The wave-CISK calculations in Lindzen (1974) assumed that cumulus heating was proportional to upward vertical velocity at the top of the mixed layer. Obviously, however, a finite minimum amplitude is necessary to lift air from the top of the mixed layer to cloud base within half a wave period. Calculations performed with Mr. Lloyd Shapiro suggests that under certain circumstances, a vertical wavelength $L \approx 2.5z_c$ will minimize the amplitude necessary to achieve this lifting. This opens the possibility that a 10 m equivalent depth might exist for reasons other than those suggested in Lindzen (1974).

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