On the Establishment of Stationary Waves in the Northern Hemisphere Winter

ARLINDO M. DA SILVA and RICHARD S. LINDZEN

Center for Meteorology and Physical Oceanography, Massachusetts Institute of Technology, Cambridge, Massachusetts

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ABSTRACT

The establishment of stationary waves in the Northern Hemisphere winter is investigated using stationary and time-dependent linear primitive equation models. Confirming the results of Nigam and Lindzen, we find that small displacements of the subtropical jet can cause significant changes in the stationary-wave response. The time scale for stationary establishment is found to be on the order of 5 days, both in the troposphere and in the lower stratosphere. The exception is for a northward shift of the subtropical jet, in which case the establishment of the new stationary solution in the stratosphere occurs on a longer time scale, which is mainly determined by dissipation. Implications for low-frequency atmospheric variability and mid- and long-range weather forecasting are discussed.

1. Introduction

This paper is concerned with the time evolution of stationary waves in response to changes in the basic flow. In a recent paper, Nigam and Lindzen (1989, hereafter referred to as NL) have investigated the sensitivity of stationary waves in the Northern Hemisphere winter to changes in the zonally symmetric zonal wind. Using a linear primitive equation stationary-wave model, they showed that shifts of the subtropical jets of only a few degrees produced substantial changes in the stationary-wave pattern. They also conducted an extensive search for other zonal-wind-sensitive zones in the Northern Hemisphere troposphere. They concluded that wind changes in the subtropical jet region are by far the most influential region for stationary-wave propagation. The main reason for this sensitivity is the fact that the jet's position determines the path for waves excited by the Tibetan plateau. In this study, using a steady and time-dependent LPE stationary-wave model, we extend their work by investigating the time scale for the establishment of stationary waves, both in the troposphere and in the lower stratosphere.

The primary motivation in NL was the need to explain the stationary-wave intensification in the Northern Hemisphere high-latitude stratosphere prior to the initiation of a sudden warming. Their results suggested that the small displacements of the subtropical jet could indeed account for this wave intensification in the lower stratosphere, and also contribute significantly to the low-frequency variability of the troposphere.

Another application of the stationary-wave dependence on the position of the subtropical jet is to short- and medium-range weather forecasting. Tibaldi and Molteni (1990) used seven years of operational analysis and forecasts to assess the skill of the operational ECMWF model in forecasting atmospheric blocking. From their composite of blocked cases it is concluded that blocking is associated with the enhancement of the amplitude of the quasi-stationary planetary-scale waves. Further, a deterioration of the skill is found for the blocked cases, which appear to be associated with the inability of the model to amplify stationary waves. The ensemble-mean error of the zonally averaged geostrophic wind for their blocking composite shows that the model zonal flow is shifted northward with respect to observations, very much in agreement with the predictions of NL. Miyakoda and Sirutis (1990) reached similar conclusions with the GFDL model for one-month forecasts.

Da Silva and Lindzen (1987), hereafter referred to as dSL, have studied the transients excited by the adjustment of the stationary waves in a barotropic model.

* Present affiliation: Department of Geosciences, University of Wisconsin–Milwaukee, Milwaukee, Wisconsin.

Corresponding author address: Dr. Arlindo da Silva, Department of Geosciences, University of Wisconsin–Milwaukee, P.O. Box 413, Milwaukee, WI 53201.

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1 Tibaldi and Molteni (1990) used a variant of Léjenas and Okland's (1983) blocking index, which is based on the gradient of geopotential height at fixed latitudes, with no regard to persistence. Therefore, it is likely that their index is sampling traveling as well as stationary waves. It is conceivable, however, that their composite of blocked and nonblocked cases filter out the traveling waves.
It was shown that wind changes associated with shifts of the tropical critical line could account for the excitation of ultralong Rossby waves in that model. A similar analysis of transients is attempted here. Namely, we specify a given stationary-wave variation due to a change in the zonal flow and analyze the transients excited in the time-dependent model as a function of dissipation and wave period. We anticipate that the wind changes considered here (shifts of the subtropical jet) are very efficient at generating changes in stationary waves in mid- and high latitudes, but not ultralong waves. The search of optimal wind changes for excitation of ultralong Rossby waves will be explored elsewhere.

Another problem of interest, and closely related to the excitation of transients, is the establishment of the new stationary solution. Due to hydrodynamical instabilities it is generally very difficult, if not impossible, to attain a stationary solution by integrating a linear time-dependent model from an arbitrary initial condition. In order to circumvent this problem, we initialize the model with a particular basic state and the stationary solution associated with this flow. Changing the flow from one configuration to another, we monitor the establishment of the new stationary solution by looking at the differences of the solution at time \( t \) from the solution at the initial time. Using this procedure, we can study the time scale for vertical and meridional stationary-wave propagation (section 4). Unlike the barotropic calculation of dSL, in this paper we concentrate on the horizontal structure and vertical propagation of the transients/stationary waves rather than their spectral structure.

This paper is organized as follows. In section 2 we describe the models and the model input parameters. Next (section 3), we investigate the stationary-wave sensitivity to variations in the subtropical jet, focusing on its horizontal and vertical structure, as well as the effect of dissipation. For the sake of clarity we reproduce some of NL's results. In section 4 we use the time-dependent model to study the establishment of the new stationary solution, both horizontally and vertically. A study of the excited transients then follows (section 5), where we discuss the lack of Rossby wave excitation. A summary along with some concluding remarks appear in section 6.

2. Model description

The stationary and time-dependent models are based on a linearization of the primitive equations in sigma coordinates, around a zonally symmetric basic state. Although the model allowed for the specification of a Hadley circulation, the results presented here do not use this feature. Model equations and the numerical procedure are given in appendices A and B, respectively. The control basic-state zonal wind is taken from observations and the basic-state potential temperature is computed from it using a thermal wind relation. Model dissipation is in the form of Rayleigh friction and Newtonian cooling. The basic-state fields and the dissipation parameters are discussed.

The fields presented in the next sections are expressed in terms of the \( \log \sigma \) coordinates described in appendix B. This system of coordinates is roughly equivalent to geometrical height away from the surface. In this coordinate system, we use the quantity

\[
\hat{\Phi} = (\Phi_{\text{sigma}} + G_0 \theta_0 \Pi)
\]

(1)

to display model results; \( \theta_0 \) is the basic-state potential temperature, \( G_0 \) is a function of the basic-state surface pressure [Eq. (27)], and \( \Pi \) is the perturbation surface pressure. This quantity will be referred to as the geopotential. Notice that in \( \log \sigma \) coordinates the wind fields are in approximate geostrophic balance with \( \hat{\Phi} \).

a. Model inputs

1) Basic state

We specify the zonal wind \( U_0 \) and compute the basic-state potential temperature using the thermal wind relation, assuming a constant \( \Pi_0 \). From the \( \nu \)-momentum equation and the hydrostatic equation one has (see appendix A)

\[
\frac{1}{a} \frac{\partial \Phi_0}{\partial \phi} = -\left( f + \frac{U_0}{a} \tan \phi \right) U_0
\]

(2)

\[
\theta_0 = \frac{1}{R_0} \frac{\partial \Phi_0}{\partial \sigma}.
\]

(3)

Integrating Eq. (2) with respect to \( \phi \) and substituting in (3) we find the expression of \( \theta_0 \) as a function of \( U_0 \):

\[
\theta_0 = \frac{1}{R_0} \frac{\partial}{\partial \sigma} \left[ \frac{-f}{\tan \phi} \left( f + \frac{U_0}{a} \tan \phi' \right) \int_{-\pi/2}^{\phi} U_0 d\phi' + X(\sigma) \right]
= \theta_u(\sigma, \phi) + X(\sigma).
\]

(4)

The integration constant \( X(\sigma) \) is determined by specifying the basic-state temperature at 45°N.

In Fig. 1 we show the control basic-state fields computed as above. The zonal wind is the monthly average for January 1979 plotted from the FGGE level III-b gridded data prepared by the European Centre for Medium-Range Weather Forecasts (ECMWF) (Bengtsson et al. 1982). Most basic states considered in this paper will be taken as a perturbation superimposed on this control case.

The time dependence of the basic state is specified as in dSL. Namely, the zonal wind is taken as changing slowly in time from an initial field \( U_1 \) to another \( U_2 \), in a characteristic time interval \( \tau \), centered at \( t = t_\tau \):

\[
U_0(\phi, t) = \alpha_1(t) U_1(\phi) + \alpha_2(t) U_2(\phi)
\]

(5)
where $r_1$ is specified as in Jacqmin and Lindzen (1985) plus a boundary-layer drag

$$r_1 = \left[a_1 + a_2 \exp\left(\frac{p - p_{surf}}{\beta p}\right)\right] \left(1 - \frac{2}{3} \tanh^2 \frac{\phi}{\phi_w}\right).$$

(8)

The parameters $a_1$, $a_2$, etc., are given in Table 1 for the three cases to be considered in this paper: standard, strong, and no boundary-layer damping.

The term $r_2$ is either zero or a Simmons-like critical-layer damping. It will be used to check the model sensitivity to strong critical-layer dissipation. It is given by

$$r_2^{-1} = \max\left[1/r_{2\text{max}}, \frac{1}{2} \left(\frac{\cos \phi}{U_0}\right)^2\right].$$

(9)

In Eq. (9), $U_0$ should be given in units of meters per second. To avoid reflection from the lid at the upper boundary, a sponge layer was introduced in the uppermost levels extending to about 25 km. For the standard case the average damping time scale is 20 days in midlatitudes, with a higher damping near the surface to mimic a boundary-layer drag. The strong damping case is defined such that the dissipative time scale in midtroposphere is about half the standard case, the boundary-layer drag remaining constant.

The Newtonian cooling is given by

$$\gamma^{-1} = \cos \phi \left[c_1 + c_2 \ln \frac{p}{p_{surf}}\right] \times \left(1 - \exp\left(\frac{p - p_{surf}}{p_w}\right)\right) + c_3.$$

(10)

The parameters $c_1$, $c_2$, etc., are

$c_1 = 117$ days

$c_2 = 24$ days

$c_3 = 1$ day

$p_w = 600$ mb.

In midlatitudes, the inverse Newtonian cooling goes from about 1 day at surface, reaches 60 days at the

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Standard</th>
<th>No boundary layer</th>
<th>Strong</th>
<th>No critical layer</th>
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<tbody>
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<td>$1/\alpha_1$ (days)</td>
<td>10</td>
<td>10</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
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<tr>
<td>$r_{2\text{max}}$ (days)</td>
<td>9999</td>
<td>9999</td>
<td>9999</td>
<td>9999</td>
</tr>
</tbody>
</table>

Table 1. Parameters for standard, no boundary-layer, strong, and critical-layer damping.
upper troposphere/lower stratosphere, and decreases to below 5 days above 25 km.

3. Stationary-wave sensitivity

We start investigating how the topographically forced waves depend on wind changes in the subtropics. NL have conducted an extensive search for other regions of sensitivity. They have shown that plausible changes in the polar night jet (9° displacement north and south of its climatological position) has only a modest impact on the waves in the stratosphere. Since the Himalayas are the primary source for topographic forced waves in the Northern Hemisphere, it is natural to expect that changes in the vicinity of this forcing will affect the path taken by the waves. Waves traveling different paths have different patterns as well as amplitudes, since more or less wave activity can reach critical layers or travel near the surface, where strong damping is present. As we will see, wind changes in the subtropics will affect waves not only in the troposphere but also in the stratosphere, in agreement with the findings of NL.

a. Control changes

We show in Fig. 2 our two control basic states. The wind change consists of a dipole centered around 30°N, going from 15° to 45°N. The result of these changes is a shift in the subtropical jet from 38°N to about 27°N, with virtually no change in its magnitude (42 m s⁻¹). These two basic states were constructed as follows: wind 1 (2) is equal to the January 1979 monthly mean zonal wind (see section 2a) minus (plus) half the dipole changes. Notice that since our subtropical jet is stronger than the jet in NL, we need a stronger dipole to produce the same jet shift.

A few words about the plausibility of the assumed wind changes are in order. Nigam and Lindzen have examined the interannual variability of zonal flow for six winters (1980–85) using analyzed ECMWF data. They found that wind changes capable of producing jet shifts as assumed in their calculation are indeed observed. Nigam (1990) used principal component analysis to further document the interannual variability of the zonally average zonal flow. Compositing with respect to selected coefficient amplitudes, he finds jet shifts that would result in sizable stationary-wave anomalies (Nigam 1990). Da Silva (1989) has showed that jet shifts of this nature also occur on the synoptic time scale, a typical example being an event in the period 9–13 January 1979. On the seasonal time scale, the northward migration of the subtropical jet from winter to summer produces a stronger wind change, but the concurrent change in the forcing is probably as important as the mechanism proposed here.

The topographic stationary-wave response for each one of the winds, along with the difference field, is shown in Fig. 3. The first column depicts the eddy geopotential height [\( \Phi / g \), Eq. (1)] at 300 mb; the second and the third show the same fields at 150 mb and 25 mb, respectively. At 300 mb and 150 mb we see clearly that the wave train excited by the Himalayas has distinct paths for winds 1 and 2. As a consequence, the second stationary solution now has a noticeable Aleutian high, which is practically absent from the stationary wave computed with wind 1. The stationary field as a whole has increased in mid- and high latitudes, with structural changes concentrated in the vicinity of the Tibetan plateau. The stationary waves in the stratosphere (25 mb) have very little change in phase, but amplitudes have increased substantially (about three times around 180°). Away from the main forcing region (Tibetan plateau) the difference field resembles the stationary wave field computed with wind 2, at all levels. The difference field at the troposphere gives us the impression of a tropically excited wave train. Notice that the difference field in the troposphere has a shorter meridional scale than the difference fields in the barotropic calculation of dSL.

The vertical structure of the stationary waves and difference field is shown in Fig. 4 on a zonal-height cross section for 21°N, 47°N, and 61°N. (One should be cautious about interpreting the results at the lowest levels. The identification of our vertical coordinate with the geometric z coordinate is not reliable near the surface, particularly in the vicinity of large mountain ranges like the Himalayas.) At 21°N, we see a wave train emerging from the Himalayan region, propagating upward but not reaching levels above 15 km. Notice that above 15 km the zonal wind is quite small which causes dissipation to become increasingly important, leading to wave absorption there. At mid- and high
latitudes every tropospheric feature has a stratospheric continuation. This is consistent with the westward tilts, which are indicative of upward propagation for a single wave. It is interesting to note that for 47°N the tilts are more pronounced for altitudes above 14 km. This is consistent with the observational results of Wallace (1983), who displayed zonal-height cross sections below 100 mb and found virtually no westward tilt. Because of this lack of tilt and an amplitude maximum around 300 mb, Wallace argues that the waves at 40°N are probably vertically trapped below the tropopause; this conclusion is not entirely supported by our results. The vertical structure of the difference field is very similar to the vertical structure of the stationary waves themselves, particularly for the zonal wind 2. They peak in the troposphere (around 10 km), and clearly have a stratospheric continuation.

The horizontal wave flux activity first introduced by Plumb (1985) is depicted in Fig. 5 at 150 mb. [Since this flux was derived under the assumption of quasi-geostrophy its meaning at the tropics is questionable. See also Karoly et al. (1989).] This figure confirms
Fig. 4. Zonal-height cross sections of the stationary-wave geopotential heights and difference field (control case). First, second, and third columns correspond to latitudes 21°N, 47°N, and 61°N, respectively. First and second row correspond to the stationary waves computed with $U_1$ and $U_2$ (Fig. 2). The third row shows the stationary waves computed with $U_1$ (first row) minus the stationary waves computed with $U_1$ (second row). Contour interval is 20 m; negative values are shaded.
Fig. 5. Plumb's horizontal wave flux (control case) at 150 mbar: (a) stationary waves computed with $U_1$, (b) stationary computed with $U_2$, and (c) difference of the wave fluxes in (a) and (b).
what we already have seen in the previous figures: the Tibetan plateau is the main source of wave activity in the Northern Hemisphere. For zonal wind 1, the wave activity originating at the Himalayas almost completely propagates equatorward. For zonal wind 2, the wave activity is stronger and part of it propagates northward to mid- and high latitudes. The difference in wave activity flux is suggestive of waves excited at the tropical latitudes. Of course, there is no tropical source in our model.

We have examined a simplified version of the stationary-wave sensitivity problem using the linear barotropic vorticity equation on the sphere. For an idealized basic state a similar jet shift produces a stationary wave change qualitatively in agreement with the linear PE model presented here. It appears that changes in index of refraction around 45°–50°N are probably as important as the changes over the jet region (30°N). Because of the small-scale wind changes and the associated large number of partial reflections, attempts to use WKB theory to predict the stationary-wave changes were not successful. Kang (1990) in a similar barotropic calculation suggests that the location of the turning latitude plays an important role in determining the amplitude of the remote response. Kang’s arguments based on qualitative WKB considerations did not apply to the wind changes used in our calculation. Of more concern is that the sensitivity of the barotropic model is somewhat different from the baroclinic model. We have found cases in which equatorward shifts of the jet actually produce reduced wave activity in mid- and high latitudes, while the baroclinic model produced the usual increase in wave activity. In view of this, we feel that the barotropic model is probably not adequate to study the stationary-wave sensitivity discussed in this paper. In addition, by selecting a single external mode the barotropic model of dSL tends to produce substantially more ultralong Rossby waves than in the present baroclinic calculation (see section 5c).

We have also tested the sensitivity of the stationary waves to the shift of the subtropical jet in the presence of thermal forcing. The thermal forcing was taken as in Jacqmin and Lindzen (1985). In the troposphere the difference field is generally small and has a very short meridional scale. This is hardly surprising since the waves are apparently forced by latent heat release in the Pacific storm track and are not very much affected by wind changes in the subtropics. The difference field in the lower stratosphere is about the same order as the difference field for the topographic waves.

b. Effect of dissipation

Since dissipation is generally a very ad hoc factor entering linear models, it is always worthwhile to check the dependence of the results on damping. In the absence of boundary-layer drag, the phase of stationary waves and the difference field (not shown) is very similar to the control case. However, amplitudes are about 50% higher on average away from the forcing region. This is characteristic of waves propagating horizontally near the ground, rather than the effect of damping on the local response (see NL). The strong damping case (twice the control case) showed again phases very similar to the control case, and an overall amplitude reduction. On average, amplitudes are about 80% of the amplitudes shown in Fig. 5. The inclusion of critical-line damping (see section 2a) did not introduce any noticeable modification of the control-case results.

c. Spectral structure

Following dSL, we now present the spectral structure of the difference field. This is done in order to determine whether Rossby waves are available in the stationary-wave differences, which is a necessary condition for excitation of these modes in the time-dependent model (see dSL).

The method used to analyze ultralong Rossby waves is similar to Ahlquist’s (1982). We simply average the fields vertically in the troposphere and lower stratosphere, having the Lamb wave structure $p^-$ as weight. We examined the sensitivity of the results to the highest level considered. No qualitative change was found, even though amplitudes tend to be slightly smaller (10%) when all levels are included in the average. The horizontal structures were expanded in terms of Hough modes, as in dSL.

Comparison of the horizontal structure of the difference field in the barotropic calculation of dSL and the difference field here shows that the difference field amplitude for the barotropic model is just about 30% higher than the baroclinic case, with phases roughly in agreement in the North Pacific region. However, the difference field in the baroclinic model has a shorter meridional scale. This is apparent in the Hough coefficient distribution (not shown). Projection onto modes with meridional wavenumber $\leq 2$ is virtually zero. Modes with meridional wavenumber equal to 3, the so-called 16-day waves, have on average less than one-third the amplitude found in the barotropic calculation. The highest amplitude for these waves is about 75 m, found for zonal wavenumber 3, which is just half that of the barotropic case (see dSL). As a consequence, Rossby waves are not excited in the baroclinic time-dependent model.

4. The establishment of the new stationary wave

The establishment of stationary waves is an interesting problem in its own right. The time scale for meridional and vertical propagation of stationary-wave energy is easily estimated using the group velocity computed from the Rossby wave dispersion relation (see, for example, Karoly and Hoskins 1981). Even though energy propagation speeds can be estimated in
a steady model with dissipation, to the best of our knowledge this calculation in a baroclinic time-dependent model has not been done yet. One reason for this is the fact that hydrodynamic instabilities make it virtually impossible to integrate the equations from a zero initial condition to the full establishment of the forced solution. An alternative to this procedure is to start with a zonal wind and its steady solution, shift the wind from one configuration to another, and monitor the establishment of the new stationary solution. For large zonal and meridional scales baroclinic instability is not likely to play a major role.

The key quantity to be used in this section is the geopotential height at time $t$ minus the initial condition. We will call this quantity the anomaly field. If the initial condition were some sort of climatology, the solution at time $t$ would be an excursion from the climatology, that is, an anomaly. Unless otherwise noted, the term anomaly will be used in this sense.

a. Checking the steadiness of the stationary solution

The stationary and time-dependent models were designed to have a high degree of compatibility. Unfortunately, it is not always possible to do so exactly, but only to the order of accuracy of the numerical schemes. In order to check the steadiness of the stationary solution in the time-dependent model, as well as the possible effect of instabilities, we considered an initial-value problem with no wind changes. The model was initialized with the stationary solution associated with the wind 1 of the control case. Although the stationary solution is not exactly steady, the departures from steadiness are fairly small compared with the changes that occur when the wind is allowed to vary. By day 7, the amplitudes at the troposphere are at most 12 m and about 5–6 m in the stratosphere.

b. The control case

The control case for the time-dependent model consists of the same wind changes as for the control stationary case (see previous section), and a transition period $\tau = 7$ days, in which the wind shifts from one configuration to another. Damping and resolution are also as in the stationary model ($\Delta \phi = 2$ degrees and $\Delta z \approx 1.8$ km). The value of the parameter $\tau$ was chosen so that at the end of the transition period the new stationary solution is a little more than half established. The dependence on $\tau$ will be discussed later in this section.

Figure 6 presents the time evolution of horizontal structure of the anomaly field at 300 mb and 25 mb, from day 1 to day 7, every other day. The initial condition is chosen to be the stationary solution associated with the wind 1. At day 1, we see amplitudes in excess of 10 m only at 300 mb, right downstream of the Tibetan plateau. One day later (day 2), the amplitude at this center has increased to about 25 m at 300 mb. By day 3, the centers at 300 and 150 mb continue to develop and by now there are disturbances in the stratosphere (25 mb) of about 10 m. From day 4 to day 7 the disturbances at all levels appear to grow in place, with only a slight eastward–northward propagation. Comparing Fig. 6 with Fig. 3 (beware of different contour intervals), we see that at day 7 most of the centers have already developed in the right place, but amplitudes are only about 60%–70% of their final value. There is also a tendency for centers to be slightly east of their final position. Notice that meridional energy propagation is almost always accompanied by zonal propagation. This development is similar to what one would get if a vorticity source were specified at the Himalayan region with the wind kept constant (e.g., Hoskins and Karoly 1981).

Figure 7 depicts the zonal–height cross section of the same quantity presented in Fig. 6 at 47$^\circ$N. At day 1, there is no noticeable disturbance at this latitudes, which is consistent with the initial development seen in Fig. 6. By day 2 (not shown), there is indication of a wave train emanating from the Himalayan region (80$^\circ$–120$^\circ$E) and propagating eastward at 47$^\circ$N, in the troposphere. Amplitudes are only slightly in excess of 15 m. At 21$^\circ$ and 61$^\circ$N (not shown) amplitudes are on average less than 5 m at this time, with a minor negative center at 61$^\circ$N, right downstream of the Himalayas. One day later (day 3), the wave train at 47$^\circ$N starts propagating upward and its amplitude is now about 25 m. Sign of upward propagation is also seen at 61$^\circ$N, while the disturbances at 21$^\circ$N are beginning to develop. Subsequent development is marked by amplification of the tropospheric centers accompanied by upward propagation, except in the subtropics (21$^\circ$N, not shown). Comparison of the time evolution at different latitudes indicate that the stationary solution is more rapidly established in middle and high latitudes. As seen in the horizontal structure, by day 7 the centers are slightly to the east of their final position.

c. Reversing the wind shift

Now we investigate how the development of the new stationary solution is affected when we force a transition from the state 2 to state 1 of the control case. Namely, we swap wind 1 with wind 2 in the control run, and initialize the model with the stationary solution of the old wind 2. The stationary-wave difference field, of course, is just the negative of the fields shown in Figs. 3 and 4.

In Fig. 8 we show the zonal-height cross section of the anomaly field at 2-day intervals. The same qualitative development noted in Fig. 11 takes place in the troposphere. However, there is a substantial reduction of propagation to the stratosphere. Until day 3 the two developments are very similar, both showing more of a tropospheric structure. After day 3, the control case
Fig. 6. Time evolution of the horizontal structure of the anomaly field (control case) at 300 mb (left column) and 25 mb (right column). The fields are shown from day 1 (top) through day 5 (bottom), at two-day interval. Contour interval is 5 m; negative values are shaded.
We can think of it as isolating the troposphere from the stratosphere. So, the waves in the stratosphere shift toward the second steady solution primarily through the local dissipation, which has a longer time scale than the usual Rossby wave dispersion. Therefore, in this case the establishment of stationary waves is chiefly determined by dissipation, rather than by the dispersive properties of stationary Rossby waves.

d. Dependence on $\tau$

Now we investigate whether the picture emerging from the control case is particular to the value of $\tau$ chosen. We consider two cases: 1) a faster wind change

starts showing increasingly upward propagation to the stratosphere, which does not happen for this case; even in the troposphere the amplitudes are about 60%–80% smaller than in the control case.

This seemingly contradictory behavior can be understood when we recall that initially the stationary waves in the stratosphere are associated with propagation from below. At any point, the stationary-wave amplitude is determined by the balance

\[
\text{wave flux in} = \text{wave flux out} + \text{dissipation}.
\]

When we switch from wind $U_2$ to wind $U_1$, the corresponding effect is to shut off the vertical propagation.
Fig. 9. Time evolution of the anomaly field at 30 mb for $\tau = 3$ days (left column) and $\tau = 14$ days (right column). The fields are shown from day 1 (top) through day 5 (bottom), at two-day interval. Contour interval is 5 m; negative values are shaded.
(τ = 3); and 2) a slower wind change (τ = 14 days). We show in Fig. 9 the anomaly field at 300 mb for the case τ = 3 days (left panel) and for the case τ = 14 days. As in the control case, the anomaly field shows the same sign of Rossby wave energy propagation from the Tibetan plateau. The main difference here is the rate at which energy propagates downstream of the Himalayas. For example, by day 3 the negative center around 150°E has amplitude of about 55 m for τ = 3 days, about 40 m for τ = 7 days, and about 30 m for τ = 14 days. The phase of the anomaly field is fairly independent of τ, with the centers slightly displaced eastward for longer τ.

In this section we have established that there is no qualitative difference between locally changing the forcing or locally changing the medium. In both cases, information propagates with the group velocity of stationary Rossby waves.

We have thus far concentrated on the establishment of the new stationary solution during the transition period, that is, the period it takes the wind to go from one configuration to another. Since in the control case most of the features are already established by this time, we proceed to present the evolution of the transients from that point on.

5. Transients excited as adjustment of the stationary waves

In a traditional forced problem with dissipation, the term transient is used to designate those disturbances that preceded the establishment of the forced steady solution. An unambiguous definition is possible only when the basic state does not vary in time. The experiments designed here are intended to circumvent this problem. Wind changes are allowed to occur in an interval τ, followed by a period of steady winds, where transients are analyzed. In the last section we clearly see the propagation of information from one point to another, consistent with the notion of group velocity in a dissipative medium. The difficulty of defining the transients in this case is purely technical. During the wind transition period we concentrated on the departures from the initial condition, and through comparison with the difference in stationary solutions, we implicitly analyzed the transients.

In this section we turn to consider the transients explicitly as in dSL. For t > τ, transients are defined as the solution at time t minus the stationary solution computed with zonal wind 2. Notice that in the limit t → ∞ the transients approach zero, provided no instabilities are present. In this limit, the second stationary solution would be fully established. The problem of practical interest is to determine how large t must be taken for this.

a. The control case

In Fig. 10 we show the horizontal structure of the transients at 300 mb, from day 7 to day 13. Notice that the contour interval used in this figure is 20 m, and not 5 m as in the previous section. By day 7 the transients are reminiscent of the difference in stationary solution (Fig. 3). Notice that the transient centers are located slightly eastward of the same centers in the difference in stationary solutions. This signature was also seen in the previous section in the anomaly field. The amplitudes are about 60% of the difference in stationary solution, but the centers are slightly displaced northward. The time evolution of the transients is marked by a slow eastward/northward displacement, with amplitudes decaying in time. A simple inspection indicates that this very low phase speed is consistent with the scale of the disturbances, through the classical Rossby wave formula \( c = U_0 - \beta/(k^2 + l^2) \). The amplitude decay follows a simply \( e^{-\tau} \) law, where \( \tau \) is an averaged Rayleigh damping coefficient. Notice that there is no sign of disturbances propagating westward, which is consistent with the absence of ultralong Rossby waves (which we discuss later).

The zonal-height cross section of the transients (not shown) is similar to the difference in stationary solutions with individual features slightly shifted eastward. The decay of the transients in all levels is as inferred from the horizontal structure.

b. Dependence on transition period and dissipation

As suggested by the simple theory of dSL (see also da Silva 1989), there are two parameters determining the nature of the transients. They are the disturbance period and the damping time scale, both normalized by the transition period. The disturbance phase speed accounts for the effects of propagation during the adjustment process. For example, chances are that for transition periods matching half the wave period destructive interference will likely occur; however, the effect depends strongly on the particular way the wind shift occurs. For the simple Rayleigh friction assumed, the effect of dissipation is straightforward: transients are dissipated as they are produced. For the cases considered here, the observed wave periods are in the range [80, 200] days, which normalized by τ fall in the nondimensional range [5, 65]. According to the theory of dSL, dissipation will be the major player in determining the amplitude of transients right after the wind shift takes place (da Silva 1989).

In order to illustrate this dependence on the ratio dissipation/transition period we show in Fig. 11 a scatter diagram of the observed transient efficiency, along with the dissipative limit of the transient efficient function:

\[
\xi_{\text{dissip}}(r\tau) = \frac{1 - e^{-r\tau}}{r\tau}.
\]

The observed transient efficiency is determined as follows. For each of the runs, we visually inspect the 300 mb/150 mb contour plots of the transients at \( t = \tau \)
and by comparing with the corresponding difference in stationary solution, determine the ratio $100 \times \frac{\text{transient amplitude}}{\text{difference in stationary-wave amplitude}}$. Since linear damping is not constant over the whole domain, the determination of the normalized damping parameter is not a trivial task. For all cases shown we used the damping parameter at the location of the feature analyzed. This is expected to underestimate the effective dissipation that acted on the transients. There are two groups of runs in Fig. 11: the ones marked with "**" have boundary-layer dissipation as in the standard case; the ones marked with "0" have a reduced or absent boundary-layer drag. The agreement with the theoretical curve is rather good, especially if one takes into account the fact that cases with boundary drag should have its abscissa decreased by 1 or 2 units.

c. Remarks on the excitation of ultralong Rossby waves

As we see from the time evolution of transients, there is no signature of westward-propagating disturbances. This is confirmed by the plots of the Rossby wave amplitude and phase as a function of time (not shown). The only mode with significant projection is the 16-
day wave with zonal wavenumber 3, and even then amplitudes are about one-third of those in the barotropic model. After day 7 (end of the transition period) amplitudes are smaller than 20 m; the phase associated with this mode does not show any sign of westward propagation. This suggests that the transient field is mostly composed of the continuum modes; the projection onto ultralong Rossby modes is probably due to the loss of orthogonality associated with shear in the zonal flow (e.g., Held 1985; Farrell 1988).

The absence of Rossby wave propagation is not particular to the control case. In a wide range of transition parameters, dissipation, and subtropical wind changes the same behavior was observed. The main difference from the barotropic model of dSL is the structure of the stationary-wave difference field, which is mainly determined by the wind changes and the possibility of vertical propagation. As we have seen, during the transition period (control case) transients are associated with the establishment of the new stationary wave. This is characterized by zonal and meridional as well as vertical propagation. These features are very distinct from the global westward-propagating Rossby waves. After this establishment period, the transients become generally small and no regular westward propagation is observed.

The possibility of Rossby wave excitation with different wind changes has not been thoroughly investigated. The wind changes in the subtropics, as considered here, are very efficient at generating changes in stationary waves in middle and high latitudes, but not the ultralong waves. As hinted by the barotropic calculation of dSL, zonal-wind changes in the tropics may generate large-scale stationary-wave changes in these latitudes, which indeed can excite the ultralong waves. The search for optimal wind changes for excitation of ultralong Rossby waves will constitute future research with this model.

6. Summary and concluding remarks

As discussed in dSL, the analysis of the stationary-wave adjustment in the time-dependent model requires knowledge of the difference in stationary waves. We have therefore documented the sensitivity of the stationary waves to wind changes in the subtropics, guided by the experience of NL. Despite the differences in model formulation and basic state, the main results in NL are reproduced. The main points are:

- Shifts in the subtropical jet of only a few degrees (\(\sim 5^\circ\)) can produce changes in the topographically forced stationary waves of 120 m. This difference field is a large-scale feature, mainly concentrated downstream of the Tibetan plateau and extending up to Europe. As in NL, we find that shifting the jet equatorward increases the stationary-wave response in mid- and high latitudes, as well as in the stratosphere. Conversely, shifting the jet poleward decreases the wave amplitudes in these regions.

- In the subtropics (21°N), stationary waves and differences are trapped in the troposphere. In midlatitudes (47°N), the increased stationary waves and the difference field reach a maximum amplitude in the troposphere, with very small westward tilts. However, tilts are pronounced above 15 km and there is clear continuation of these disturbances into the stratosphere. These tropospheric-trapped disturbances are associated with short meridional scales. In high latitudes (60°N), amplitudes have their maximum in the stratosphere and with virtually no westward tilt.

- The sensitivity of thermally forced waves to changes in the subtropical jet is generally smaller than the sensitivity of topographically forced waves, at least in the troposphere. The difference field has very short meridional scale and little indication of propagation from the tropics. On the other hand, in the stratosphere the difference field associated with diabatic heating is of the same magnitude as its topographic counterpart.

- Consistent with other studies (e.g., NL), in the troposphere the stationary waves downstream of the Tibetan plateau are very sensitive to boundary-layer drag. Removing this dissipation completely increases the amplitude by about 50% in the troposphere; the sensitivity is reduced in the stratosphere. The doubling of Rayleigh damping has the effect of reducing the amplitudes to 80% of their original value, both in the troposphere and in the stratosphere. Critical-line dissipation was found to have a negligible effect on our results. For all cases, dissipation leaves the phase of the disturbances essentially unchanged.

- For this particular zonal-wind change, the projection of the difference field in Rossby waves is much smaller than that found with the barotropic model. Thus, Rossby wave excitation is not likely in the time-dependent model. The only candidate for excitation is the 16-day wave associated with zonal wavenumber 3. The subtropical wind shift was chosen because it is the
*optimal* wind change that affects stationary waves in mid- and high latitudes. As we have seen, this is not the optimal wind change to excite Rossby waves. A search for the best wind change for Rossby wave generation has not been attempted here.

When the wind is allowed to change in a finite time interval, there is a lag until the new stationary solution is established. The main points emerging from this study are:

- The stationary and time-dependent models are fairly compatible and instabilities do not play any role in the experiments described here.
- In the troposphere, the effect of changing the subtropical jet is first noted just downstream of the Himalayas, and subsequently this information propagates eastward and northward. This behavior is the same we would get if a vorticity source were specified at the Himalayan region with the wind kept constant. In the stratosphere, the new stationary waves appear to be established without sign of horizontal propagation from the Tibetan plateau. It is indeed indicative of propagation from below.
- The establishment of the new stationary solution is pretty much restricted to lower levels in the first days, and mostly to latitudes just downstream of the Tibetan plateau. This period is followed by vertical propagation. Upward propagation is well defined in the control case (wind shifting equatorward). In the anticontrol case (wind shifting poleward), propagation to the stratosphere ceases slowly, leaving the establishment of the new stationary solution in the upper levels to be dictated by the local dissipation there.

The spatial structure of the transients is reminiscent of the difference in the stationary solutions. The transients generated by shifts of the subtropical jet are quasi-stationary and there is no excitation of ultralong Rossby waves.

There is a qualitative agreement between the time evolution of our *anomaly* field and the anomaly field of Dole (1982): both present features of stationary Rossby waves propagating downstream with the group velocity (see da Silva 1989). The evolution of the vertical structure of the model anomalies also resembles observations: initially there is an eastward tilt with height that decreases as the anomalies evolve. In our model, for almost every tropospheric feature there is a stratospheric continuation.

Perhaps the most serious limitation of our model when compared with the persistent anomalies of Dole is the fact that our basic state is the zonally averaged zonal wind. With this kind of basic state, the changes in stationary waves occur in all longitudes. As mentioned in the last section, there is indication (e.g., Weickmann et al. 1985; Lau and Boyle 1987) that fluctuation in convection in the maritime continent is accompanied by changes in the flow over the eastern coast of Asia. For such a localized wind change, in light of the results of this section, one expects that changes in stationary waves would occur only downstream of the region where the wind is changing. Preliminary observational work by R. Black (1989, personal communication) suggests anomalous zonal wind in the eastern coast of Asia prior to the development of the Pacific cases; however, anomalies in the zonally averaged mean flow are too small to affect the stationary waves in any significant way.

One result that emerges from our calculation is the possibility that the anomalies have a stratospheric continuation. At present there is no available observation of this extension. Such work would help sort out some of the existing theories. Strictly localized theories do not predict any significant stratospheric extension.

As demonstrated by Tibaldi and Molteni (1990) and Miyakoda and Sirutis (1990), the inability of the operational models in simulating blocking (or enhanced stationary-wave) activity is associated with systematic errors in the zonal-mean wind. Model subtropical jets tend to be displaced too far to the north. This zonal-flow/stationary-wave relationship is very much in agreement with the predictions of NL and the results of section 3. Parameterizations of orographic gravity wave drag have been attempted as a fix for model zonal-wind bias, but the existing schemes do not appear to provide a substantial improvement of blocking simulation (Miyakoda and Sirutis 1990). Our results suggest that unless this zonal-wind bias is corrected, models will not be capable of representing stationary-wave intensification, and skill deterioration in a time scale of five days is expected.

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**APPENDIX A**

**Model Equations**

We start from the hydrostatic nonlinear primitive equations in sigma coordinates, on the sphere, in the presence of linear dissipation:

\[
\frac{du}{dt} = \left( f + \frac{u}{a} \tan \phi \right) v + \frac{1}{a \cos \phi} \left( \frac{\partial \Phi}{\partial \lambda} + c_p \theta \frac{\partial \rho'}{\partial \lambda} \right) + ru = X \quad (12)
\]

\[
\frac{dv}{dt} + \left( f + \frac{u}{a} \tan \phi \right) u + \frac{1}{a} \left( \frac{\partial \Phi}{\partial \phi} + c_p \theta \frac{\partial \rho'}{\partial \phi} \right) + rv = Y \quad (13)
\]

\[
\frac{d\theta}{dt} = \frac{1}{\cos \phi} \left( \frac{\partial \Phi}{\partial \phi} + c_p \theta \frac{\partial \rho'}{\partial \phi} \right) + m
\]

\[
\frac{d\rho'}{dt} = -\frac{1}{a} \left( \frac{\partial \Phi}{\partial \phi} + c_p \theta \frac{\partial \rho'}{\partial \phi} \right) + n
\]

\[
\frac{d\rho}{dt} = -\frac{1}{a} \left( \frac{\partial \Phi}{\partial \phi} + c_p \theta \frac{\partial \rho'}{\partial \phi} \right) + n
\]
\[ \frac{d\Pi}{dt} + \frac{\Pi}{a \cos \phi} \left( \frac{\partial u}{\partial \phi} + \frac{\partial}{\partial \phi} (v \cos \phi) \right) + \Pi \frac{\partial \sigma}{\partial \sigma} = 0 \] (14)
\[ \frac{d\theta}{dt} + \gamma \theta = p^{-*} \frac{Q}{c_p} \] (15)
\[ \frac{\partial \Phi}{\partial \sigma} = -c_p \theta \frac{\partial \sigma}{\partial \sigma} \] (16)
where
\[ \Pi = \frac{p - p_{\text{top}}}{\sigma}, \quad \frac{\partial \Pi}{\partial \sigma} = 0 \] (17)
and
\[ \frac{dA}{dt} = \frac{\partial A}{\partial t} + \frac{u}{a \cos \phi} \frac{\partial A}{\partial \phi} + \frac{v}{a \cos \phi} \frac{\partial A}{\partial \sigma} + \gamma \frac{\partial A}{\partial \sigma}. \] (18)

Most of the symbols have their conventional meteorological meaning. The exception is the potential temperature, defined here as
\[ \theta = p^{-*} T. \] (19)

The boundary conditions are the usual constraint of no flow across the boundaries:
\[ \sigma = 0, \quad \text{at} \quad \sigma = 0, 1. \] (20)

In order to minimize reflection from the upper boundary, a sponge layer is specified in the uppermost levels. The preceding equations are linearized around a zonally symmetric basic state,
\[ u = U_0(\phi, \sigma, t) + u', \]
\[ v = V_0(\phi, \sigma, t) + v', \]
\[ \sigma = \hat{\sigma}_0(\phi, \sigma, t) + \hat{\sigma}', \]
\[ \Pi = \Pi_0(\phi) + \Pi', \]
\[ \theta = \Theta_0(\phi, \sigma, t) + \theta', \]
\[ \Phi = \Phi_0 + \Phi'. \]

For wavelike perturbations of the form
\[ u' = u'(\phi, \sigma)e^{-i\lambda t}, \quad \cdots \] (21)
the dimensional time-dependent form of the linear equations is\(^2\)
\[ \frac{\partial u}{\partial t} + \frac{V_0}{a} \frac{\partial u}{\partial \phi} + \frac{\xi_0}{a} \frac{\partial u}{\partial \sigma} + \left( i \frac{U_0s}{a \cos \phi} - \frac{V_0}{a} \tan \phi + r \right) u \]
\[ - \left( f + \frac{U_0}{a} \tan \phi \right) v - \frac{1}{a} \frac{\partial U_0}{\partial \phi} v + \frac{\partial U_0}{\partial \sigma} \sigma \]
\[ + \frac{\xi s}{a \cos \phi} (\Phi + G_0 \theta_0) = X \] (22)
\[ \frac{\partial v}{\partial t} + \frac{V_0}{a} \frac{\partial v}{\partial \phi} + \frac{\xi_0}{a} \frac{\partial v}{\partial \sigma} + \left( f + 2 \frac{U_0}{a} \tan \phi \right) u \]
\[ + \left( i \frac{U_0s}{a \cos \phi} + \frac{1}{a} \frac{\partial V_0}{\partial \phi} + r \right) v + \frac{\partial V_0}{\partial \sigma} \sigma \]
\[ + \frac{1}{a} \frac{\partial \Pi}{\partial \phi} + G_0 \frac{\partial \theta_0}{\partial \phi} = Y \] (23)
\[ \frac{\partial \theta}{\partial t} + \frac{V_0}{a} \frac{\partial \theta}{\partial \phi} + \frac{\xi_0}{a} \frac{\partial \theta}{\partial \sigma} + \frac{1}{a} \frac{\partial \Theta_0}{\partial \phi} v + \frac{\partial \Theta_0}{\partial \sigma} \sigma \]
\[ + \left( i \frac{U_0s}{a \cos \phi} + \gamma \right) \theta + H_0 \frac{\partial \Pi}{\partial \phi} = Q_0 \] (24)
\[ \frac{\partial \Phi}{\partial t} + \Pi_0 \frac{\partial \Phi}{\partial \sigma} + i \frac{U_0s}{a \cos \phi} \Pi + \frac{\Pi_0}{a \cos \phi} \left( i \frac{\xi s}{a \cos \phi} u + \frac{\partial}{\partial \phi} v \cos \phi \right) = 0 \] (25)
\[ \frac{\partial \Phi}{\partial \sigma} = R_0 \Phi + S_0 \Theta_0 \Phi. \] (26)

The quantities \( G_0, R_0, S_0, H_0, \) and \( Q_0 \) are functions only of the basic-state variable \( \Pi_0 \). They are defined following:
\[ P_0 = \Pi_0 \sigma + p_{\text{top}} \]
\[ G_0 = R P_0^{-1} \sigma \]
\[ R_0 = c_p \frac{\partial P_0}{\partial \sigma} \]
\[ S_0 = -R \frac{\partial}{\partial \sigma} \sigma P_0^{-1} \] (27)
\[ H_0 = \frac{\kappa \sigma}{P_0} \]
\[ Q_0 = \frac{P_0^{-*}}{c_p}. \] (28)

The time-dependent model is solved numerically with the equations in their dimensional form as just given. The stationary model, however, produces a better-conditioned matrix when the equations are cast in non-dimensional form. Therefore, we nondimensionalize the equations using the following factors:
\[ (x_*, y_*) = a(x, y) \]
\[ t_* = \Omega^{-1} t \]
\[ (u_*, v_*) = \Omega a (u, v) \]
\[ \sigma_* = \Omega \sigma \]
\[ \theta_* = \frac{\Omega^2 a^2}{R \Pi_0} \]
\[ \Phi_\ast = \Omega^2 a^2 \]
\[ \Pi_\ast = \Pi_0 \Pi. \]

In the preceding equations the asterisk subscript denotes a dimensional quantity. The stationary, nondimensional form of Eqs. (22)–(26) can be found in da Silva (1989). Even though it is not apparent from these equations, the topographic forcing enters the momentum equations. It can be easily seen if we write the hydrostatic equation in integral form

\[ \Phi = \Phi_T - \int_1^1 (R_0 \theta + S_0 \Pi_0) \, d\sigma \]
\[ = \Phi_S + \tilde{\Phi}, \]

where \( \Phi_S \) denotes the topographic height multiplied by the gravity constant, properly nondimensionalized. Upon substitution into the momentum equations we get

\[ V_0 \frac{\partial u}{\partial \phi} + \cdots + \frac{\nu_S}{\cos \phi} \left( \Phi + G_0 \Pi_0 \right) = - \frac{\nu_S}{\cos \phi} \Phi_T \]
\[ V_0 \frac{\partial v}{\partial \phi} + \cdots + \frac{\Phi_T}{\partial \phi} + G_0 \theta_0 \frac{\partial \Pi}{\partial \phi} = - \frac{\partial \Phi_T}{\partial \phi}. \]

(30)

(31)

APPENDIX B

Numerics

a. Time-dependent model

The dimensional linear primitive equations (22)–(26) were integrated in time using an explicit Matsuno scheme (Matsuno 1970). The prognostic and diagnostic steps were implemented in the standard way (Haltiner and Williams 1980). Since the basic state does not depend on longitude, a semispectral method was used in which the equations are independently integrated for each zonal wavenumber. Finite differences were used for the meridional and vertical directions. A fourth-order implicit Padé derivative formula was used to approximate the spatial derivatives. Boundary conditions at the polar latitudes were specified exactly, using the hemispheric continuation method described in dSL. Numerical boundary conditions at the top and bottom of the model were deduced as a 3-point uncentered Padé formula.

A uniform grid was used for the meridional direction. For most of the experiments, the meridional resolution was taken to be 2 degrees. Grid refinement experiments performed with the stationary model (see below) indicated no sizable changes when the resolution was increased to 1 degree. Since stationary waves have their scales better determined in geometric coordinates, a nonuniform \( \sigma \)-coordinate grid was specified. This grid was designed to be approximately uni-

form in log-\( p \) coordinates. Namely, we define the new coordinate

\[ z = -H_0 \ln \frac{p}{p_0} = -H_0 \ln \left[ \frac{\Pi_0 \sigma + p_{\text{top}}}{\Pi_0 + p_{\text{top}}} \right] \]

(32)

where \( H_0 \) is a constant defined such that \( z = 30 \) km at 10 mb. In this coordinate system we define a uniform grid:

\[ z_k = (k - 1) \Delta = \frac{k - 1}{K - 1} H_0 \]

(33)

for \( k = 1, \cdots, K = 18 \). With 18 levels, \( \Delta z \approx 1.7 \) km. The \( \sigma \) grid is obtained by inverting Eq. (32). All quantities in this paper are displayed with \( z \) as the vertical coordinate. Notice that it is equivalent to a log-\( \sigma \) co-

ordinate system. The top and bottom of the model are specified at 1013 mb and 10 mb, respectively.

A time step of 2 minutes was used to integrate the time-dependent model. With this step no filtering was necessary. Early implementations of the model were tested with an extension of the prescribed solution technique described in Dee and da Silva (1986). The code was further tested with the initial condition being the stationary solution (see section 4).

b. Stationary model

As in the time-dependent model, the stationary model was solved separately for each zonal wavenumber and spatial discretization was performed with a fourth-order Padé scheme. This was done in order to achieve a maximum degree of compatibility between the two models. As mentioned before, the stationary model was implemented with the equations in nondimensional form.

In order to avoid a lengthy report of the details of implementation, we give here only a schematic description. First, the dependent variables \( \Phi \) and \( \sigma \) are eliminated using the hydrostatic and thermodynamic equations. The new system contains only three complex bidimensional variables \( (u, v, \text{and} \theta) \) and a single one-dimensional variable \( \Pi \). After discretization is performed, the coupled system of equations can be put in the form

\[ M_x = f \]

(34)

where \( M \) is a block tridiagonal matrix, with each block having the dimension \( (3K + 1)^2 \), \( K \) being the number of vertical layers (18). This system is solved using a standard Gaussian elimination procedure. In order to save storage, the subdiagonal blocks were not precomputed and stored. Instead, the blocks were computed as the Gaussian elimination advanced. As a result, we needed only to store \( 2J(3K + 1)^2 \) entries of the matrix \( M \).

Grid and resolution were taken as in the time-depen-
dent model. Besides the compatibility test with the
time-dependent model (section 4), an artificial rhs was specified to force the solution to be simple functions. This procedure was very helpful in isolating errors early in the implementation.

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